DISCUSSION ON MAGNETISM

FERROMAGNETISM AND HYSTERESIS

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ABSTRACT. The author reviews briefly his early work on hysteresis in the ferromagnetics, discusses his symbolic models of magnetic structure, and suggests that ferromagnetism affords the most promising source of clues to the outstanding problems of magnetic theory.

As a hopelessly old-fashioned magnetician, I find it an embarrassing compliment to be asked to open this discussion. You are taking down a dusty piece of apparatus from the upper shelf on which it was long ago put, and inviting it to function. It can do so only in pretty much the old way. I have nothing new to offer. Perhaps the best service I can render is to recall some aspects of the subject which were problems when I began to study magnetism fifty years ago, and are problems still.

My work dealt exclusively with the curious special group of substances we call ferromagnetics—iron, nickel, cobalt, and their alloys. To these we add a potential fourth member, manganese, because its alloys show ferromagnetic properties. Briefly, the distinguishing characteristics are (1) an immense readiness to be magnetized, (2) saturation, and (3) hysteresis.

Nowadays you are concerned with magnetism as a function of all kinds of matter. But I wish to emphasize the gulf that divides ferromagnetics from other substances. It is a difference in kind, not simply in degree. Of the distinguishing features none is more significant than hysteresis. In my own early experiments it forced itself on my attention at every turn. I became soaked in hysteresis, and was led to invent that name*, feeling the need of a word that should be sufficiently wide to include not only the phenomena of magnetic retentiveness, but other manifestations of what seemed to be essentially the same thing, though some of them were not associated with any visible magnetic change. I found, for example,

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* *Proc. R.S.* **33**, 22 (1881); **34**, 39 (1882); **36**, 123 (1883); *Phil. Trans.* **176**, 524 (1885); **177**, 365 (1886).

that when a piece of iron or other ferromagnetic metal was subjected to a cycle of stress, as by hanging loads on a wire, and removing and reapplying them repeatedly, the quality of the metal went through a cycle of consequential changes, but irreversibly, being very different at corresponding points in the loading and unloading processes. This is true not only of magnetic quality but of other qualities as well; and it occurs with stresses that are far too small to produce permanent strain. The cyclic changes of stress appear to cause some kind of internal changes of structure, which are cyclic also but lag statically behind the changes of stress, exhibiting a sort of physical conservatism on the part of the metal—a tendency towards persistence of previous state.

That is only one example of the kind of phenomenon which the word hysteresis was intended to cover.

What is the physical cause of this curious lagging? Here is a problem which you have still to solve. From the fact that it occurs only in ferromagnetic solids one conjectures that it is closely associated with the effects of retentiveness which are observed in the process of magnetization.

The fact of saturation has long ago taught us to think of that process as an orienting of the elementary magnets first imagined by Weber. Each atom has its fixed place in the space-lattice, and we do not now think of the atom itself as turning, for in that case we should expect the cohesion of the substance to be affected, and we know it is not. I have tried the experiment of strongly magnetizing a piece of iron in a testing machine, while it carried a load which very nearly made it yield, and have found that repeated reversals of the magnetism did nothing to reduce the elastic resistance. We must rather think of something within each atom that can turn in response to an applied magnetic force, something which permanently possesses a magnetic moment sufficient to provide the saturation value when in every atom it is brought into parallelism with the applied force.

What controls its turning? When we begin to apply a magnetic force and gradually increase it we find three stages, always fairly distinct, and capable, under special conditions, of being very sharply separated from one another.

Interpreted as a turning of the Weber elementary magnets, the stages are, first, a small amount of deflection of a reversible kind in which the response to the applied field is slight and is proportional to the field, and there is no hysteresis; next, a big break away with irreversible tumbling into new positions of stable equilibrium, a process which involves much dissipation of energy. There may be more than one successive tumbling from one stable position to another on the way towards saturation. The last stage is again reversible; it is the final deflection from a stable position into complete parallelism with the applied force. In all this operation it is the tumbling from one position of stability to another that is the essence of hysteresis. It accounts for residual magnetism and for the expenditure of energy in a cycle of magnetization.

Forty years ago* I showed that all the characteristic features of the process could be very closely reproduced in a model made up of pivoted compass needles,

* *Proc. R.S.* **48**, 342 (1890).

regularly spaced like the atoms in a crystal lattice, and controlling one another simply by their mutual magnetic forces. Models have gone out of fashion now. My magnetic model was never more than a crude way of showing how hysteresis might be ascribed to the mutual action of parts of the interatomic magnetic system. More recently* I brought it a little nearer actuality, perhaps (though no atomic model can claim to be more than a piece of symbolism), by treating the Weber element as a part of each atom, capable of turning within the atom and controlled by magnetic interaction between itself and the other parts, which were regarded as fixed with respect to the neighbours in the lattice. Here again the control is imagined to depend upon magnetic forces, and we picture each Weber element as being able to respond to an externally applied field, first by a small reversible displacement, then by swinging violently into a new configuration in which it is again stable, but from which it can be displaced by a further increase in the field.

This notion, crude and vague though it is, seems to me still to point the way towards a clearing of our ideas about ferromagnetism. Shortly after it was first suggested it received an unexpected confirmation, of which I venture to remind you because it has escaped notice in most of the modern books. An acute critic, Mr James Swinburne†, took exception to my idea of magnetic control because it should imply the absence of loss of energy through hysteresis when a piece of iron has its magnetism reversed by the revolution of a very strong field, a field strong enough to secure saturation. For in that case the Weber elements would be held steadily pointing in the direction of the field and would have no opportunity of tumbling from one position of stability to another. It seemed most improbable that there should be so big a difference, in respect of dissipation of energy, between the reversal of magnetism which is brought about by the revolution of a constant strong field and that which is brought about by the reduction of the field to zero and its reapplication in the opposite sense. But soon afterwards it was discovered, through the experiments of Prof. Baily‡, that this difference does in fact exist. He measured the hysteresis loss caused by a revolving field, and found that for fields of moderate strength there was much loss, but that when the field was made stronger the loss passed through a maximum and rapidly fell almost to zero as the condition of saturation was approached. The whole action turned out to be just such as magnetic control of the Weber element would demand.

These reminders of the past will serve their purpose if they induce some of you, who are naturally engrossed by newer aspects of magnetic theory, to revert to a study of the ferromagnetic group. After all, it is there that we have the magnetism of the atom revealing itself on a relatively grand scale, there that we find an attractive tangle of conspicuous magnetic phenomena, and it is there, I think, that we may most profitably search for clues to what is now obscure.

* *Proc. R.S.E.* **42**, 97 (1922).   † J. Swinburne, *Industries*, Sept. 19, 1890.
‡ F. G. Baily, *Phil. Trans.* A, **187**, 715 (1896).

MAGNETISM IN THE TWENTIETH CENTURY

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ABSTRACT. The state of magnetism at the beginning of the century is indicated and the work up to 1913, mainly that of Langevin and Weiss, is briefly described. The outlook during the quantum-principle period (1913–25) and after the reformulation of the quantum theory is sketched. Against this background the theoretical and experimental progress made in connection with dia-, para- and ferromagnetism is considered.

The diamagnetic susceptibility of atoms and ions is in good agreement with the charge-distribution indicated by wave mechanics; the susceptibilities of molecules give information as to their sizes, and of crystals as to the sizes and shapes of the molecules in the crystals.

Paramagnetic theory applies particularly to ions of the various transition series of elements. For the rare earths the observed magnetic moments agree with those corresponding to the ground states deduced by Hund's method. For the other series the susceptibilities support the view that the effective electrons (those in incomplete groups), being on the outside of the ions, take part in interaction with neighbouring ions.

Ferromagnetism is considered in the light of the molecular field hypothesis, and the following topics are discussed: spontaneous magnetization, the nature of the "carriers," specific heat, the Barkhausen effect, single crystals, Heisenberg's theory of the molecular field, ferromagnetic materials.

The importance of the work on strong magnetic fields is indicated, and the magnetic properties of the elements are briefly discussed.

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§1. RANGE OF SUBJECT

IN so far as magnetism can usefully be regarded as an autonomous branch of physics, it is mainly concerned with the magnetic properties of materials. The field aspect of magnetism is best treated as a part of electromagnetic theory, modified by relativity, or embodied in a more general relativity theory. The study of magnetic materials does, however, lead unavoidably to field theories, for the ultimate interpretation of magnetic properties is largely in terms of the magnetic properties of the electron, and the consideration of these necessarily demands some kind of relativistic quantum electrodynamics.

In its more limited aspect magnetism is concerned with the effect of magnetic fields on materials, as affecting primarily the intensity of magnetization, and secondarily optical (light emission and transmission), mechanical and electrical properties. From a magnetic point of view, substances may be divided into three classes. In paramagnetics the magnetic intensity is proportional to the field applied (except in high fields at low temperatures) and in the same direction; in diamagnetics it is in the opposite direction. The susceptibility—the ratio of the intensity to the applied field—of dia- and paramagnetics is usually very small compared with that of ferromagnetics, such as iron. Ferromagnetic substances differ from paramagnetics

in that under certain conditions they may be "spontaneously" magnetized in the absence of an applied field. The general character of the variation of the magnetization with the field is well known. Usually there are marked hysteresis effects. The three classes of materials differ further in the way in which the magnetic properties depend on the temperature. Above a certain critical temperature (the Curie temperature) ferromagnetics become paramagnetic.

Up to ten years or so ago, except for specialists, magnetism meant little more than the study of certain properties of iron. This was partly due to the difficulties in technique on the experimental side, and on the theoretical to the difficulties in interpretation of results: a susceptibility, as long as it is a mere susceptibility, can give little intellectual satisfaction. Yet for chemists, in the widest sense of this term, as well as for physicists, the magnetic properties of substances of all kinds are of the greatest interest and importance.

In recent years progress has been rapid, and the value of magnetic investigations has been much more fully realized. It is perhaps not too much to say that the study of the effects of magnetic fields on radiation from atoms in the Zeeman effect has proved the master key for the elucidation of the structure of atoms, and paved the way for the later developments of the quantum theory. The study of susceptibilities gives information about the sizes and shapes of molecules, and about the nature of ions in liquids and solids. Further work on the effects of magnetic fields, particularly the strong fields used by Kapitza, promises to shed light on some of the still baffling problems as to the nature of crystals, and on the characteristics of the types of interaction between the aggregating atoms.

In this paper a brief account will be given of the three main stages in the development of outlook during the present century, in its bearing on magnetism. The properties of dia-, para- and ferromagnetics will be considered, and some of the outstanding problems will be indicated. This introductory sketch is intended to serve as a background to the more special investigations which are described in other papers.

§2. THE THREE PERIODS

Progress under a cloud (1900–13). At the beginning of the century, owing to the work of Ewing and many others, much was known of the magnetic properties of the ferromagnetic elements, particularly iron. Curie's extensive investigations had led to the conclusion that in general the specific susceptibility of diamagnetics was approximately independent of temperature, while the susceptibility χ of paramagnetics decreased as the temperature T increased. For oxygen the susceptibility was found to be inversely proportional to the absolute temperature, and this was also approximately true for a number of paramagnetic solutions. The result is embodied in Curie's empirical law for paramagnetics

$$\chi = C/T.$$

The electron had been discovered, and attempts were being made to extend the electron theory, on the basis of classical conceptions, to cover a wider range of phenomena. The revolutionary theory of discontinuity of radiation had been put

forward by Planck, but in spite of the success of this quantum theory in certain directions, in connection for instance with temperature radiation and later with the photoelectric effect and specific heats, it was at first regarded rather as a desperate hypothesis, unlikely to have any ultimate validity. But it was realized that there were fundamental difficulties, for classical conceptions alone were unable to account for the properties, particularly radiational, of atoms built up of electrons, or even for their existence. Theoretical work on the structure and properties of atoms was therefore necessarily of a tentative and provisional character, carried out, so to speak, under a cloud of uncertainty.

In 1905 appeared Langevin's classical paper on dia- and paramagnetism. His investigations applied primarily to a gas, the molecules of which contained electrons rotating in orbits. The stability of such structures had to be assumed. An electron orbit, like a current circuit, behaves as a magnet. If the moments of the orbital magnets "balance out," the molecule as a whole will have no magnetic moment. The effect of a magnetic field is equivalent to the superposition of an angular velocity—the Larmor precession—and this gives rise to an intensity of magnetization oppositely directed to the field; the treatment is in essentials similar to that of Weber, who considered the hypothetical Ampèrian molecular current induced by an applied field. For the atomic susceptibility the expression obtained may be written

χ_{At}

$$\chi_{At} = - (e^2/4mc^2) \sum_n \overline{r_1^2},$$

r_1
n

where $\overline{r_1^2}$ is the time mean square of the radius of the resolved electronic orbit perpendicular to the field, the summation being taken over the n electrons in the atom.

μ

When the molecule has a resultant magnetic moment μ there will be a second effect, usually much larger. The molecules will tend to orient themselves so that the magnetic axes lie in the field direction; if the conditions in the gas are such that a state of equilibrium can be attained (a question of some delicacy) statistical

$\overline{\mu}$

treatment shows that the mean resolved magnetic moment $\overline{\mu}$ will be given by

$$\overline{\mu}/\mu = \coth a - 1/a,$$

a

where $$a = \mu H/kT.$$

When $\mu H/kT$ is small—which means, in practice, in the case of paramagnetics, for any attainable fields except at low temperatures—the expression reduces to

$$\overline{\mu}/\mu = \mu H/3kT,$$

giving for the gram-molecular susceptibility

χ_M

$$\chi_M = \sigma_M^2/3RT,$$

σ_M

where σ_M is the gram-molecular saturation moment, that is, the magnetic moment when all the molecular magnetic axes are parallel to each other.

Langevin's treatment thus accounts for the salient generalizations from Curie's work as to the variation with temperature of dia- and paramagnetic susceptibilities. It further shows how the results may be interpreted. Diamagnetic susceptibility

gives an indication of the size of the electronic orbits; from paramagnetic suscepti-
bility the molecular magnetic moment may be calculated.

The next step was taken by Weiss. In ferromagnetics the magnetization in
small applied fields is much larger than can be accounted for by the action of the
external field alone. It is necessary to suppose that there is some mutual inter-
action between the molecules. Weiss suggested that there was a molecular field
proportional to the intensity of magnetization, I, the total field H being given by

$$H = H_e + NI,$$

where H_e is the external field and N the Weiss molecular field constant. This is
best regarded as a purely formal hypothesis, and as such its value in correlating
a wide range of observed phenomena can hardly be overestimated. Although the
molecular field is equivalent to a magnetic field in its effects, fields of the magni-
tudes required cannot be due to purely magnetic or to associated electrostatic
interaction; and for this reason the hypothesis is perhaps better stated in the less
committal form that there is internal energy $(-\frac{1}{2}NI^2)$ associated with magnetization
of intensity I.

In Langevin's theory the effect of any mutual molecular interaction due to
orientation was neglected. With the assumption of Weiss the expression for the
susceptibility of a paramagnetic becomes

$$\chi_M = \sigma_M{}^2/3R\,(T - \theta) = C_M/(T - \theta),$$

where $\theta = N\rho\sigma_M{}^2/3MR$, ρ being the density and M the molecular weight. This
equation is an adequate expression of the variation of susceptibility with temperature
observed for a large number of paramagnetics.

If N is positive there arises the possibility of spontaneous magnetization (in the
absence of an external field) below a critical temperature θ (the Curie point), the
value of which may be found by eliminating a between the equations

$$\sigma/\sigma_0 = \coth a - 1/a,$$

and

$$\sigma/\sigma_0 = a\,MRT/\sigma_M{}^2\,\rho N.$$

Normally ferromagnetics in bulk are not spontaneously magnetized. This fact
is attributed by Weiss to the material being built up of domains (which may be
perfect microcrystals) whose properties resemble those of elementary crystals of
pyrrhotite. While the domains may each be spontaneously magnetized to the degree
appropriate to the temperature, their directions of magnetization are distributed at
random. The true spontaneous magnetization may be obtained by extrapolating
to zero field the magnetization observed in large fields, where the variation is
linear. The variation of the spontaneous magnetization with temperature found in
this way is in good agreement with the equations; while the general character
of the variation of magnetization with field in small fields can be qualitatively
accounted for.

For paramagnetics the gram-molecular magnetic moment can be deduced from
the measured susceptibility when its variation with temperature is known; for ferro-
magnetics the moment may be simply found from the spontaneous magnetization

Marginal notes: I, H H_e, N C_M θ, ρ, M

at absolute zero (whose determination involves only slight extrapolation) when the magnetic "carriers" are parallel to each other. From the results of his own measurements, particularly on iron and nickel, and such others as were available, Weiss concluded that magnetic moments were all integral multiples of a definite unit, the Weiss magneton, the value of which has usually been taken as 1123·5 per gram-molecule*. For paramagnetics the atomic (or ionic or molecular) magnetic moment may be expressed as p Weiss magnetons, calculated from the formula

p

$$p = \frac{\sigma_M}{1123\cdot5} = \frac{1}{1123\cdot5} \sqrt{\{3R\chi_M (T - \theta)\}}$$
$$= 14\cdot07 \, C_M^{\frac{1}{2}}.$$

σ_A

From the saturation-moment σ_A per gram-atom of ferromagnetics the magneton value is

$$p = \sigma_A/1123\cdot5.$$

While there can be little doubt that this unit is not of fundamental significance it is a very convenient one, and for this reason will no doubt continue to be used; moreover, the idea that there was such a unit proved a most valuable incentive to further research.

In the period under review the names of Langevin and Weiss stand out. Langevin developed the theory for certain ideal cases. Weiss extended the theory and showed its wide application; he carried out pioneer work on ferromagnetic crystals; and he initiated lines of investigation which are still being actively pursued and continue to yield most valuable results†.

The quantum principle (1913–25). For magnetic theory, the quantization of angular momentum in Bohr's application of the quantum principle to the problem of atomic structure is of fundamental importance. If the angular momentum of an orbital electron can assume only values which are multiples of $h/2\pi$, a unit of magnetic moment is also provided, namely

μ_1

$$\mu_1 = eh/4\pi mc.$$

The value per gram-molecule (μ_1 multiplied by Avogadro's number) is 5593, just under 5 times the Weiss unit. Later, consideration of the Zeeman effect, in the light of the postulate that the frequency of radiation emitted is equal to the difference between the energies of the atom in its initial and final states divided by h, showed that an unexpected modification was necessary in Langevin's treatment of paramagnetism. Langevin had supposed that a magnetic carrier could assume any orientation in an applied field. Now the energy depends on the orientation, and the mere fact that in the Zeeman effect discrete lines are obtained shows that the magnetic moments of the atoms resolved in the direction of the field can only assume certain discrete values. There was much confusion in the earlier attempts at interpreting the Zeeman effect. It is unnecessary to retell that story in any

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* The latest value is given as 1125·6.

† Some of these are described in his paper contributed to the present discussion.

detail. It must suffice to say that, as a result of experiment, generalizations and guesses, a scheme was eventually worked out which correlated the observations in a satisfactory or almost satisfactory manner. Chronologically the spatial quantization theory of Debye and Sommerfeld led to the remarkable experiments of Gerlach and Stern on the magnetic deviation of atomic rays. These provided direct evidence of "discrete orientation," and enabled the value of the Bohr magneton to be confirmed. From the Zeeman effect it was found that if the resolved angular momentum of the atom in Bohr units of $h/2\pi$ was m, the corresponding magnetic moment was not m, but mg (in Bohr units $eh/4\pi m_0 c$), where g is the Landé splitting factor—a function of the various quantum numbers defining the atomic state. An approach to a satisfactory interpretation of the g formula, and the clearing up of many other difficulties, was arrived at by the spinning-electron hypothesis of Goudsmit and Uhlenbeck—that the electron itself had an intrinsic spin $\frac{1}{2}h/2\pi$ ($\frac{1}{2}$ in Bohr units) and a magnetic moment $eh/4\pi mc$ (1 in Bohr units).

m

g

Schemes were evolved (Hund is largely responsible for the details) by which the quantum state of an atom could be worked out when the quantum numbers of the electrons in it were known. The resultant orbital moment l, the spin moment s, and the total resultant j can be found by combination in the appropriate manner of the quantum numbers of the constituent electrons. In the treatment of paramagnetism it is a complete knowledge of the quantum nature of the ground state of atoms (and ions and molecules) which is required. This will be considered in more detail later. Here only the character of the magnetic deviation experiments of Gerlach and Stern will be outlined. In these experiments a stream of atoms, delimited by slits, is passed between pole pieces which give a non-homogeneous magnetic field, and received on a plate, where a trace is formed. In the absence of a field the trace is a single line. When the field is applied traces are obtained which do, in fact, correspond to the possible mg values (that is the magnetic moments in the field direction) of the atom under investigation. Thus Cu, Ag, Au, Na, K, H, give a double trace corresponding to $mg = \pm 1$. In all cases where there are no disturbing effects the pattern observed is compatible with that anticipated from spectroscopic observations, and from the correlating scheme. This work gives perhaps the most vivid experimental demonstration of the necessity of a quantum interpretation of natural phenomena.

l

s, j

Reformulation (1925...). With the aid of the spinning-electron hypothesis the scheme built up was satisfactory, but only in a formal sense. Considering only the difficulties which bear on magnetic theory, we may say that atomic models, at the best, led to a g formula containing j^2, l^2, s^2 in place of $j(j+1)$, ... in the empirical formula. The orbital model of the atom was much more definite than the experimental facts warranted. Moreover there was fundamental inconsistency in that a quantum postulate and restriction were arbitrarily superposed on a system following, up to that point, classical dynamics. In Heisenberg's new quantum kinematics an attempt was made to formulate relations between quantities which were essentially observable. This was the starting point of the development in the

last few years of a self-consistent quantum dynamics approached from quite different points of view, but with equivalent results, in matrix and wave mechanics*. In some respects magnetic theory has undergone little modification. In so far as it was based on a formal scheme, much of the same treatment still applies, as in connection with quantum states of atoms, but the interpretation of the formal scheme is radically changed. In other directions, however, quantum dynamics has led to conclusions which have meant a fundamental advance in the understanding of certain magnetic phenomena.

A definite position and velocity cannot be assigned to an electron simultaneously. The electron orbits lose their definiteness. In an atom there is only a certain probability of an electron being in a given volume. Alternatively, but rather less rigorously, the electrons in an atom may be regarded as fused into a continuous distribution of space charge, which in its reaction to external fields can to a certain extent be treated *as if* it were a classical charge distribution.

The mathematical treatment of the fundamental wave equations leads naturally to the recognition of a new type of "interchange" interaction, which has been shown by Heitler and London to account for homopolar molecule formation; while Heisenberg has shown that it is capable of accounting for the effects correlated, but unexplained, by the molecular field hypothesis. In this connection a prominent part is played by the Pauli exclusion principle, which in its wave-mechanical no less than in its older formulation is one of the "stubborn facts" of nature.

In the hands of Dirac and others the spinning-electron hypothesis has lost much of its arbitrary nature, for it is shown to be formally necessitated by the combination of the wider principles of wave mechanics and relativity. But many difficulties remain, and an adequate quantum electrodynamics is only gradually being built up†.

This brief outline of the developments since 1913 will perhaps give a sufficient indication of the background against which more specific magnetic problems may be considered.

§3. DIAMAGNETISM. SIZES OF ATOMS AND MOLECULES

The diamagnetic susceptibility of a centrally symmetrical atom or ion is given by

$$\chi_{At} = -\frac{e^2}{6mc^2} \sum_n \overline{r^2}.$$

This expression will hold for the quantum orbital model. If the ion has a magnetic moment the paramagnetic effect will predominate, so the expression is of direct importance only for atoms or ions with no resultant moment (that is, in a $^1S_0$ quantum state). In dealing with experimental results on material in bulk, it is convenient to consider first those from which values for quasi-independent ions or atoms can be deduced with the minimum of uncertainty. The ideal substances are monatomic gases; but fairly definite conclusions can also be drawn from solutions of salts (containing definite ions) and from solid salts which are definitely ionic.

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* See the paper contributed to this discussion by H. S. Allen.
† See the paper contributed by C. G. Darwin.

A number of accurate measurements of the susceptibilities of alkaline and alkaline earth halides are now available. (Ikenmeyer, Hocart, Reichender.) These give values for $\overline{r^2}$ compatible with other evidence. The susceptibility increases fairly regularly in series such as Na^+, K^+, Rb^+; F^-, Cl^-, Br^-. For ions with the same number of electrons the susceptibility decreases as the nuclear charge increases, as in Cl^-, K^+, Ca^{++}. Earlier measurements suggested that the inert gases were anomalous, but the measurements of Wills and Hector show that they fall into line, argon, for example, falling between Cl^- and K^+.

If the orbital model is replaced by the space charge model, it is possible to calculate very easily the equivalent value of $\overline{r^2}$ from the distributions for spherically symmetrical ions given by Hartree's self-consistent field method (Stoner). The agreement with experiment is good for the inert gases and positive ions, but for the negative ions there are considerable discrepancies which suggest that the charge in the outer parts of the ions is overestimated in Hartree's method.

It may be noted that the orbital model gave values which were much too large in cases where approximately complete calculations could be made, namely for He and H. It was shown by Van Vleck that the new mechanics led to satisfactory values. He has extended the treatment to the hydrogen molecule, and the agreement with experiment is very satisfactory.

For molecules generally, the complete calculation of the effect of a magnetic field obviously presents formidable difficulties. The diamagnetic susceptibility does, however, give an indication of the extent of the space charge distribution. The effect of double bonds and various kinds of linkages in organic molecules which is indicated by Pascal's extensive work, when interpreted in the light of modern views, seems capable of throwing valuable sidelights on problems of chemical constitution.

In crystals the problem is more complex. There is a regular arrangement of the positive centres of forces, and the distributed electronic charge linked with one or more of these may have a different effective area in different directions. The diamagnetic susceptibility will then have different values in different directions. Such directive effects have been known for some time; careful measurements have been made on a number of organic crystals (Raman, Bhagavantam) giving results of considerable interest.

The diamagnetism of a number of metallic elements is best considered separately, as in these the part played by free electrons has also to be taken into account.

Although the main lines of the theoretical treatment are clear, there are a number of points of detail to be cleared up. Experimentally there remains much that can usefully be done. To give simple examples, in addition to those indicated implicitly: there are very few accurate measurements available as to change of susceptibility with change of state or change of structure, or as to the difference in ionic susceptibility in solids and solution.

§ 4. PARAMAGNETISM. IONIC MAGNETIC MOMENTS

Atoms or ions containing incomplete groups of electrons will have a magnetic moment due to electron spin or orbital moments or both. In a free ion the spin and orbital moments give a resultant mechanical moment j; possible resolved moments (m) in the direction of an applied field are then $j, j - 1, \ldots - j$. For an assembly of such ions, the theoretical value of the Weiss magneton number, p, calculated from the susceptibility as described above, is

$$p = 5g\sqrt{\{j(j+1)\}}.$$

This would apply ideally to the monatomic vapours of the alkali metals, but owing to the experimental difficulties it has so far only been confirmed as to order of magnitude for potassium (Gerlach). It has, however, been found to apply to the trivalent ions of the rare earths. In these it is the underlying N VI and VII groups ($n = 4$, $l = 3$) which are incomplete, and the moments deduced for the ions (from the susceptibilities of solid salts and solutions, measured by Cabrera, St Meyer and others) are, with one exception, in remarkable agreement with those corresponding to the spectroscopic ground states obtained by Hund's method. This is somewhat surprising, for it implies that the magnetic "carriers," even in the solid salts, are quasi-independent; and that their resolved moment in the field direction can change.

In accordance with the Bohr-Ladenburg view, paramagnetic properties appear in the salts of the various transition series of elements. Except for the rare earths, it is the $l = 2$ groups (which, when complete, have 10 electrons) which are involved. The first transition series has been studied most completely: this is the iron series, with from 18 to 28 electrons in the ions. There is considerable variability in the magneton values found for some of the ions, but for ions with the same numbers of electrons the values cluster closely round fairly well-defined means. These, however, do not correspond with those calculated with the j formula from the anticipated ground states of the ions; nor is the agreement greatly improved when account is taken of the effect of temperature in modifying the statistical distribution of the ions in different possible states (Laporte and Sommerfeld). It seems here as if the orbital moment l and spin moment s are best considered separately. A fair agreement between theory and experiment is obtained in a number of cases by supposing the spin moments only to be effective (Bose); practically all the cases are covered by supposing that the orbital moments also may be partly concerned (Stoner). The ions behave as though they were free with respect to the spin moment, but not with respect to the orbital moment. Qualitatively, many of the previous difficulties as to variability may thus be satisfactorily explained, but there remain many problems of detail. One of the most important of these is connected with the values found for θ in the expression $\chi = C(T - \theta)$. The value of θ depends on the anion as well as the cation, being different in different salts; and although much valuable experimental work has been done, notably by Cabrera, the significance of such regularities and irregularities as have been observed is not clear. Heisenberg's

theory of the molecular field will undoubtedly prove the starting point for the solution of some of these problems.

In the ions of the first transition series, in contradistinction to those of the rare earths, the "magnetic" electrons are those in the outer group, and they presumably take part also in the interaction of a quasi-chemical character with neighbouring ions, whether in solids or solutions. It is for this reason that they do not behave as magnetically free. In the palladium and platinum families the ionic nature of the salts is much less pronounced; the magnetic moments found do not fit in with any of the usual theoretical expressions; and the variation with temperature is peculiar.

Molecules in general are diamagnetic; the outstanding exceptions are oxygen and nitric oxide. For these the moments correspond to the ground states of the molecules ($^3\Sigma$ for O_2, $^2\Pi$ for NO); the variation with temperature of the moment of NO predicted by Van Vleck has recently been confirmed (Bitter).

With respect to complex salts, many of which have recently been investigated by Welo, it must suffice to mention here that Sidgwick's conception of the "effective atomic number" of the central atom is a valuable aid in interpreting the magnetic results. To a first approximation the moments observed correspond to possible spin moments for ions with a number of electrons equal to the effective atomic number; but this is only a first step towards a satisfactory explanation of the magnetic properties of these salts.

§5. FERROMAGNETISM. PROBLEMS OF INTERACTION

Spontaneous magnetization. Ferromagnetism is a special case of paramagnetism. The value of the molecular field hypothesis lies in the fact that, in spite of its formal character, it serves as a trustworthy guide through the complexities of ferromagnetic behaviour. It is an essential consequence of this hypothesis that a substance containing magnetic carriers may be spontaneously magnetized below a certain critical temperature if the molecular field is positive; further, since an ordinary piece of a ferromagnetic is not necessarily spontaneously magnetized as a whole, the magnetization in one direction must be assumed to extend only throughout limited regions, or domains. An external field has the effect of making this spontaneous magnetization apparent, as the directions of magnetization in the domains tend to parallelism. By extrapolating to zero field the magnetization observed in high fields, the value of the spontaneous magnetization appropriate to the temperature (that is due to the molecular field alone) may be found. The variation of this magnetization with temperature has been investigated for a number of ferromagnetics. The most accurate results are those of Weiss and Forrer on nickel, and more recently on a number of ferromagnetics at low temperatures*. From the extrapolated magnetization at absolute zero, when the carriers will be parallel to each other, the magnetic moment per atom may be found; and from the character of the variation with temperature some interesting conclusions can be drawn.

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* See the paper contributed to this discussion by P. Weiss.

The nature of the "carriers." In experiments on the gyromagnetic effect predicted by Richardson in 1908 the ratio of the magnetic to the mechanical moment of the magnetic carriers can be measured. For all the ferromagnetics which have been investigated, the ratio is found to be twice that for a classical "orbital" electron. From this the conclusion seems inevitable that it is the intrinsic spin moment of the electron that gives rise to the magnetic moment. This does not mean necessarily that the carriers are "free electrons." They may be free, or they may be electrons in atoms free to change their direction of spin; free, that is, to change their orientation in respect of the *s*, but not of any *l* moment.

The theoretical curve for the variation of magnetization with temperature when the carriers can only assume two orientations in a field* is quite distinct from that when the carriers can assume any orientation, as in the Langevin-Weiss theory. The experimental curves agree closely with the theoretical curves corresponding to electron spin. There are still, however, near absolute zero and near the Curie point slight discrepancies which remain to be explained.

Specific heat. The variation of specific heat and its sudden drop at the Curie point provide confirmation of the view that a ferromagnetic is spontaneously magnetized, and that the negative energy $(-\frac{1}{2}NI^2)$ associated with the magnetization decreases more and more rapidly up to the Curie point and then disappears. If it is assumed that electron spin only is concerned, the number of effective electrons per atom can be estimated from the drop in the specific heat at the Curie point. Now, since the magnetic moment of the electron (1 Bohr magneton) is known, the number of effective electrons per atom can also be estimated from the low-temperature saturation-moment per atom. The numbers obtained in these two ways are in good agreement: 0·60 and 0·57 for nickel; 2·20 and 2·27 for iron.

The variation in the thermo-electric power of nickel against platinum with temperature near the Curie point has been measured by Dorfmann, Jaanus and Kikoin. They have interpreted their results as indicating that the magnetic electrons in nickel are also the conduction electrons. This raises a number of difficulties which, however, cannot be adequately discussed here†. At the Curie point there are also changes in volume and in elastic properties‡.

Above the Curie point. Above the Curie point, ferromagnetics become paramagnetic. The graph of $1/\chi$ against T gives a number of approximately straight lines. From these the values of p and θ are obtained in the usual way. The values of p, however, are such that the magnetic moment cannot be attributed to electron spin alone; in γ-iron, for example, about 10 electrons per atom would be required. At these temperatures the orbital moments must also play a part. This raises what seems to be a very baffling problem, which is part of the wider problem of the metallic state generally. While it is possible to account for the observed moments by assuming the presence of specific ions as "carriers," there is no independent

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* Possible values of m when $s = \frac{1}{2}$ are $\pm\frac{1}{2}$.
† See the paper contributed to this discussion by W. Gerlach.
‡ See the papers contributed by F. C. Powell and L. F. Bates.

evidence for the existence of these in the metal. In some ways it might be prefer-
able to treat the crystal as a whole; but this treatment does not at present show
promise of accounting quantitatively for the values found for the susceptibilities.

The Barkhausen effect. Single crystals. In an ordinary magnetization curve,
the steep part corresponds to the occurrence of irreversible changes of direction
of magnetization throughout domains. The occurrence of discontinuities in the
magnetization, when the applied field changes gradually and continuously, was
first demonstrated by Barkhausen. In later investigations the number and magni-
tude of such changes have been observed. In some cases (as in a specially treated
nickel wire investigated by Forrer) a single discontinuity may account for a great
part of the whole hysteresis jump, but in general the discontinuities correspond to
domains of small size, of the order of 0·1 mm.[3]. The domains may be filament-
shaped. It is not justifiable to identify the domains with grains or microcrystals.

Much progress has recently been made in the investigation of the magnetic
properties of single crystals of ferromagnetic metals. These are not so simple as
might have been anticipated, but this fact is partly due to the large effect produced
by small quantities of impurities, and by even very slight mechanical distortion.
In cubic iron crystals (Webster, Gerlach) there are slight directional effects,
magnetization occurring most readily along the tetragonal axis; but the differences
are small except in the region just below saturation. These directional properties
correspond to internal fields of the order of a few hundred gauss, and can be
accounted for, at least qualitatively, as arising from the classical magnetic inter-
action of elementary magnets placed at the lattice points (Peddie, Mahajani). The
initial permeability is very high and the remanence low, though there is some
uncertainty in the determination of the true remanence. Gerlach suggests that
a single crystal may not consist of a single domain; but the state of things
might possibly be due to the unavoidable imperfections in the crystals. The mag-
netostriction results on single crystals are simpler and more significant than those
on ordinary material.

Heisenberg's theory. The classical and classical-quantum theories could give
no satisfactory explanation of molecular field phenomena. Heisenberg has now
shown how these may arise from a type of interaction characteristic of quantum
mechanics. Consider an idealized ferromagnetic consisting of an aggregate of atoms
each containing one electron free to change its orientation, the remainder of the
atom having no magnetic effect. Each atom can interact with its neighbours. For
two hydrogen atoms at a given distance apart there are two possible energy states,
in one of which the electrons are parallel and in the other anti-parallel. The differ-
ence in energy between these two states far exceeds that due to the purely magnetic
interaction. Similarly, in the idealized ferromagnetic, the energy will depend
indirectly on the mutual orientation of the electrons in neighbouring atoms. The
energy must be assumed to be less when the electrons are parallel. (For hydrogen,
the anti-parallel state has least energy and corresponds to molecule formation.) A
complicated statistical treatment is applied to the crystal as a whole, and expressions

J_0, z

are obtained for the Curie temperature and the variation of magnetization with temperature, in terms of the interchange interaction J_0 and the number z of neighbours of each atom. As a first approximation, the expressions obtained agree with those from the Weiss theory (modified for the 1-Bohr-magneton case) and so are in agreement with experiment; the detailed expressions, however, are unsatisfactory and indicate that some modification in the treatment is necessary. Heisenberg's idea represents a very great advance, for molecular field phenomena are now embraced within the scope of the general theory.

Ferromagnetic materials. The variation of the magnetization of ferromagnetics in relatively small fields has not been considered here. The general nature of the effects is well known, but the variety in detail in the behaviour of different materials is so wide as to make a brief summary practically impossible. The properties in question are of the highest technological importance, and mention can only be made here of recent progress in the production of materials having characteristics which are desirable for different purposes, such as iron-nickel alloys, particularly permalloy with very high initial permeability, ferrocobalt with a saturation magnetization greater than that of iron, and cobalt steels with high coercivity combined with high remanence. While it may be known how the presence of impurities and thermal or mechanical treatment may affect the magnetic properties of a particular material, so many factors are involved that it is in general by no means clear why such effects are produced. An explanation of the properties of a complex ferromagnetic material can hardly be given when there are still difficulties in interpreting the properties of, say, a relatively simple iron crystal. The general problem, which involves metallurgy and crystallography, is one of the most interesting in magnetism. From a theoretical point of view the study of the simpler binary alloys, with one or both constituents ferromagnetic, and in particular the determination of the variation of Curie point and saturation moment with composition, shows promise of yielding results of great value.

§ 6. MISCELLANEOUS

Strong fields. There remain a number of topics which should at least be briefly mentioned in this introduction. One of these, which would require fuller treatment were it not to be specially discussed, is the study of the effect of very strong magnetic fields by the method developed by Kapitza. The importance of these investigations lies in the fact that the fields used, approaching half a million gauss, are of the same order of magnitude as the pseudo-magnetic fields, arising possibly from some kind of interchange interaction, already existing in solids. The measurement of the change of electrical resistance in such fields has led to entirely unexpected results of great importance for the theory of conductivity. The method seems to be the most powerful one available for the experimental investigation of outstanding problems in connection with interatomic forces and crystal structure*.

* See the paper contributed to this discussion by P. Kapitza.

Susceptibilities of elements. In discussing dia- and paramagnetism a somewhat arbitrary selection of materials was considered, namely those which were "normal." Among the elements, however, there are many which are not normal, in that they may have a diamagnetic susceptibility varying with temperature, or a paramagnetic susceptibility which varies very little. Among the paramagnetics the susceptibility may be very small, so that its origin must be in some way different from that of normal susceptibility. If there are free electrons with a magnetic moment in metals these might be expected to give rise to strong paramagnetism. It was shown, however, by Pauli, by the application of the Fermi statistics, that the electrons will normally balance magnetically in pairs, and that when a field is applied the magnetization produced will be small; moreover, that the susceptibility will vary little with temperature. This treatment accounts qualitatively for the susceptibilities of the alkali metals, and removes one of the outstanding difficulties in connection with the magnetic properties of the elements. But many others remain. It should also be noted that the free-electron explanation cannot apply to other substances, such as potassium bichromate, which have a small constant paramagnetism. The recent progress in formulating more satisfactory theories of electrical conductivity, and the new facts brought to light by Kapitza's investigation of the effects of strong fields on a wide range of elements, will give an entirely new interest to the study of the susceptibilities of the metallic elements.

MAGNETISM AND THE QUANTUM THEORY

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ABSTRACT. The results of the modern form of the quantum theory are reviewed in relation to the explanation which they afford of magnetic phenomena. Reference is made to the magnetic moment of the spinning electron which appears to play an important part in the new quantum mechanics, and to the explanation of the molecular field of Weiss as originating in the quantum interaction between electrons. The necessity for taking into account the principle of relativity is emphasized in connection with the possible structure of the electromagnetic field: the quantum theory may demand four-dimensional tubes of force in space-time. Heisenberg's principle of indeterminacy is briefly discussed.

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§1. THE NEW QUANTUM MECHANICS

FRESH light has been thrown on the problems of atomic magnetism by the new quantum mechanics associated with the names of Heisenberg and Schrödinger. Although the subject was approached by entirely different routes the matrix mechanics and the wave mechanics form a self-consistent scheme, and from the mathematical standpoint may be regarded as equivalent. There is a remarkable, and even puzzling, difference between the starting point of the matrix mechanics and that of the wave mechanics. The former, like the earlier quantum theory, is based on discontinuities such as stationary states and quantum transitions. Schrödinger's work is based on the ideas of continuity and causality familiar in classical physics. While Schrödinger himself sought to interpret the field scalar, ψ, of his fundamental equation in terms of the density of the electrical charge, Born* interpreted it as a probability in the statistical sense. There seems to be little question that the statistical interpretation lies closer to the real truth, and this is the view emphasized by Condon and Morse in their recent book on *Quantum Mechanics*. The question of being able to establish agreement between the electromagnetic theory and the new wave mechanics has been discussed also by Bateman† and by Louis de Broglie‡. "It is possible that the classical values of the potentials and of the fields may be only the mean values of the real magnitudes."

It is true that the wave picture of the atom becomes less definite than that of Bohr, in which planetary electrons circulate about a massive nucleus in definite orbits, but where it has been possible to institute exact comparison the results are in every case favourable to the new model. The hydrogen atom may be regarded as a nucleus surrounded by a distribution of space charge determined by the quantum

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* M. Born, *Zeit. für Phys.* **37**, 863 (1926); **38**, 803 (1926).
† H. Bateman, *Nature*, **118**, 839 (1926).
‡ Prince Louis de Broglie, *Comptes Rendus*, **184**, 81 (1927).

numbers, which emerge in a natural way from the equations. An alternative treatment, which has been favourably received by many theorists, regards the hydrogen atom as a centre of force surrounded by a field of probability, namely the probability of the electric charge which constitutes the negative electron being found within any assigned region. In this case the quantum numbers define the characteristics of the probability field. It is necessary to emphasize the fact that it is only in dealing with the one-electron problem that space of three dimensions can be employed in wave mechanics. In the general case of several electrified particles it is necessary to employ spaces of many dimensions, the number of dimensions corresponding to the number of degrees of freedom of the particles.

For each electron in an atom it is found that there is a total quantum number n and a subsidiary quantum number l which is less by unity than the azimuthal quantum number k of the orbital picture, so that $l = k - 1$. Also to each electron there must be assigned a quantum number s, defining the spin of the electron. For a single electron $s = \pm \frac{1}{2}$, so that the angular momentum of spin is $\frac{1}{2}(h/2\pi)$, where $h/2\pi$ is the quantum unit of angular momentum. The spinning electron was invoked to account for certain discrepancies between theory and experiment, the so-called "duplexity" phenomena, the observed number of stationary states for an electron in an atom being twice the number given by theory. Dirac* has shown that the earlier treatment employing the new quantum mechanics was incomplete in so far as it was not in agreement with relativity, or, alternatively, with the general transformation theory of quantum mechanics. His investigation, which satisfies the requirements of both relativity and the general transformation theory, leads to an explanation of all duplexity phenomena without further assumption. "All the same there is a great deal of truth in the spinning electron model, at least as a first approximation."

In a recent paper on the interaction of electric charges Eddington† has investigated the angular momentum of the electron, and finds that this is made up of two parts: "the second term is the angular momentum recognized in the older quantum theory; the first term is the 'spin of the electron' which has to be added to the orbital momentum. It will be seen that the so-called spin is deeply rooted in the matrix theory, being inherent in the transformation properties of the vectors and independent of the physical entities (mass, positive or negative electric charges) that may ultimately be located at the point (x). We obtain it purely from relativity considerations without introducing dynamical conceptions or even a wave-equation."

Associated with the mechanical moment there is in each case a magnetic moment. The Bohr magneton, $eh/(4\pi m_0 c)$, represents the magnetic moment due to the motion of an electron of mass m_0 in an orbit with angular momentum $h/2\pi$. For the intrinsic spin of the electron, the ratio of the magnetic to the mechanical moment is double that for the orbital moment, so that the magnetic moment of the spinning electron is one Bohr magneton.

n
l
k
s

m_0

* P. A. M. Dirac, *Proc. R.S.* A, **117**, 610 (1928).
† A. S. Eddington, *Proc. R.S.* A, **126**, 696 (1930).

    The total angular momentum of an atom is to be found by taking the vector sum of the spin and orbital moments. This resultant is represented by the quantum number j.

    In a magnetic field a single lj energy level separates into a number of component levels. They can be specified by a magnetic quantum number m, which may be regarded as the resolved value of j in the direction of the field. This magnetic quantum number can take the values $j(j-1)(j-2), \ldots -j$. The resolved magnetic moment of the atom expressed in Bohr units is given by mg, where g is the so-called "splitting factor" of Landé. The new quantum mechanics gives for g the formula

$$g = 1 + \frac{j(j+1) + s(s+1) - l(l+1)}{2j(j+1)},$$

in agreement with the results deduced from observations on the Zeeman effect.

    Results of very great interest from both the theoretical and the practical standpoints have been reached by the attribution of angular momentum to the nucleus of the atom. The results are more especially striking in the case of hydrogen, for which an analysis of the band spectrum indicates an angular momentum of one half a quantum unit. The outcome of work on the specific heat of hydrogen has ultimately led to the experimental proof of the existence of two kinds of hydrogen molecules, in one of which the wave functions are symmetrical in the rotational wave function and in the other anti-symmetrical. The nuclear magnetic moment corresponding to nuclear spin must in general be very small. This conclusion is interesting in connection with an attempt made by the writer several years ago to investigate the effect on the spectrum of the magnetic field of the core of an atom.

    The structure of the spectrum of helium for a long time eluded interpretation on the basis of the quantum theory, but Heisenberg was able to give an explanation based on the mutual interaction of the electrons in the atom. There is close correspondence between this problem and a classical resonance problem, and it may be said that Heisenberg's theory depends on a new type of quantum resonance. Another way of expressing the theory is to say that as there is nothing to distinguish between the two electrons, the effective result must be the same as if they were interchanged.

    The problem of the formation of molecules on the basis of wave mechanics has been attacked by Heitler and London[*]. When two neutral atoms, say two atoms of hydrogen, combine to form a molecule, the homopolar compound produced is due to resonance phenomena which occur in the Schrödinger waves. The calculated value of the energy of combination of the hydrogen atoms was found to be in agreement with the empirical value.

    Heisenberg[†] has advanced a theory of ferromagnetism which explains the molecular field of Weiss as originating in "the exchange degeneracy of the electrons in the different electronic systems of the lattice." His theory has been amplified by Fowler and Kapitza[‡], who have shown how it accounts in a natural way for the large change of specific heat in passage through the Curie point, the relatively large change

    * W. Heitler and F. London, *Zeit. für Phys.* **44**, 455 (1927).

    † W. Heisenberg, *Zeit. für Phys.* **49**, 619 (1928).

    ‡ R. H. Fowler and P. Kapitza, *Proc. R.S.* A, **124**, 1 (1929).

of volume at the same point, and the phenomena of magneto-striction. These effects are far too large to be explained by magnetic forces, but the exchange effects of Heisenberg's theory are adequate to give at least a qualitative account of them. "The magnetization compels a new selection from the various possible energies of interaction, and this results in comparatively large changes in the forces of attraction and repulsion at given distances, and so in changes of size in the magnetized crystal."

A concise account of this quantum interaction theory of the molecular field may be found in E. C. Stoner's recent book*, which forms a valuable summary of modern work based on the quantum theory.

§2. LINES AND TUBES OF FORCE

Let us picture an imaginary experiment in which two observers are in uniform motion relatively to one another. The first is provided with a charged body delicately suspended; the second with a suspended magnetic pole (or, if objection be taken to the conception of an isolated pole, with a suspended magnet). The first observer on noticing a deflection of his charged particle concludes that it is acted on by an electric field, the second observer supposes that his magnetic pole is acted on by a magnetic field. Thus the field which one observer regards as electric, the other regards as magnetic. Which is right? According to the principles of the theory of relativity it is impossible to say that one is more right than the other. " How can the same body both give and not give a magnetic field?" asks Eddington†, and he supplies the answer: "On the classical theory we should have had to explain one of the results as an illusion. On the relativity theory both results are accepted. Magnetic fields are relative."

If our two observers adopt the view that a field implies the existence of lines of force, the first observer will postulate the existence of lines of electric force, the second will conclude that his magnetic pole is affected by lines of magnetic force.

It is well known that Faraday attached great importance to the conception of lines of force, and emphasized the physical nature of such lines. This remark applies both to magnetic and also to electric lines of force. Quotations might be multiplied to illustrate his insistence on the view that lines of force should be regarded as, in some sense, physical realities. Later investigators stressed the importance, sometimes of one set of lines, sometimes of the other, and it might almost be said that there was a conflict as to which was the more fundamental. As in so many other cases the conflict ended in the decision that both views were right, or that both sets of lines should be regarded as particular cases of a more general conception. In the four-dimensional world of Minkowski, as E. T. Whittaker has shown, we have to consider a covariant family of surfaces, which may be called tubes of force in the electromagnetic field or *calamoids*. The Faraday electric and magnetic tubes are not distinct and rival things, but two limiting cases of the same thing. In fact the words of Minkowski with regard to space and time may be adapted so as to refer to the

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* E. C. Stoner, *Magnetism* (Methuen, 1930).
† A. S. Eddington, *The Nature of the Physical World*, p. 22 (Cambridge, 1929).

lines of force of classical theory, and we may say: "Henceforth electric lines in themselves and magnetic lines in themselves shall sink to mere shadows, and only a union of the two shall preserve an independent existence."

The present writer*, following S. B. McLaren, has drawn attention to the relation between lines of electric and magnetic force in three dimensions and the quantum constant. The angular momentum of McLaren's magneton or of the ring electron in its unitary condition is given by $(1/2\pi) N_e N_m$, where N_e is the corresponding number of tubes of electric induction, N_m the number of tubes of magnetic induction. Taking $h/2\pi$ as the quantum unit of angular momentum we find, on equating the two expressions,

$$N_e N_m = h.$$

§3. TUBES OF FORCE AND WAVE MECHANICS

It may be suggested that there is a connection between the hypothesis of quantum magnetic tubes and the wave mechanics of Schrödinger. Each quantum tube of magnetic induction, defined by the ratio of Planck's constant to the electron charge, is associated with an amount of energy $h\nu$, where ν is a frequency characteristic of the tube. The condition of the tube was formerly pictured as an internal spin or vorticity, but this representation may require some modification to suit the new wave model.

Schrödinger† points out that, on his view, in the unperturbed normal state of a conservative system either the distribution of electricity is constant in time or there is a stationary current distribution. "We may in a certain sense speak of a return to electrostatic and magnetostatic atomic models." Consider, for example, the rotator with fixed axis‡, in which a charged particle of mass m_0 moves in a circle of radius a. The allowed energy levels E_n are given by

$$E_n = \frac{h^2}{8\pi^2 m_0 a^2} n^2 \qquad\qquad n = 0, 1, 2 \ldots,$$

and the charge density (charge per unit angle) comes out to be simply $e/2\pi$, that is, the charge functions as though it were distributed uniformly over the circle. The magnitude of the current associated with the nth state is

$$\frac{eh}{2\pi m_0} \cdot \frac{n}{2\pi a^2}.$$

The external effect of this current distribution is a magnetic field equivalent to that due to a small magnet of magnetic moment $n \dfrac{eh}{4\pi m_0 c}$ in electromagnetic units, that is, n Bohr magnetons.

It is interesting to notice that this result is exactly analogous to that for the "ring electron" which formed the subject of a discussion at a meeting of the Physical

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* H. S. Allen, *The Quantum and its Interpretation* (Methuen, 1928).
† E. Schrödinger, *Wave Mechanics*, p. 123 (Blackie, 1928).
‡ E. U. Condon and P. M. Morse, *Quantum Mechanics*, p. 96 (McGraw-Hill, 1929).

Society* and we may apply to this model the conception of quantum magnetic tubes which the author has employed in connection with the ring electron.

We thus find
$$N_e N_m = nh,$$

where $N_e = e$ and is the number of tubes of electric induction, while N_m is the number of tubes of magnetic induction, associated with the model. This result points to the existence of discrete tubes of magnetic induction, the unit tube being defined as h/e.

It would seem probable† that this result applies also to the general case of central-force motions as in the hydrogen atom. Fermi‡ has shown that by adopting Schrödinger's interpretation of the field scalar the correct expression for the magnetic moment of a hydrogen-like atom may be deduced. He employs the expression for ψ found by Fock and then integrates the wave equation. It appears that the component magnetic moment in the direction of the field is equal to the product of the Bohr magneton and the magnetic quantum number. In this problem the lines of current are circles situated in planes perpendicular to the axis, with their centres on the axis.

§4. THE PRINCIPLE OF INDETERMINACY

Heisenberg§ has formulated a relation which imposes a limit on the maximum accuracy with which simultaneous measurements of the position and momentum of a particle can be made. This principle has also been discussed by Dirac and by Bohr.

If p and q denote two conjugate quantities in the Hamiltonian sense, for example the momentum and corresponding coordinate of a particle, then there will always be in the simultaneous observation of these quantities an uncertainty given by the rule that $\Delta q \cdot \Delta p$ is of the order of h (Planck's constant). Thus if we seek to determine with great accuracy the position of a particle, it becomes difficult to determine at the same time its velocity or momentum. Greater accuracy in the determination of position is compensated by a greater inaccuracy in the measurement of momentum. If, on the other hand, we strive to measure the momentum more accurately, we can only do so at the expense of the determination of position.

There seems to be something analogous to this in the relation $N_m N_e = h$, which has been considered in the preceding sections. We may associate the determination of the position of an electron with the determination of the electric lines of force which constitute its electric field, and we may associate the determination of its momentum with the determination of the magnetic lines of force which arise from its motion. The product of the two quantities N_m and N_e remains constant in a given state, and represents an integral multiple of the quantum of action.

It may be pressing a formal resemblance too far to suggest that Heisenberg's

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uncertainty relation, which is concerned with *errors* in the determination of p and q, may be associated with the difficulty, referred to in § 2, of distinguishing between the existence of an electric and that of a magnetic field. But if there is any physical significance in the resemblance, it points to the necessity of considering the uncertainty relation in the light of the theory of relativity.

Broadly speaking, there seems to be a necessity to interpret this relation in connection with a universe of four, or even of five dimensions, as is indicated at the end of Flint's *Wave Mechanics** in the discussion on the physical significance of the wave equation and the wave function.

* H. T. Flint, *Wave Mechanics*, p. 89 (Methuen, 1929).

THE POLARIZATION OF THE ELECTRON

By Prof. C. G. DARWIN, F.R.S.

ABSTRACT. A short account is given of the discovery of the spinning electron and of its analogies with and distinctions from polarized light. The question of the direct observation of the magnetism of the free electron is reviewed from the theoretical standpoint, and a comparison is made with the results of recent experiments.

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§ 1. THE ORIGIN OF THE THEORY

THE "spinning electron" was invented in 1925 and by now, as far as we may judge, its properties are fairly completely known. I shall therefore have nothing new to say on fundamentals, but shall chiefly discuss some minor points of interest. The starting point of the whole theory was of course the mysterious fact that every spectrum gave twice as many levels as it should. This doubling was explained in the old quantum theory by the idea of the spinning electron, a convenient term to describe the quality of possessing a definite angular momentum and magnetic moment as well as charge. The model had certain rather unnatural features, and needed some doctoring to make it work, but we need not criticize these as the interest in them has disappeared.

Turning to the wave mechanics we may make use of the picture of the atom drawn by de Broglie and Schrödinger, provided that we remember very emphatically that it answers only half the question, and is to be supplemented by rules of "interpretation" as to what will be observed. With this limitation we make a rough picture of the energy levels of the atom by imagining it to be a sphere of glass of variable consistency with refractive index increasing strongly towards the centre. A beam of light in the glass is bent strongly round the centre, and may be unable to escape. As it goes its distribution will of course alter and the natural analysis is into the normal modes of harmonic vibration, each of which corresponds to one of the energy levels. But now suppose that we look a little more closely and allow for the polarization of the light. Mathematicians have wisely refrained from the troublesome study of the passage of light in media of variable refractive index, but it is easy to show that even isotropic media will produce double refraction—the unequal reflections at an interface are really a special case of this. The double refraction splits each normal mode into two, in which the frequencies and regions of disturbance are slightly different. Now the doubling of the number of levels was the cardinal difficulty of the older theory, so that as soon as Schrödinger's theory was discovered, it suggested that it would be worth while to see whether this doubling could not be produced by something analogous to the polarization of light.

This was the idea that led me to attack the problem in 1926. It was not of course to be expected that the electron waves would be exactly like the electromagnetic, since for the electron the two polarized components are anti-parallel instead of perpendicular. The attempt to invent a suitable wave system by the use of the machinery of tensors met with no success, but finally it proved possible to form a pair of simultaneous differential equations which yielded the hydrogen levels approximately. The equations had one defect, in that the S-levels came in the wrong places; subsequent history showed that a very little more ingenuity would have put them right, and this in a way that would have been most suggestive for further progress. The equations were quite asymmetrical in appearance and it was even surprising that in fact they were independent of the choice of the axes x, y, z. Now relativity theory is concerned with systems independent of the choice of axes, and from this condition develops the conception of tensors apparently as a matter of necessity, and so it seemed clear that the equations ought to be put in tensor form. This I therefore did, but wrongly as subsequent events proved—we return to this later. My work had been empirical, but Pauli showed how the equations could be derived from the dynamical principles of the quantum theory by means of the spinning electron. He of course got the S-levels wrong too, and we can therefore say that the electron has dynamical qualities definitely not represented by the idea of spin. It is for this reason that I prefer to speak of the "polarized electron," for the word "polarization" is already used in several different senses, and one more will not add to the confusion.

§2. DIRAC'S EQUATIONS

The important step in the theory was made by Dirac by means of his principle that the equations must be of the first order in t, and so, to conform to relativity, in x, y, z also. The consequence is a system of four equations, or alternatively a single equation involving four-rowed matrices. The matrices can be chosen in many ways, so that the four equations can be put in a large variety of forms, but all are hopelessly asymmetrical, so that no inspection, but only detailed testing, can verify that they are independent of the choice of axes. Many attempts have been made to overcome this trouble by expressing them in tensor form, but on the whole I think it must be said that they are not of much use. To get the tensor form the four equations must be replaced by sixteen, relating together certain invariants, vectors and tensors of second rank. Moreover, the observable properties depend only on four quantities, so that we can never determine unique values for the sixteen. The fact is that six years ago Weyl showed that there were quantities, other than vectors, which possess the property of invariance for orthogonal changes of axes. They may roughly be described as half-order tensors; for when the axes are rotated through an angle, they change in a way that depends on half that angle. Dirac's variables are of this character. The property can only exist for orthogonal transformations of the coordinates, and if we demand the full gravitational theory of relativity we must use the sixteen equations, as has been done by several writers. If the present theory

were final, we should certainly be forced to do this; but I have a very strong impression that the whole thing will ultimately take a new form, and that for the present it is best to put up with the asymmetry of the four equations, in spite of the intolerable amount of algebra to which it leads in any but the most general theorems.

§3. THE MAGNETIC MOMENT

Though the four ψ's cannot conveniently be thrown into vector form, there are quadratic functions of them which can, and as the observed quantities all depend on these we must consider them. The chief of them is the current density $\psi^* a_1 \psi$ (where a_1 is one of Dirac's four-rowed matrices) with time component $\psi^* \psi$ for the density. There are also others which one can construct by writing down expressions such as $\psi^* a_1 a_2 \psi$ and $\psi^* a_1 a_2 a_3 \psi$, and finding how they transform for changes of axes. The complete set is two invariants, two four-vectors of which one is the current, and one six-vector. I overlooked the existence of the second four-vector in my paper on the subject. Its physical meaning is most easily seen if a wave-packet be constructed for an electron moving freely. The vector is closely related to the density of "null-moment," that is, of the magnetic moment as it would appear to an observer moving with the packet; its time component is the component of null-moment in the direction of the motion. The six-vector obeys the same rules of transformation as do electric and magnetic forces. If the electron is reduced to rest, three components vanish and the others give the magnetic moment. Now a moving magnet certainly gives rise to electric forces and so can be said to have an electric moment—this idea was developed long ago by Frenkel in connection with the old spinning electron—so it is the natural interpretation to regard the six-vector as the actual magnetic and electric moments of the moving electron. It appears that for a free electron the electric moment is always perpendicular to both the magnetic moment and to the direction of motion, and that the magnetic moment differs from the null-moment in that the component perpendicular to the direction of motion is increased in the ratio $1/(1 - v^2/c^2)^{\frac{1}{2}}$.

There are certain questions about this, however, which are not quite clear, for Dirac found no electric moment except an imaginary one, and it is not very evident what that means. I am inclined to think that the discrepancy is only a matter of definition. Dirac was discussing the action of a force on an electron, whereas here we have the force exerted by the electron on a set of distant electrometers. The two phenomena are of course related by dynamics in a perfectly precise way, but nevertheless the chosen interpretations may be different. If for example a non-uniform electric force cannot split a stream of electrons into two, we might say that they have no electric moment, whereas there might still be, in the distant electric field, inequalities which could be attributed to an electric moment. This suggestion is only tentative, and perhaps the matter ought to be cleared up. It would I think be mainly a question of classical electrodynamics, but would involve rather troublesome considerations of the electric fields of moving media.

§4. THE DETECTION OF THE MOMENT OF FREE ELECTRONS

It is interesting to consider what are the prospects of directly observing the magnetic moment of the electron. Both the general wave theory and the magnetic moment were first discovered as matters of indirect inference from the theory of spectra, but the first was soon verified for free electrons by experiments on their diffraction. The magnetic moment of the electron has of course been verified by the device of loading the electron with the neutral core of a silver atom and doing a Stern-Gerlach experiment, but this may be considered only indirect, and what we would like is to be able to exhibit the moment of a *free* electron.

A number of ingenious experiments of the Stern-Gerlach type have been suggested for the purpose, but on closer examination they have all revealed fallacies of some kind. Finally Bohr was led to the tentative suggestion that the magnetism of the free electron might be intrinsically unobservable. By one of those simple arguments which only he can produce, he showed that a Stern-Gerlach experiment is useless, because a non-uniform field must have a cross component which exerts an uncertain force on the electric current. Attacking the question from the other end, he imagined that a polarized electron had been somehow produced and considered how it would be observed by a magnetometer. He showed that again the uncertainty of position and speed would lead to a greater variation of the magnetic force than would the intrinsic magnetism. Other work has shown that Bohr's tentative principle was wrong, and this emboldens me to disagree with the last conclusion, by the suggestion that there is no reason why we should not have several magnetometers surrounding the track of the electron; then the uncertainties of position will have different effects from the magnetic moment so that by a survey of the magnetic field the moment could be determined. At all events it is certainly possible to construct a wave-packet for an electron polarized in any direction, and to show that the distant field depends on the polarization. The argument is perhaps not quite conclusive, because the essential feature of the uncertainty principle is that the observation disturbs the experiment, and no allowance is made in Bohr's argument for the reaction of the magnetometer or magnetometers on the electron. But as the matter has been settled anyhow in the negative, it is not worth further enquiry.

When the diffraction of electrons was first observed, several experimenters tried without success to find a polarization. I was myself led to examine what their prospects were—this was before Bohr had discussed the subject. The coherent scattering by a crystal offers greater intensities for the experimenter than the scattering by an irregular assembly of atoms and it also gives easier mathematics. I therefore discussed this case, but simplified the problem a little by using a grating instead of a lattice; the extension to a crystal lattice would be very easy. There was also another simplification, which, as it turns out, has an important effect, and this was the assumption that the electric field was everywhere small, whereas in fact it of course becomes infinite near the nuclei of the atoms. The regularly spaced electric and magnetic forces can be analysed into Fourier series, and each term then is responsible for one spectrum. The study of one such spectrum will suffice. If a beam

of electrons already polarized falls on the grating, the electrons in the lateral beam will have the direction of their magnetization altered. It is a true polarization effect, for the change of direction depends only on the quality and not on the magnitude of the forces in the grating. When the forces are only electric the effect is a pure rotation through an angle depending only on the speed, and a similar result is true for some types of magnetic force, though here the rotation is irrespective of the speed. In these cases, if the original beam is unpolarized, the diffracted beam will be so too, since all its components are simply rotated like a solid body. But there are cases involving both electric and magnetic forces where this would not be so. It appeared that they could hardly be realized in practice, and as I was not aware of Bohr's suggestion at the time, I merely recorded the way they must be treated and verified that they would really give the effect, but without any estimate of magnitude. Recently I have worked out one or two cases, so as to see what polarization might be expected. One of the most favourable is something like a sheet of atoms magnetized normally. The amount of polarization is roughly $(vH/cE).\sin(\frac{1}{2}\epsilon)$ measured in Bohr magnetons per electron, where v is the speed, ϵ the angle of deviation, and E and H are respectively the greatest electric and magnetic forces in the grating. This result does not perhaps get us much further, because in fact the electric force becomes infinite at the nuclei, and so it is not clear what value should be chosen for E in the formula.

Mott has made a much better study of the question by solving accurately the passage of an electron past a nucleus. He showed that the electric forces could produce a partial polarization without, as in my case, the necessity of a magnetic force as well; the discrepancy is presumably due to my assumption that the electric force was everywhere small. The polarization could be observed by the device of scattering the beam twice in succession. Suppose for example that both scatterings are through right angles, and that we compare the intensities of the beams going parallel and anti-parallel to the original beam; then Mott showed that with the most favourable velocity, 3/4 that of light, there would be found a difference in the proportional number of electrons of amount about $(Z/96)^2$, where Z is the atomic number of the scattering nuclei. In arriving at this number certain rather doubtful assumptions were made, but the conclusion is quite definite that the result ought to be observable by the use of heavy elements and high speeds, but not with light elements or low speeds.

Very recently Rupp has published an account of some experiments on the subject, in which he claims to have detected the effect. He used two reflections at successive mirrors, set at almost glancing incidence. There is of course no essential difference between reflection at a plate of atoms, and scattering at a single atom; the plate merely concentrates the intensity into one direction at the expense of others, while the quality of the rays depends only on the influence of the single atoms. By rotating the second mirror round the direction of the rays, Rupp found an asymmetrical effect of Mott's type. It was 10 per cent. for gold when the electrons had a high speed, but not when they were slow, and it could not be detected for beryllium even at high speeds. The work was evidently carried out with very great skill and

care, and the experimental results look most convincing. But they do not support the theoretical work on account of their magnitude; for Mott showed that the asymmetry should be proportional roughly to $\sin^3(\theta/2)$, where θ is the angle of deflection at each mirror, and for Rupp's angle of 20′ the value should be quite insensible. It is indeed common sense that when the electron is only slightly deflected, the forces are weak and so we should expect the effect on the polarization also to be small. To clear the matter up it would be desirable therefore to have experiments at broader angles, but the intensities would become very small, and no doubt the technical difficulties would be enormous. To conclude, I do not think we can reconcile the theory and the experiment. If the theory is right the experiment should have shown no effect; while if the result of the experiment is right, as it seems to be, and if it is really due to the magnetism of the electron, then we must suppose some of the effects disregarded in the theory are more important than we should have anticipated.

THE GYROMAGNETIC EFFECT
AND PARAMAGNETISM

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ABSTRACT. The importance of determinations of the gyromagnetic effect is discussed in relation to paramagnetism, though the gyromagnetic amplitude is so minute as to render accurate determination difficult in some cases. (i) In the case of ferromagnetism the value of $g \doteqdot 2$ found experimentally is in accord with theory. (ii) In the case of paramagnetic substances only one accurate determination has yet been made, namely that for Dy^{+--}: it gives $g = 4/3$, in good agreement with both susceptibility measurements and theory. Preliminary data on Mn^{++} and Cr^{+++} suggest, in agreement with susceptibility results, that both these ions are in S-states. (iii) Experiments in progress on Eu^{+++} taken in conjunction with the Van Vleck theory should yield useful information.

§ 1. INTRODUCTION

ACCORDING to the Langevin theory of paramagnetism, deduced for a free magnetic gas, the susceptibility χ varies with temperature according to the law $\chi T =$ constant, where T is the absolute temperature. For many paramagnetic solids and solutions, the variation of susceptibility is given by the modified Curie law

$$\chi_M (T + \Delta) = C_M,$$

where χ_M is the gram-molecular susceptibility and C_M the Curie constant per gram molecule.

The magnetic moment is usually expressed such that the number, p, of Weiss magnetons ($\mu_W = 1123$) is given by

$$p = 14 \cdot 07 \, C_M^{\frac{1}{2}} \qquad \ldots\ldots(1).$$

This assumes that the magnetic carrier is as free as in the gaseous state. No satisfactory theory has been put forward to account for the varying values of Δ which are found for paramagnetic substances.

On the quantum theory the magnetic moment of an atom or ion will have only discrete values, and expressions for the magnetic susceptibility have been deduced on this assumption. Hund* showed that the number of Weiss magnetons could be determined from the spectroscopic state and that

$$p = 4 \cdot 97 g \, \sqrt{\{j \, (j + 1)\}} \qquad \ldots\ldots(2),$$

where g is the Landé splitting factor, and j the quantum number specifying the total angular momentum. He calculated the magneton numbers for the rare earth

χ
T

Δ

χ_M, C_M

p

g, j

* F. Hund, *Zeit. für Phys.* 33, 855 (1925).

group and, with the exception of Sm ⁻⁻⁻ and Eu ⁻⁻, the results showed close agreement with the experimental data. The expression was later extended by Laporte and Sommerfeld*, who pointed out that there would be a distribution of the ions among the different possible j-states.

Recently Van Vleck† has developed a comprehensive theory of magnetism on the basis of the quantum mechanics. He obtains an expression for the susceptibility

$$\chi = \frac{N\Sigma_j \{[g^2 j\,(j+1)/3kT] + \alpha\,(j)\}\,(2j+1)\,e^{-W/kT}}{\Sigma_j\,(2j+1)\,e^{-W/kT}} \qquad \ldots\ldots(3),$$

W where $W - hc\Delta\nu_j$ and is the difference in energy between the particular j level concerned and the ground level.

In the absence of experimental data, the overall multiplet separation is deduced from the formula

$$\Delta\nu = R\alpha^2 \frac{(Z-\sigma)^4\,(2L+1)}{n^3 l\,(l+1)\,(2l+1)},$$

R where R is the Rydberg constant,

α α the Sommerfeld fine-structure constant,

L, l and L the resultant of all the individual l's.

This expression differs from that obtained by Laporte and Sommerfeld in the addition of the term $\alpha\,(j)$. The two terms in the numerator arise from the fact that the angular momentum and magnetic moment vectors are not parallel. The first term includes the contribution of the part of the magnetic moment which is parallel to the angular momentum j, whilst the other, a second order term, is due to the component of magnetic moment perpendicular to the angular moment vector, and gives rise to second order Zeeman terms. This $\alpha\,(j)$ term is particularly large where the l and s vectors are large, with the resultant j small. The two cases in which this holds are Eu⁻⁻⁺ and Sm ⁻⁻, and Van Vleck claims that the previous discrepancy from Hund's formula in the case of these ions is completely removed by the inclusion of this term.

§ 2. THE GYROMAGNETIC EFFECT

M A change M in the magnetic moment of a substance should be accompanied
U by a change U in the angular momentum, the ratio being given by $M/U = g\cdot e/2m$. Thus whilst, on the one hand, susceptibility determinations should give a mean value of $g^2 j\,(j+1)$, on the other hand, the gyromagnetic ratio enables one to calculate the mean value of g.

The author and others have measured this change of angular moment, which is small, for ferromagnetic bodies, and for all substances investigated $g = 2$ within the limits of experimental error. This, interpreted on modern theory, indicates that ferromagnetism is due to the spin of the electron. For paramagnetic materials, theory and experiment indicate that the magnetic moment is due, in general, to

* O. Laporte and A. Sommerfeld, *Zeit. für Phys.* **40**, 333 (1926); Laporte, *ibid.* **47**, 761 (1928).
† J. H. Van Vleck, *Phys. Rev.* **31**, 587 (1928); *ibid.* **34**, 1494 (1929).

he resultant of orbital and spin moments. The writer has recently developed a method for measuring this ratio for paramagnetic substances* and results have been obtained on the Dy^{+++} ion in Dy_2O_3. The state of this ion according to Hund is ^6H, the j value being $(l + s) = 15/2$†. The number of Weiss magnetons calculated by means of equations (1) and (2) from the above state is 53, whereas susceptibility measurements give 52. The measurements of the gyromagnetic ratio for this ion give the value of g as $1\cdot28 \pm 7$ per cent., in excellent agreement with the theoretical value $4/3$.

§ 3. COMPARISON WITH EXPERIMENT

It remains to discuss other members of the several groups and to compare theory with experimental data in so far as they exist. In the case of the first transition series, none of the above theories of susceptibility shows any agreement with the experimental data. In order to explain the magneton numbers, one has to make the arbitrary assumption that all the ions are in S-states‡.

Preliminary measurements on the gyromagnetic ratio have been made by the writer on Mn^{++} and Cr^{+++}, though owing to the relatively small susceptibilities of these substances as compared with that of Dy^{+++}, it will be necessary to obtain greater sensitivity before more exact results can be obtained. The amplitude of the angular momentum varies inversely as g and the measurements made so far for these two ions give $g > 2$. In the case of Mn^{++} all theories agree well with the susceptibility measurements, there being d^5 electrons which give a ^6S-state for which $g = 2$. For Cr^{+++} the original Hund theory gives g as $2/5$ whilst calculation from the Laporte-Sommerfeld expression gives a mean value of g as $1\cdot4$. Cr^{+++} is, therefore, another ion for which a direct determination of the gyromagnetic effect is required.

For the rare earth group, Van Vleck made his comparison of theory with experiment by taking the susceptibility results at room temperature and assuming the Curie law χT = constant. But Cabrera and Duperier§ have measured the susceptibilities of most of these ions, in some cases both for the oxide and the anhydrous sulphates, and found that in general the susceptibilities follow the law

$$(\chi + K)(T + \Delta) = C \qquad \qquad ......(4),$$

where K is independent of the temperature.

If these are classified according to increasing number of $4f$ electrons, the susceptibilities fall naturally into three types. Of the first six, Ce^{+++} has not been measured, and the ion corresponding to the element 61 is as yet unknown. The remaining four have a negative value of K, corresponding to a constant superposed paramagnetism which increases to a maximum at Eu^{+++}. For Sm^{+++} the $(\chi + K)$

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* *Proc. R.S.* (in press).

† For ions in the second half of the rare earth group, which includes Dy^{+++}, the Van Vleck values do not differ appreciably from those of Hund. This is due to the fact that the multiplets, which are wide ($\sim 10,000$ cm.$^{-1}$), are inverted, so that the Boltzmann factors for their components of lowest j are negligible.

‡ For a complete discussion of this point see E. C. Stoner, *Phil. Mag.* **8**, 250 (1929).

§ B. Cabrera and A. Duperier, *Comptes Rendus*, **188**, 1640 (1929).

value is so small and Δ so large that the total susceptibility is approximately in-dependent of temperature and only preliminary values are given. For the next five, which have the greatest susceptibilities, $K = 0$, and the modified Curie law is followed. For Gd^{+++}, which occupies the position analogous to that occupied by Mn^{++} in the iron group, $\Delta = 0$. For the two remaining substances, Tu^{+++} and Yb^{+++}, K is positive. In general, the K values for the first type, i.e. in the first half of the group, are numerically greater for the sulphate than for the oxide, whilst in the third type the reverse is the case.

Van Vleck applied his theory to the room-temperature susceptibility-measure-ments on the rare earths, and completely removed the discrepancy for Eu^{+++} and Sm^{+++}, which were the two exceptions to the Hund equation, and improved the agreement in the case of the other members of the first half of the group. In view of the measurements of Cabrera and Duperier mentioned above, it is worth while to apply equation (3) to the experimental results at different temperatures. For $Eu_2(SO_4)_3$ the agreement between 150 and 600 abs. is quite close, the cal-culated susceptibility varying from the experimental values by only 1 or 2 per cent. At 300° abs., the $\alpha(j)$ term is responsible for about 75 per cent. of the susceptibility, this proportion decreasing with increasing temperature*. For the oxide, the χ-T curve lies below that of the sulphate by a constant difference of about 10 per cent. of the susceptibility at room temperature.

In the case of Sm^{+++}, equation (3) gives an approximately constant suscepti-bility between 300° and 600° abs., but there is an upward trend of the susceptibility with decreasing temperature, whereas experiment† shows an approximately constant value. In order that equation (3) may yield a constant susceptibility the first term must disappear. This obtains when the majority of the electrons are in a lowest level which has j or $g = 0$, unless it is an S term. Until more definite information about the shape of the χ-T curve is available, it does not seem profitable to discuss it further.

As has already been pointed out, K is negative in the first half of the group, and positive in the second half. This result may be connected with the fact that owing to the Boltzmann factor the apparent magneton number increases with temperature for erect terms, and decreases for inverted terms.

Considering the assumptions involved the agreement appears to be fairly satis-factory. In the first place, there is little or no experimental confirmation for the electron distributions, and further, there is the uncertainty of deducing the correct multiplet separations and screening constants. For other ions, the Van Vleck and Hund expressions agree sufficiently well with susceptibility determinations for the difference to be attributed to experimental error.

Measurements of the gyromagnetic ratio on Eu^{+++} should be of interest, and experiments are in progress on Eu_2O_3. The Sommerfeld-Laporte theory gives a mean value of $g = 3.2‡$. On the assumption that Van Vleck's $\alpha(j)$ term in

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* The temperature range of Cabrera and Duperier's measurement is not specified.
† E. H. Williams, *Phys. Rev.* **12**, 165 (1918).
‡ The ground level is ^7F and all states except the lowest have $g = 3/2$.

equation (3) does not contribute towards the angular momentum measured, the effective mean value of g should be about 6·4. Measurements of the gyromagnetic amplitude should throw light on this point.

Finally we may consider the susceptibilities of the ions of the second and third transition series. The chlorides have been investigated by Cabrera and Duperier*. Only one ion, Ru^{+++}, shows appreciable paramagnetism (specific susceptibility $= 9·5 \times 10^{-6}$), the rest having positive or negative susceptibilities of the order 10^{-7}. Ru^{+++} is the only one which has an odd number of electrons.

Measurements for these six ions were made over a range of temperature of 100° and are expressible by the empirical formula (4) above, K being positive for Ru^{+++}.

On the other hand, the measurements have been made over a limited range of temperature so that it is difficult to determine accurately the deviation from the usual law $\chi (T + \Delta) = \text{const.}$

In spite of the difficulty of assigning the electron distribution to these ions in the absence of spectroscopic evidence, it is obvious that they are quite different from the members of either the first or fourth transition series. Even in the case of Ru^{+++} the deepest levels deduced from the distributions d^5 and sd^4 both give susceptibilities greater than the experimental value.

On the whole it appears that the modified Curie law is the exception rather than the rule for the four transition series. Generally speaking, it is only in cases where the susceptibility is large that the law is obeyed. Even in these cases, the departure from this law may be masked by the magnitude of the magnetization. Stoner† notes two exceptions in the iron group, e.g. Ni^{+++} and Cu^{++}, and these are two which have smaller susceptibilities. It seems essential to make measurements over as wide a temperature-range as possible in order to throw light on this point.

* B. Cabrera and A. Duperier, *Comptes Rendus*, **185**, 414 (1927).
† *Loc. cit.*

THE CHANGE IN SIZE OF A FERROMAGNETIC AT THE CURIE POINT

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Communicated by R. H. Fowler, F.R.S.

ABSTRACT. The changes of size of iron and nickel at their Curie points are too great to be explained by purely magnetic forces. On Heisenberg's theory of ferromagnetism, magnetization is accompanied by a change in the strength of binding between atoms, and the resulting changes of size of crystals of iron and nickel at their Curie points are of the same order of magnitude as those observed.

The calculations for iron make use of a modification of Heisenberg's original theory, in which the restriction that each atom possesses only one electron concerned in magnetic effects is removed. A consideration of the specific heat discontinuity at the Curie point shows, however, that the theory is far from satisfactory.

The change in size which accompanies magnetization varies continuously from the Curie point to saturation, but does not affect measurements of magnetostriction at ordinary temperatures.

§ 1. INTRODUCTION

WHEN the temperature of a ferromagnetic crystal passes through the Curie point, the crystal shows an anomalous expansion or contraction. This phenomenon could not be interpreted on Weiss's molecular field theory, since in order to account for it one required to know how the strength of the molecular field depends upon the size of the crystal, and this could not be determined, the physical origin of the molecular field being quite unknown.

In Heisenberg's* theory of ferromagnetism the molecular field is replaced by the "exchange" interaction of the quantum theory, which leads to homopolar binding between the atoms in the crystal. The change of size at the Curie point may be directly interpreted as the strain produced by the change of strength of binding which accompanies the change-over from the paramagnetic state (electron spins orientated at random) to the magnetized state (electron spins orientated parallel).

Attention was drawn to the phenomenon by Fowler and Kapitza†, who calculated the change of size on Heisenberg's theory, and made a rough estimate of its magnitude in iron. The experimental values which they used were, however, incorrect‡, and it is the main object of this paper to give the correct data. The data

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* W. Heisenberg, *Zeit. für Phys.* **49**, 619 (1928).
† R. H. Fowler and P. Kapitza, *Proc. R.S.* **124**, 1 (1929).
‡ This was pointed out by Dr W. L. Webster, to whom I am much indebted for references to the correct data.

quoted by Fowler and Kapitza relate not to the Curie point, but to the temperature at which an allotropic transformation occurs (Fe β, body-centred cubic $\rightarrow$ Fe γ, face-centred cubic). In the purest specimens this transition temperature lies about 160° above the Curie point, but in others the interval can apparently be very much smaller, so that the two temperatures can easily be confused. The volume change at the higher temperature is discontinuous, whereas that at the Curie point is continuous, and much smaller than the former. The magnitude of the effect is, in fact, almost of the same order as the ordinary magnetostriction, and it is necessary to show that the effect is not entirely due to the latter. The conclusion is reached that the effect cannot be entirely due to purely magnetic forces.

It is also necessary to revise the theoretical estimate of the value. In doing this, we make use of the modified theory, which has recently been given by Heisenberg*, for the case when there is more than one magnetic electron per atom (iron clearly requires such a theory). It is only possible to estimate the change of size roughly, but the correct observed values for iron and nickel are found to be of an order of magnitude acceptable to the theory. The larger erroneous values which Fowler and Kapitza had to account for would have put a great strain on the possibilities of the theory.

Finally, the relation of the Curie-point change of size to the magnetostriction at ordinary temperatures is considered. Data are given for iron and nickel.

§2. EXPERIMENTAL DATA

Magnetic measurements are most directly interpretable when made on single crystals, but unfortunately no experiments appear to have been made on the change of size of single crystals at the Curie point. We must therefore make use of measurements made on polycrystalline specimens.

(a) *Iron.* Benedicks† measured the linear expansion of a strip of pure electrolytic iron in the temperature range 700° C.–950° C. by a differential method, in which the expansion of the iron was compared with that of similar strips of gold or palladium, which have nearly the same coefficients of expansion as iron. Comparison of the curves obtained with gold and palladium shows that the expansion of these metals is quite regular in the temperature range considered. The size of the strip was 89 mm. × 4 mm. × 0·16 mm. The heating was effected by an electric furnace, which produced a magnetic field along the length of the strip of about 10 gauss (the field varied with temperature). Throughout all the measurements, the strip was under a tension of 34 gm./mm.². Rough measurements were made of the intensity of magnetization in the specimen, and the Curie point was found to be 768° C.

Part of one of the experimental curves is shown in Fig. 1. The ordinates represent the excess δl of the length l of the iron strip, over that of a gold strip, where $l = 89$ mm. It is seen that there is no discontinuity in length at the Curie point,

l, δl

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* *Probleme der modernen Physik*, p. 114 (Hirzel, Leipzig, 1928).
† C. Benedicks, *Journal of the Iron and Steel Institute*, **89**, 407 (1914).

but that in the magnetized state an increase in length, which disappears at the Curie point, is superposed upon the ordinary thermal expansion. It should be noted, however, that the coefficient of expansion does not actually become negative*. To find the magnitude of this change of length, it is necessary to produce back the expansion curves above the Curie point. This has been done for three of
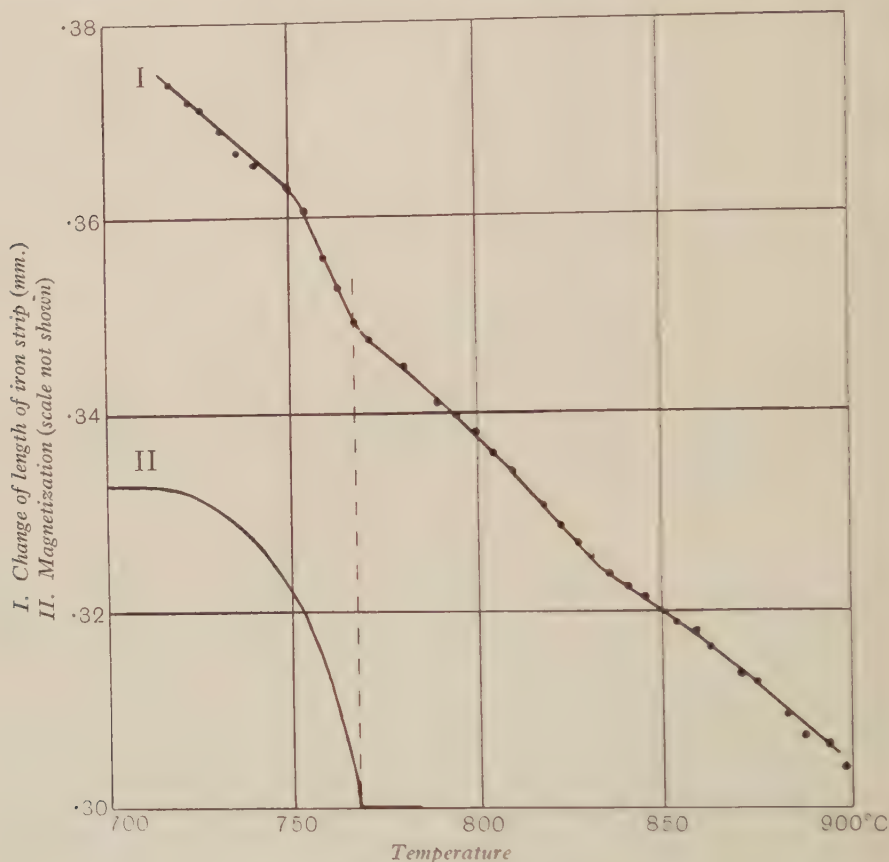
Fig. 1. Effects of temperature on iron.

Benedicks' curves, which have been produced back to 50° below the Curie point (it does not seem safe to extrapolate further), and the following value is obtained for the mean value of the increase in length in the magnetized state:

$$\delta l/l = + (1\cdot 2 \pm 0\cdot 4) \times 10^{-4}.$$

Benedicks also measured the ordinary magnetostriction at various temperatures, i.e. the change of length resulting from the application of the magnetic field. This was done very simply by the mere switching off of the current in the heating coils,

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* The well-marked change of slope at about 835° C. is interesting. Near this temperature there is also a change in the paramagnetic constants.

and observation of the change of length. As will appear later, we shall only require to know the value at $718°$ C. (i.e. $50°$ below the Curie point), which was found to be

$$\delta l/l = 6 \times 10^{-6}.$$

(b) *Nickel.* The coefficient of expansion of nickel in the neighbourhood of the Curie point was measured by Colby*, who used an electrolytically prepared specimen, $8\cdot2$ mm. in thickness, with a Curie point at $374°$ C. The specimen was heated by means of an electric furnace, the coils of which were so arranged as to produce no magnetic field, and the coefficient of expansion was measured at various temperatures by an optical method.
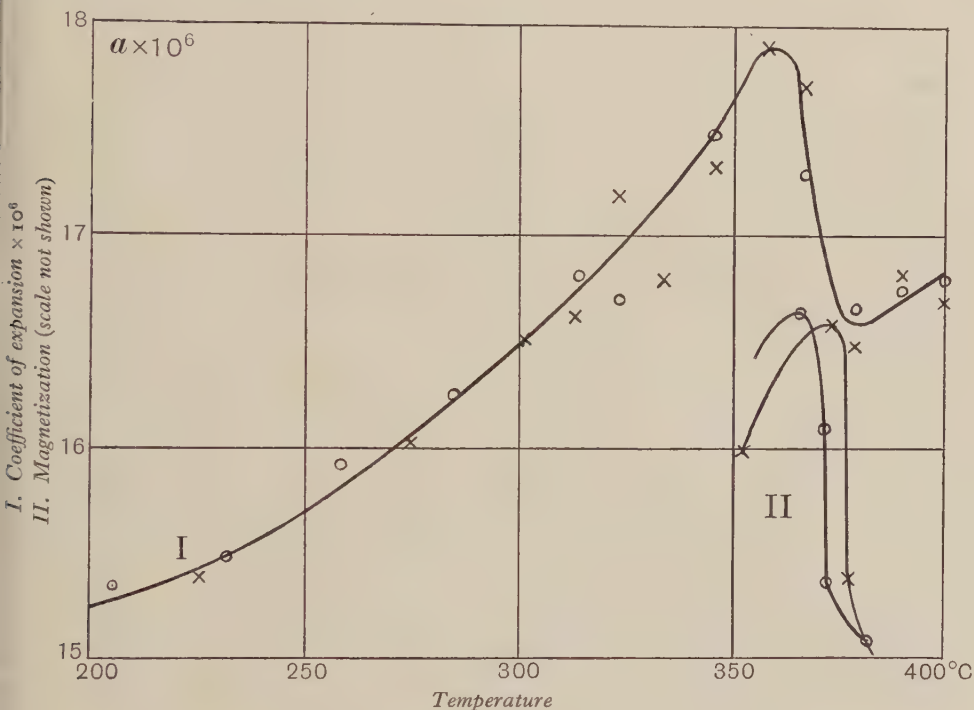
Fig. 2. Effects of temperature on nickel.

The results of two series of experiments are shown in Fig. 2. The ordinates represent the coefficient of expansion. It is seen that as in the case of iron, below the Curie point, nickel shows an anomalous change of length which disappears at the Curie point. But whereas iron shows an anomalous expansion, nickel shows an anomalous contraction in the magnetized state. The length is given by the area under the expansion curve, and the anomalous change of length is given by the area of the hump in the curve. At $220°$ C. the estimated change in length per unit length is as follows:

$$\delta l/l = - (0\cdot9 \pm 0\cdot3) \times 10^{-4}.$$

* W. F. Colby, *Phys. Rev.* **30**, 506 (1910).

§ 3. CORRECTION FOR MAGNETIC FORCES

The above results show that the Curie-point change of length is larger than the change produced by the application of a field at ordinary temperatures (the ordinary magnetostriction) but of the same order of magnitude. The variation of the latter change in different directions in a single crystal of iron is as great as 35×10^{-6} per unit length.

In this paper the assumption is made that the magnetostriction at ordinary temperatures arises from the purely magnetic interactions of dipoles and quadrupoles situated at the lattice points of the crystals; the dipoles and quadrupoles are assumed to be "rigidly" attached to each other with their axes parallel. Akulov[*] has shown that the magnetic interactions between atomic dipoles will account for the magnetostriction in single crystals in cases of saturated magnetization; and Mahajani[†] has shown that the assumption of orienting quadrupoles, rigidly attached to the dipoles, accounts satisfactorily for the observed deviations of the direction of magnetization in single crystals of iron from the direction of the applied field.

We have therefore to calculate the change of length which will occur at the Curie point when the ferromagnetic possesses (i) dipole energy and (ii) quadrupole energy, in addition to the usual (iii) elastic energy and (iv) thermal energy. In calculating the length changes arising from (i) and (ii), it is permissible to neglect (iv).

(i) *Dipole energy.* We consider the dipole energy of a polycrystalline specimen, and will suppose that it is split up into a large number of small regions in each of
I which the magnetization is saturated to intensity I, and approximately uniform. These regions will not necessarily be the small single crystals of which the specimen is composed; in fact, experimentally there appears to be little connection between their volume and the crystalline "grain size"[‡]. The dipoles in the neighbourhood
H_i of a lattice point will be approximately parallel, and the force H at the point will be given by

$$H_i = H_s + \tfrac{4}{3}\pi I + H,$$

H, H_s where H is the (bulk) magnetic field, and H_s is the magnetic field arising from the dipoles inside a small sphere surrounding the point.

U_d The energy U_d of the system is given by

$$U_d = -\tfrac{1}{2} \int (I, H_i)\, d\tau.$$

H itself arises from the system of dipoles, and will be proportional to I, so that we
α may write

$$|H| = \alpha |I|.$$

The value of α will vary from point to point, and will depend upon the shape and arrangement of the small fully magnetized regions. If, for example, the regions are infinite plane laminae, magnetized alternately parallel and anti-parallel to their common normal, then

$$\alpha = -4\pi.$$

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[*] N. Akulov, *Zeit. für Phys.* **52**, 389 (1928–9); **59**, 254 (1930).
[†] G. S. Mahajani, *Phil. Trans.* **228**, 63 (1929).
[‡] This conclusion is reached through a study of the Barkhausen effect.

But such an arrangement is very unlikely, as we should expect the regions to be such as to possess minimum magnetic energy, subject, perhaps, to conditions of minimum volume or maximum surface area per unit volume. The magnetic energy is actually a minimum when the regions are long filaments, magnetized along their length, for α then vanishes. In any case, we may replace α by its average value, which almost certainly lies between 0 and -4π. We obtain, therefore, for the energy

$$U_d = -\tfrac{1}{2}\int (I, H_s + \beta I)\, d\tau,$$

where β is a constant lying between $\tfrac{4}{3}\pi$ and $-\tfrac{8}{3}\pi$.

For a cubic crystal (as in the cases of iron and nickel), H_s will vanish. But in order to investigate the stresses arising from the dipole energy, we need to know the energy of an arbitrarily strained crystal, for which H_s will no longer vanish. H_s will, in general, be a linear function of the strain components, and the stress will be obtained by differentiation of U_d with respect to these strain components. In general, therefore, the stress, and hence the resulting strain, will be anisotropic, in correspondence with the fact that the magnetostriction varies in different directions in a single crystal.

Now consider, in particular, a polycrystalline specimen, unmagnetized in bulk. We may suppose that the crystal axes are oriented at random, and further that the directions of magnetization in the small magnetized regions are also at random. In this case, the strain produced by the magnetic forces must be isotropic, since all directions are equivalent, and we may therefore take

$$H_s = 0.$$

The energy is then

$$U_d = -\tfrac{1}{2}\beta I^2 V,$$

where V is the volume of the specimen.

The elastic energy ω is given by

$$2\omega = \frac{1}{\kappa_0}\left(\frac{\delta V}{V}\right)^2 V,$$

where κ_0 is the compressibility. The size is determined by

$$\frac{\partial}{\partial \delta V}(\delta U_d + \omega) = 0,$$

whence

$$\delta V/V = -\tfrac{1}{2}\kappa_0 \beta I^2,$$

and

$$\delta l/l = -\tfrac{1}{6}\kappa_0 \beta I^2.$$

For iron,

$$\kappa_0 = 0.6 \times 10^{-12},$$

$$I = 1710,$$

and therefore

$$|\,\delta l/l\,| < 2.6 \times 10^{-6}.$$

For nickel,

$$I = 510,$$

so that $\delta l/l$ is even smaller. Thus in both cases, the dipole forces produce no appreciable change of length in a polycrystalline specimen which is unmagnetized in bulk.

(ii) *Quadrupole energy.* We shall now deal with quadrupole forces, and again we need only know the quadrupole energy U_q of an unstrained cubic crystal. This has been calculated for the case of iron (body-centred cubic lattice) by Mahajani*, who found for the energy

$$U_q = \frac{\gamma}{d^5}\,(s_2^2 s_3^2 + s_3^2 s_1^2 + s_1^2 s_2^2 - \tfrac{1}{5}),$$

where s_1, s_2, s_3 are the direction cosines of the quadrupoles, d is the lattice constant, and γ is a constant.

For a polycrystal, unmagnetized in bulk, the direction (s_1, s_2, s_3) will be at random, and we must replace $(s_2^2 s_3^2 - s_3^2 s_1^2 - s_1^2 s_2^2)$ by its average value. Thus

$$U_q - \frac{\gamma}{d^5}\,(s_2^2 s_3^2 - s_3^2 s_1^2 - s_1^2 s_2^2 - \tfrac{1}{5}) = 0.$$

Thus the quadruple energy of a bulk-unmagnetized polycrystal vanishes, and so can lead to no change of length at the Curie point. We assume that this result holds also for nickel, with a face-centred cubic lattice.

We have found that the magnetic forces lead to no appreciable change of length at the Curie point when the state changes from the completely unmagnetized (i.e. paramagnetic) to the fully magnetized on the micro-scale, the specimen being still unmagnetized on the macro-scale. This was actually the change of state in Colby's experiments. But in Benedicks' experiments, the specimen was placed in a field of about 10 gauss, which at high temperatures is sufficient to produce nearly saturated bulk-magnetization; the mechanical tension in the specimen would also assist in the attainment of saturation. We must therefore add to the above correction that corresponding to the change-over from the bulk-unmagne-tized to the bulk-magnetized state. In the calculation of this change of length, it would no longer be sufficient to consider only isotropic strains, and it would be necessary to calculate U_q and U_q for an arbitrary strain†. But this calculation can be avoided, since the magnetostriction which Benedicks measured (viz. the change of length produced by application of the field) is precisely the required correction; this change of length must therefore be subtracted from the total change of length at the Curie point.

It is thus seen that the magnetic forces cannot account for the observed change of length at the Curie point. The values of this change, corrected for magnetic forces, are

(a) for iron,

$$\delta l/l = (+\,1\cdot1 \pm 0\cdot4) \times 10^{-4} \text{ at } T = 718° \text{ C.}; \quad T_c = 768° \text{ C.};$$

(b) for nickel,

$$\delta l/l = (-\,0\cdot9 \pm 0\cdot3) \times 10^{-4} \text{ at } T = 220° \text{ C.}; \quad T_c = 374° \text{ C.}$$

* *Loc. cit.*

† In the calculation of U_q, which has been made by Akulov (*loc. cit.*), H_c will no longer vanish.

§4. HEISENBERG'S MODIFIED THEORY

In its original form, Heisenberg's[*] theory applied to the case in which there is one effective electron per atom, and the changes of size at the Curie point have been calculated by Fowler and Kapitza[†], by the use of this form of the theory. Heisenberg[‡] has since modified his theory for the case of more than one effective electron per atom, and the modification can easily be introduced into the calculations of the Curie point change of length. No essentially new feature is introduced, and it is unnecessary to give the calculations in full detail. For details of the method, reference should be made to the paper by Fowler and Kapitza, whose notation is used here.

Heisenberg considers a crystal of $2n$ atoms, each with z "nearest neighbours." Each atom possesses y effective electrons, and the total number of electrons is $2N$, where

$$2N = 2ny.$$

It is assumed that the exchange interactions between electrons belonging to the same atom are much greater than those between electrons belonging to different atoms. It is further assumed that the exchange energy corresponding to a simple transposition of a pair of electrons belonging to neighbouring atoms has the same value J_0 for all such pairs, and is negligible for pairs of electrons belonging to atoms which are not closest neighbours. The mean energy $\bar{E}$ of all states of the system with the *eigen*-value s for the total spin of the electrons is found to be

$$\bar{E} = -zy\frac{s^2 + N^2}{2N}J_0 + \text{const.,}$$

and the mean square deviation $\overline{\Delta E^2}$ of the energy from its mean value is

$$\overline{\Delta E^2} = zy\frac{(N^2 - s^2)(3N^2 - s^2)}{4N^3}J_0{}^2.$$

If m is the component of s in the direction of the external magnetic field H, and μ is the mass of an electron, the energy of a state with *eigen*-values m and s is

$$\bar{E} + \overline{\Delta E} - \frac{eh}{4\pi\mu c}H.2m.$$

A Gaussian distribution of energy levels being assumed, the partition function K can now be formed, and reduced to the form

$$K = \sum_{m=-N}^{N} f_m \exp\left\{\left[\alpha + y\beta\left(1 - \frac{\beta}{z}\right)\frac{m_0}{N} + \frac{y\beta^2 m_0{}^3}{2zN^3}\right]m\right\}$$

$$\times \exp\left\{\frac{y\beta N}{2} + \frac{3y\beta^2 N}{8z} - y\beta\left(1 - \frac{\beta}{z}\right)\frac{m_0{}^2}{2N} - \frac{3y\beta^2 m_0{}^4}{8zN^3}\right\},$$

where

$$\alpha = \frac{1}{kT}\frac{eh}{2\pi\mu c}H;$$

$$\beta = \frac{zJ_0}{kT};$$

[*] W. Heisenberg, *Zeit. für Phys.* **49**, 619 (1928).   [†] *Loc. cit.*
[‡] *Probleme der modernen Physik*, p. 114 (Hirzel, Leipzig, 1928).

f_m f_m is the number of states with the *eigen*-value m for the component of s in a fixed
m_0 direction, and m_0 is the most probable value of m.

Now each atom has total spin $\frac{1}{2}y$, and its component in any direction can have
the values $\frac{1}{2}y, \frac{1}{2}y - 1, \ldots, -\frac{1}{2}y$. f_m is therefore the coefficient of ξ^m in

$$(\xi^{\frac{1}{2}y} + \xi^{\frac{1}{2}y-1} + \ldots + \xi^{-\frac{1}{2}y})^{2n}.$$

We obtain* for K,

$$K = [e^{xy} + e^{x(y-2)} + \ldots + e^{-xy}]^{2n} \cdot \exp\left[\frac{y\beta N}{2} - \frac{3y\beta^2 N}{8z} - y\beta\left(1 - \frac{\beta}{z}\right)\frac{m_0^2}{2N} - \frac{3y\beta^2 m_0^4}{8zN^3}\right],$$

x, ζ where

$$2x = \alpha + y\beta\left(1 - \frac{\beta}{z}\right)\zeta + \frac{y\beta^2}{2z}\zeta^3 \qquad \ldots\ldots(1),$$

and ζ is written for $m_0 N$, the ratio of the magnetization to the value when all the
electron spins are parallel. More conveniently,

$$\log K = 2n\left\{\log\left[e^{xy} + e^{x(y-2)} + \ldots + e^{-xy}\right] + \frac{1}{4}y^2\beta + \frac{3}{16}\frac{y^2\beta^2}{z}\right.$$

$$\left. - \frac{1}{4}y^2\beta\left(1 - \frac{\beta}{z}\right)\zeta^2 - \frac{3}{16}\frac{y^2\beta^2}{z}\zeta^4\right\}. \quad (2).$$

m_0 is given by $m_0 = \dfrac{\partial}{\partial\alpha}(\log K)$,

or $\zeta = \dfrac{1}{y} \cdot \dfrac{ye^{xy} + (y-2)e^{x(y-2)} + \ldots - ye^{-xy}}{e^{xy} + e^{x(y-2)} + \ldots + e^{-xy}} \qquad \ldots\ldots(3);$

the right-hand side is the new Langevin function. When $y = 1$ it reduces to
$\tanh x$.

Equations (1), (2) and (3) represent the modified theory, and in order to test
it I have calculated the change of specific heat at the Curie point. It will be seen
later that it is permissible to neglect all terms containing z explicitly, and with
this simplification the specific heat changes are found to be:

(a) For iron, the value of y being taken as 3, $\Delta C = 2 \cdot 2R = 4 \cdot 4$ calories per
degree per gram-atom.

The observed value† is 6·8.

(b) For nickel, the value of y being taken as 1, $\Delta C = 1 \cdot 5R = 3$ calories per
degree per gram-atom.

The observed value† is 1·7.

The agreement is unsatisfactory, and suggests that the assumptions upon which
the theory is based are unsound. In order to account for the high value for iron,
it seems necessary to suppose that the electrons are free to orient individually,
and not in groups of y as in the present theory. This condition would obtain if, as

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* Heisenberg obtains the equation

$$K = F \cdot (e^{xy} + e^{x(y-1)} + \ldots + e^{-xy})^{2n},$$

which disagrees with the expression given above. I think that Heisenberg's expression is incorrect,
since it contains both odd and even powers of e^x, although it is derived from an expression containing
only even powers.

† See Fowler and Kapitza, *loc. cit.*

as been suggested*, the electrons involved are conduction electrons, but the suggestion involves several difficulties. For example, it cannot explain the ferro-magnetism of minerals which are relatively bad conductors, although some of these also show large specific heat changes at the Curie point.

In the paramagnetic state, when ζ and x are small, the equations become

$$2x = \alpha + y\beta\,(1 - \beta/z)\,\zeta\,\}$$
$$\zeta = \tfrac{1}{3}\,(y+2)\,x \qquad \}$$

whence

$$\zeta = \frac{\alpha}{6/(y+2) - y\beta\,(1 - \beta/z)},$$

and therefore the value β_c of β at the Curie point is given by

$$\beta_c = \frac{z}{2}\left[1 - \left\{1 - \frac{24}{zy\,(y+2)}\right\}^{\frac{1}{2}}\right] \qquad \ldots\ldots(4),$$

and the corresponding Curie temperature T_c is given by

$$T_c = \frac{2J_0}{k}\left[1 - \left\{1 - \frac{24}{zy\,(y+2)}\right\}^{\frac{1}{2}}\right]^{-1} \qquad \ldots\ldots(5).$$

As a necessary condition for ferromagnetism, we must have

$$z > 24/y\,(y+2).$$

This condition is satisfied by all types of lattice when $y > 1$.

When z is large, these formulae become

$$\beta_c = \frac{6}{y\,(y+2)} \qquad \ldots\ldots(4'),$$

$$T_c = \frac{zJ_0}{k}\cdot\frac{y\,(y+2)}{6} \qquad \ldots\ldots(5'),$$

and the formula for ζ just expresses the Weiss law, with the gram-molecular susceptibility χ_M, where

$$\chi_M = \frac{n_0}{k}\frac{y\,(y+2)}{3}\left(\frac{eh}{4\pi\mu c}\right)^2\cdot\frac{1}{T - T_c},$$

n_0 being Avogadro's number. This is the usual expression.

[§ 5. THE CURIE-POINT CHANGE OF SIZE

To take into account the forces which are not involved in the magnetic energy, we use the corrected partition function K' given by

$$\log K' = 2n\left\{\log\left[e^{xy} + e^{x\,(y-2)} + \ldots + e^{-xy}\right] - \frac{1}{4}y^2\beta\left(1 - \frac{\beta}{z}\right)\zeta^2 - \frac{3}{16}\frac{y^2\beta^2}{z}\zeta^4\right\} - \frac{F}{kT},$$

where F is the rest energy of the crystal. We neglect the energy of the thermal motion.

* J. Dorfmann and R. Jaanus, *Zeit. für Phys.* 54, 277 (1929); J. Dorfmann and I. Kikoin, *ibid.* 54, 289 (1929).

The size is determined by

$$\frac{\partial}{\partial V} \log K'(H, V, T) = 0,$$

or

$$2ny^2 \frac{\partial \beta}{\partial V} \left\{ \frac{1}{4} \left(1 - \frac{2\beta}{z}\right) \zeta^2 + \frac{1}{8}\frac{\beta}{z} \zeta^4 \right\} - \frac{1}{kT}\frac{\partial F}{\partial V} = 0.$$

Above the Curie point the first term vanishes, and therefore the increase in volume in the magnetized state is given by

$$\frac{\delta V}{V} = 2nzy^2\kappa_0 \frac{\partial J_0}{\partial V} \left\{ \frac{1}{4}\left(1 - \frac{2\beta}{z}\right) \zeta^2 + \frac{1}{8}\frac{\beta}{z} \zeta^4 \right\} \qquad \ldots\ldots(6),$$

κ_0 where κ_0 is the compressibility, given by

$$\frac{1}{\kappa_0} = V \cdot \frac{\partial^2 F}{\partial V^2}.$$

§ 6. NUMERICAL ESTIMATES FOR IRON AND NICKEL

It has not yet proved possible to calculate the exchange energy J_0, which must therefore be regarded as a constant to be determined experimentally; for this purpose equation (5) is convenient. $\partial J_0 / \partial V$ is also unknown, but we can estimate its order of magnitude when the magnitude of J_0 is known. We shall use (6) to determine $\partial J_0 / \partial V$.

(*a*) *Iron*. It is clear from equation (6) that the sign of $\partial J_0 / \partial V$ determines the sign of $\delta V/V$. Now J_0 is positive, and it is surprising, as Fowler and Kapitza pointed out, to find that for iron $\delta V/V$, and hence $\partial J_0 / \partial V$, are positive also. But they did not point out an important consequence of this fact. In Heisenberg's theory it is assumed that the exchange energy corresponding to an exchange of electrons belonging to atoms other than "closest neighbours" is negligible, the supposition being that J_0 decreases rapidly (exponentially) with interatomic distance. This must certainly be true for large enough distances, but it is now seen that J_0 is, in fact, increasing at the interatomic distance of closest neighbours. Iron has a body-centred cubic lattice, and each atom has 8 closest neighbours at distance $d\sqrt{3}/2$, where d is the lattice constant. There are 50 more atoms at distance $d\sqrt{3}$ or less. If J_0 is increasing at distance $d\sqrt{3}/2$, it seems highly improbable that the effect of these 50 other atoms can be neglected. This conclusion, that at small distances the strength of homopolar binding does not decrease so rapidly as is commonly supposed, is important in problems of crystal structure.

It appears, therefore, that in the preceding formulae we must suppose z to be considerably greater than 8. Now z occurs explicitly in formulae (1) to (4) on account only of the Gaussian distribution of energy levels, and the effect of increasing z is to decrease the effect of the Gaussian distribution. To the order of accuracy which we require here, we can neglect the Gaussian distribution altogether, for the numerical errors involved are at most 10 per cent. This results in a considerable simplification of the formulae, such as was made in the construction of equations (4′) and (5′).

The data for iron are as follows:
$$T_c = 768° \text{ C.}; \quad \kappa_0 = 0.6 \times 10^{-12}; \quad 2n/V = 8.6 \times 10^{22}.$$
The most probable value of y is 3.

At $718°$ C. $\zeta = 0.4$, from (1) and (3),
$$\delta l/l = + 1.1 \times 10^{-4},$$
and therefore the volume change $\delta V/V$ is
$$+ 3.3 \times 10^{-4}.$$
Substituting these values in (6), we get
$$z\, \partial J_0/\partial V = 1.7 \times 10^{-14}.$$
From (5′)
$$z J_0 = 5.7 \times 10^{-14}.$$
The distance a between an atom and its nearest neighbour is
$$\tfrac{1}{2}\sqrt{3} \times 2.8 \times 10^{-8},$$
and the binding force $\partial J_0/\partial a$ is given by
$$z\,\frac{\partial J_0}{\partial a} = 2.1 \times 10^{-6}.$$

(b) *Nickel.* For nickel, $\partial J_0/\partial V$ is negative, but it is reasonable to suppose that in this case also we ought to take z greater than the number of *closest* neighbours, which is 12 since nickel has a face-centred cubic lattice. We shall therefore again neglect the effect of the Gaussian distribution.

The data are as follows:
$$T_c = 374° \text{ C.}; \quad \kappa_0 = 0.6 \times 10^{-12}; \quad 2n/V = 9.4 \times 10^{22}; \quad y = 1.$$
At $220°$ C.
$$\zeta = 0.7 \quad \text{and} \quad \delta l/l = -0.9 \times 10^{-4},$$
and therefore
$$\delta V/V = -2.7 \times 10^{-4}.$$
Substituting these values in (6), we get
$$z\,\frac{\partial J_0}{\partial V} = -3.9 \times 10^{-14}.$$
We have also
$$z J_0 = 1.8 \times 10^{-13}.$$
The distance a between an atom and its closest neighbours is
$$2^{-\frac{1}{2}} \times 3.5 \times 10^{-8},$$
and the binding force $\partial J_0/\partial a$ is given by
$$z\,\frac{\partial J_0}{\partial a} = -4.7 \times 10^{-6}.$$

Theoretical estimate of $\partial J_0/\partial a$. For two hydrogen atoms at distance a, Heitler and London[*] obtain for J_0
$$J_0 = \sum_{n=-1}^{3} A_n\, (a/a_0)^n \cdot e^{-2a/a_0},$$

[*] W. Heitler and F. London, *Zeit. für Phys.* **44**, 455 (1927).

a_0

where the coefficients A_n are all of the same order of magnitude, and a_0 is the "radius of the electron orbits." Let us assume an expression of the same form for the exchange energy which enters into ferromagnetism. For the outer orbits in a crystal, a/a_0 probably lies between 1 and 2, and therefore $(a/J_0)\partial J_0/\partial a$ is of order of magnitude 1. It is impossible to determine the sign, but it might well be positive.

For iron and nickel the values of $(a/J_0)\partial J_0/\partial a$ are 0·9 and $-$0·6, and therefore the changes of length of these metals at the Curie point are of the order of magnitude required by the theory.

§ 7. RELATION TO MAGNETOSTRICTION AT ORDINARY TEMPERATURES

It has been pointed out that the Curie-point change of size is a continuous change, reaching completion only when the magnetization ζ becomes 1, and therefore extending over all temperatures below the Curie point. It is necessary to consider whether it can affect magnetostriction at ordinary temperatures.

In the magnetization of a ferromagnetic by the application of an external magnetic field, the primary effect of the field is to orient the direction of magnetization in each of the fully magnetized regions we have already discussed, the completion of the process being marked by the attainment of bulk-saturation. During the process, the small-scale magnetization does not alter appreciably. Further increase of the external field leads to a very small linear increase of ζ.

The greater part of the magnetostriction takes place during the first stage, i.e. before saturation is reached. The Curie-point change of size, being isotropic, is unaffected by changes of orientation, and so makes no contribution to the magnetostriction in the first stage. After saturation has been reached, the magnetostriction varies linearly with ζ^2, the greater contribution coming from the Curie-point change. The variations of the magnetostriction above saturation ought therefore to be approximately isotropic, even in single crystals, unlike the magnetostriction below saturation. The effect, however, is small, and I have been unable to find any experimental data.

§ 8. ACKNOWLEDGMENT

THE INTERRELATIONS OF MAGNETIZATION AND TEMPERATURE IN CRYSTALS

By W. PEDDIE

ABSTRACT. The paper deals with the question of an equation of thermomagnetic state. The possible existence of such an equation, in a form analogous to Van der Waals' thermo-mechanical equation of state, was indicated by Curie, and has been explicitly formulated and tested by Dr Ashworth. Two equations of a type different from that of Ashworth are discussed.

§ 1

THE interrelations of magnetization and temperature in crystals, as in other bodies, can be worked out only in terms of the kinetic theory. In the cases of gases, liquids, or solutions, the difficulties are of no greater order than are those which beset the ordinary applications of the kinetic theory. But it is quite a different matter when we have to deal with a magnetic crystal, or a random congeries of crystals. Applications have been made by Frivold, but the mathematical difficulties were such as to compel limitation to extreme cases.

In addition to the mathematical difficulties, there is also uncertainty arising from ignorance of the constitution of molecules or atoms in respect of the origin of their magnetic qualities. Do the electrical circulations which give rise to these consist in the orbital motions of the exterior electrons, or have they a nuclear origin? Are they sub-orbital, consisting in the spin of the electrons themselves? There are some indications that the latter view may be correct.

§ 2

Just as in the initial development of the kinetic theory of gases, it was found necessary as a first step to postulate that the atoms were hard, smooth, spherical, elastic bodies, small in relation to their average distance apart; so, in magnetic theory, we have the initial postulate that the equivalent atomic magnets can be regarded as ideal magnets, of constant magnetic moment, and small in relation to their least distance apart on the crystalline lattice. And, just as these tremendously restrictive postulates of the theory of gases led, nevertheless, to important correspondences with observed results, so the immense restriction of the magnetic postulates does not prevent general applicability of the results to the properties of magnetic crystals. Difficulty in connection with the last postulate disappears if the

W. Peddie

electrical circulation is that of the inner electrons, or is on a nuclear or sub-nuclear scale. And a test of the nature of the atomic, or sub-atomic, conditioning of the circulation may be founded on a comparison of observed effects with results obtained by retention, on the one hand, of the severely restrictive initial postulates, or, on the other hand, by their modification.
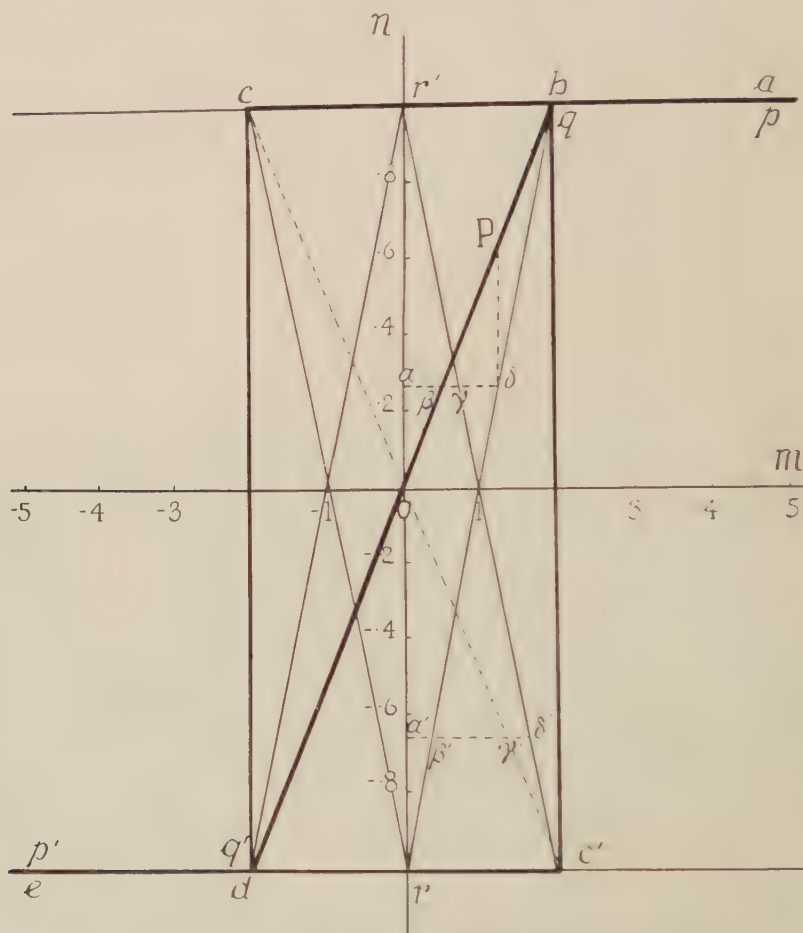
Fig. 1. Degree of saturation for varying external fields, when $l = 0$.

§ 3

The general equation of thermomagnetic state, i.e. the law relating magnetic quality to the strength of the external field and to temperature, is, as Frivold found, outside present possibilities of attainment. It is therefore necessary to proceed by a process of averaging, just as Waterston and Joule did in ordinary kinetic theory. And, just as their procedure led to qualitatively correct results, so, in the case of magnetism, qualitative correspondence with observation may perhaps be expected.

If H be the component of the external field in the direction of uniform mag-
netization, while $F \cos \phi$ is the similar component of the internal field arising from
the action of all the other co-directed molecular or atomic magnets upon any one of
their number situated on the crystalline lattice, supposed to be infinitely extended
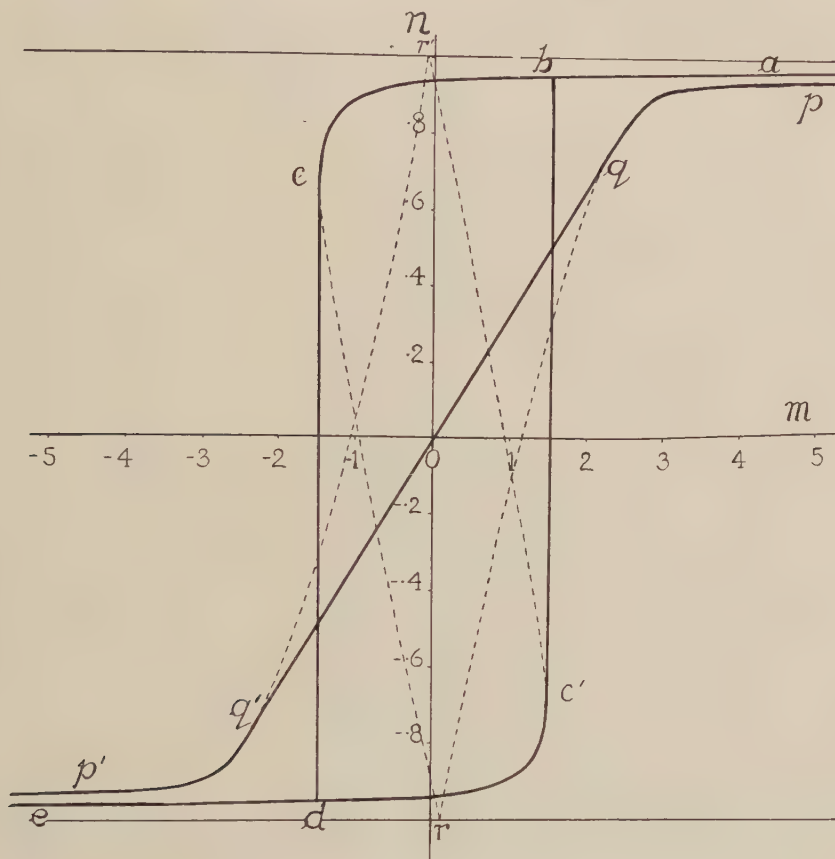
Fig. 2. Degree of saturation when $l = 0 \cdot 1$.

and homogeneous, the work done by thermal action in deflecting every magnet
from alignment with that direction through an average angle ϕ, is

$$- \int_{\infty}^{\phi} (H + F \cos \phi) \, dM_0 \cos \phi,$$

where M_0 is the fixed magnetic moment of each magnet supposed to be ideal. On
the presumption that there is equipartitioning of energy amongst the magnetic
and thermal freedoms this gives, provided that F is independent of ϕ,

$$HM_0 (1 - \cos \phi) + FM_0 \tfrac{1}{2} (1 - \cos^2 \phi) = kT,$$

where k is the corresponding atomic gas constant and T the absolute temperature.
If the condition of equipartition be not satisfied, k may differ from that constant,

27–2

and the results of observation seem to indicate that it has actually a considerably smaller value. The field F, proportional to M_0, is fixed whenever the space lattice and its mode of occupation by the atoms is given along with the direction of magnetization with reference to the lattice framework. We may therefore put

f
$$F = 2M_0 f,$$

and so obtain
$$HM_0 \left(1 - \cos \phi\right) + M_0^2 f \left(1 - \cos^2 \phi\right) = kT$$

Fig. 3. Degree of saturation when $l = 0.2$.

as the equation of thermomagnetic state on the assumption that we are dealing with an infinitely extended homogeneous magnetic crystal, and that the action of each magnetic atom is representable by that of a single small ideal magnet suitably situated.

I, I_0

Now ϕ being the average inclination of the magnets to the direction of magnetization, we have $\cos \phi = I/I_0$, where I is the intensity of magnetization and I_0 is its saturation value. Thus the equation becomes

$$HM_0 \left(1 - I/I_0\right) + M_0^2 f \left(1 - I^2/I_0^2\right) = kT.$$

The value of f is calculable whenever the lattice is specified. There is a cone of directions, symmetrically related to the lattice, in which f is zero. Within that cone it is positive, and the internal field aids magnetization; outside it, the internal field opposes magnetization.
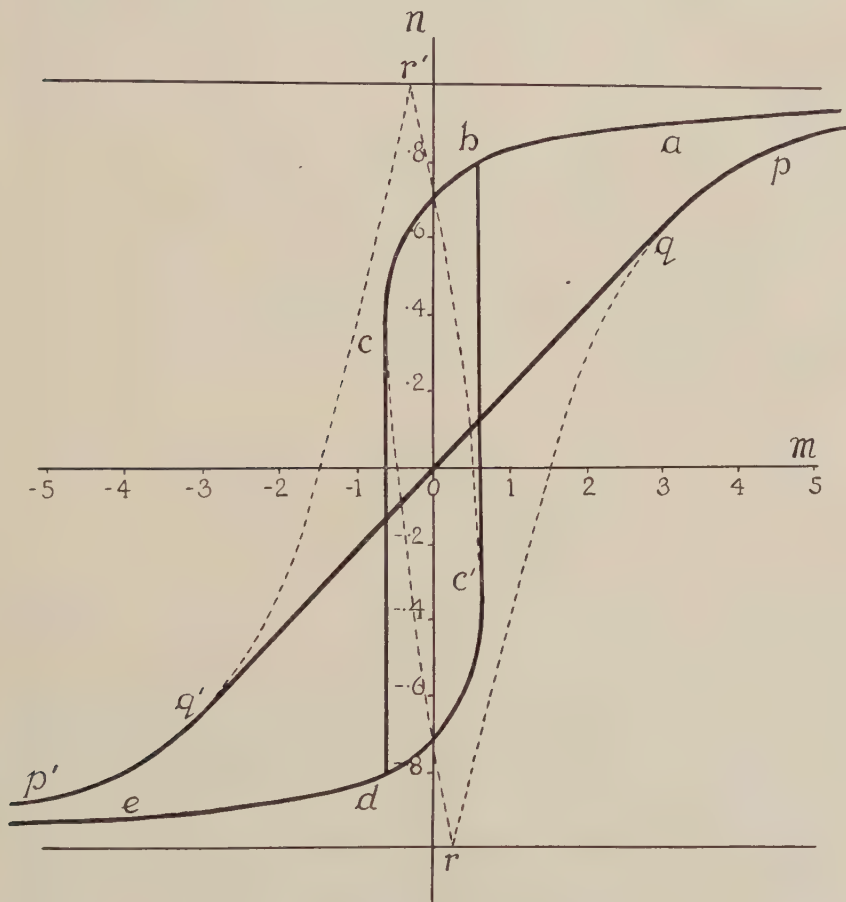
Fig. 4. Degree of saturation when $l = 0.5$.

§ 4

The above equation is representable by a hyperbola referred to oblique axes if values of I/I_0 be plotted against values of H. In the accompanying figures the curve *abcr* represents the applicable portion, which must lie between the limits $n = \pm 1$, of the hyperbola

$$n^2 + mn = m + 1 - l,$$

where $m = H/M_0 f$, $n = I/I_0$ and $l = kT/M_0^2 f$. The values of l which are used in the various figures are given thereunder. The curve *edc'r'*, obtained by rotation of *abcr* through 180° round o, applies when the field and the magnetization are

m, n, l

reversed. The portions cr and $c'r'$ of the curves, having negative slope, correspond to instability, and therefore not to actual conditions. If we take initially a strong positive field and magnetization, and gradually reduce both, the part abc is followed until, at c, instability ensues and the rectilinear part cd is instantaneously described. The ordinate Ob represents the residual magnetization, and the abscissa at c gives the coercive force. Negative magnetization proceeds by the path de; and, on withdrawal and final reversal of the negative field, by the path $edc'ba$. The closed loop $bcd'c'b$ is the hysteresis loop.

One postulate made in the evaluation of f, taken as positive in the direction of magnetization, is that all the elementary magnets are averagely co-directed. Thus a condition of zero magnetization in zero field is, at sufficiently low temperature, one of instability. In an actual crystal, however, non-homogeneity on a sufficiently small scale probably brings about a step-by-step passage of small scale groups of magnets throughout the whole volume, to final stability, in such a way as to give magnetization proportional to the external field. This is a condition observed in single magnetic crystals.

<div align="center">§ 5</div>

If, in the equation, the sign of f be reversed, we have

$$n^2 - mn = 1 + l - m,$$

the corresponding curves pqr and $p'q'r'$ being shown in the diagrams. No hysteresis can be manifested. If the hypotheses adopted corresponded to the conditions in a randomly crystalline material, it would follow that cyclical dissipation of energy through hysteresis only occurs throughout a fraction of the whole material.

Just as the portion cr of a hyperbola $abcr$ is not described by the representative point, so the portion qr of a hyperbola pqr is not described. Thus, for example, at the point r the internal field and the action of temperature tend towards positive magnetization, while the external field is either zero ($l = 0$) or also tends towards demagnetization. Again, at the point where pqr crosses the m-axis, although there is no magnetization and therefore no internal field, the positive external field prevents the crossing point from being a position of equilibrium. The external field corresponding to any given position on qr compels displacement of the representative point parallel to On until it reaches the line Oq which is tangential to pqr at q. The general conditions are readily traceable in Fig. 1 ($l = 0$).

Here the hyperbola coincides with the asymptotes qp and qr. At the point β, the positive magnetizing field $\alpha\beta$ is co-directed with the internal field $\alpha\gamma$, so that the total field $\alpha\delta$ and the negative magnetization are destroyed. At the point δ', representative of positive magnetization, the positive external field $\alpha'\delta'$ exceeds the demagnetizing field $\alpha'\beta'$, so that the resultant field $\beta'\gamma'$ causes additional magnetization, the course of which is represented by $\delta'P$. P is the position of stability, under the given external field, at which the external field is balanced by the internal field. The line qq', together with the highly stable ranges qp and $q'p'$, extending to

infinity, represent the whole course of the tracing point when the internal field opposes the magnetization at zero absolute temperature.

In Fig. 2 ($l = 0.1$) the value of the external field which corresponds to any given magnetization when the internal field opposes it is greater than that shown in the first figure. The difference of the corresponding abscissae in the two diagrams is
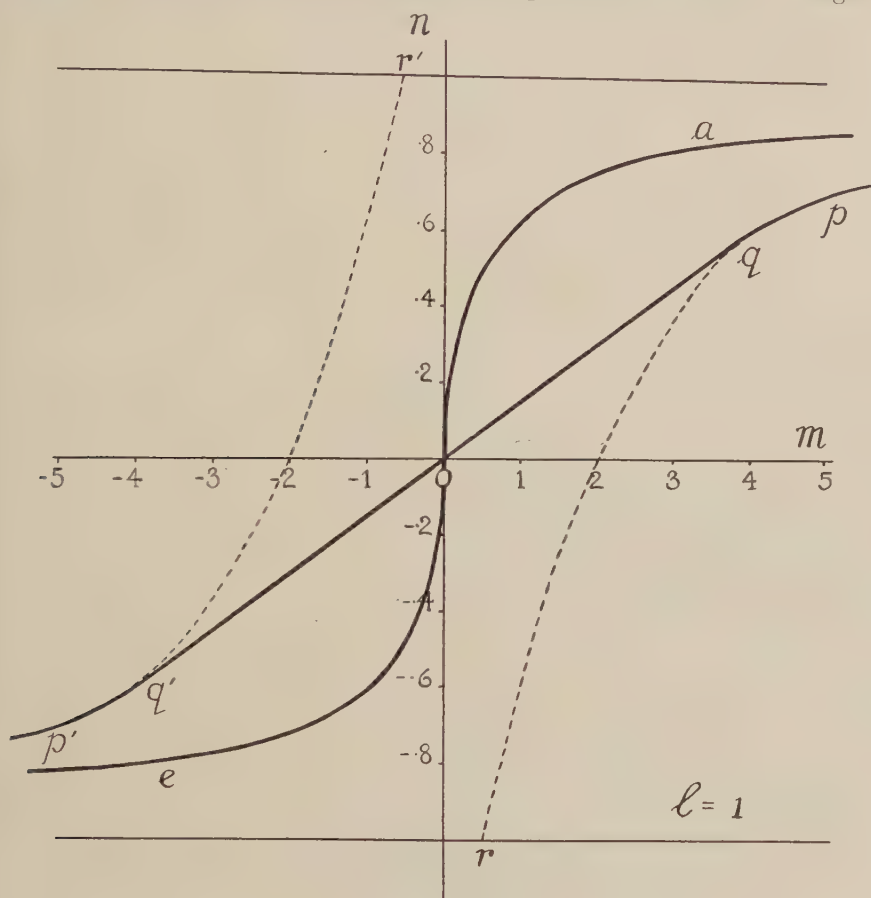
Fig. 5. Degree of saturation when $l = 1$.

the field equivalent of the temperature effect. Reasoning similar to that previously given indicates that the tracing point leaves the hyperbola at the point q and that Oq, tangential to pqr, gives the course of magnetization at weaker fields.

This seems to be in accordance with the results of observations on magnetic crystals. The parts of the observed paths adjacent to the origin appear to be linear, and to leave the curved paths tangentially.

Hysteresis still occurs when the internal field aids magnetization, but is reduced in magnitude. This reduction goes on progressively as the temperature rises until, finally, the hysteresis vanishes when, at $l = 1$, Fig. 5, the critical temperature is reached.

At a temperature higher than the critical value, e.g. l - 2. Fig. 6, both curves present the anhysteretic character and possess an initial rectilinear part.
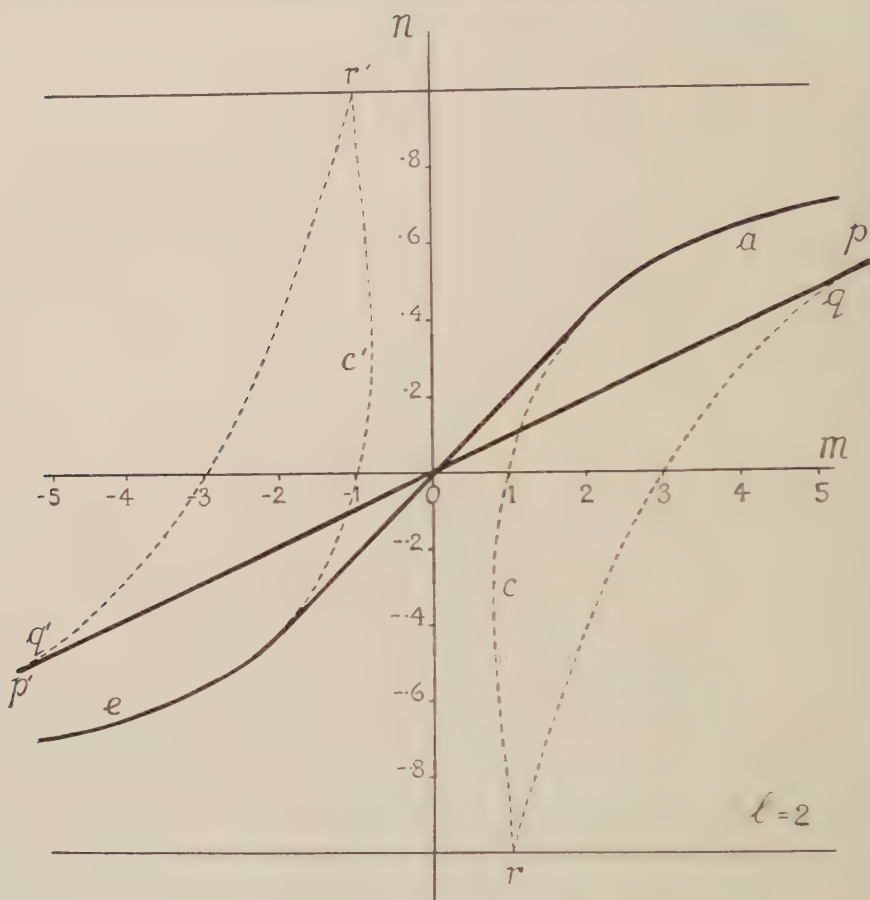
Fig. 6. Degree of saturation when $l = 2$.

§ 6

It appears that a good general account of the phenomena of magnetization in crystals, including cases involving instability, can be given by means of a postulate which involves an equation of the second degree only.

The equation

$$l = m (1 - n) \pm (1 - n^2)$$

is a reduced equation of thermomagnetic state in which the critical field is half of the maximum internal field, while the critical temperature is that at which the average molecular translatory energy is equal to the maximum work of reversal of magnetization in the critical field. Neither of these quantities is definite until the direction of magnetization in the crystal and the nature of the lattice is specified.

The mean of all the individually possible values would be the observable quantities in a randomly crystalline solid.

§ 7

It is necessary to note that the square of the quantity a_0, the semi-length of the molecular magnet, enters into the expression for the factor f in a cubic crystal. In the preceding work this has been postulated to be constant; or, rather, the postulate is that Ma_0^2, where M is the effective moment, is proportional to I, for it is made to involve $\cos\phi$. Now if we deal with an ideal magnet of moment M_0 which maintains randomly an angle ϕ with the direction of magnetization, the effective moment is $M_0 \cos\phi$. But the effective value of a_0 is also altered; and, if we take it too as being proportional to $\cos\phi$, the internal field is proportional to $\cos^3\phi$.

The effect of that assumption, together with another, is detailed in a paper published in 1927*. In equation (9) of that paper,

$$\left(m + \frac{n^2}{3}\right)\left(l - \frac{n}{3}\right) = \frac{8l}{9},$$

the factor 9 in the denominator has inadvertently been omitted, and this affects the scale of the accompanying figure.

The above equation is a generalized equation of state, which presents strong analogies to van der Waals' equation of thermodynamic state, but nevertheless exhibits fundamental differences. One of the resemblances is that the factor $8/27$ enters into the expression for the critical temperature; another is that the equation is a cubic in n. But the above equation was only put into that form by the choice of units which depend on the direction of magnetization. This limitation can however readily be removed.

If we assume that the molecular constitution is such that the internal field contains a term involving $\cos\phi$ as well as one involving $\cos^2\phi$, we can write the fundamental postulate as

$$kT = HM_0 (1 - \cos\phi) + M_0^2 a_0^2 f [\beta (1 - \cos^3\phi) + \alpha (1 - \cos^2\phi)],$$

and so obtain, by putting

$$\cos\phi = n, \quad H = mM_0 a_0^2 f\beta, \quad kT = lM_0^2 a_0^2 f\beta,$$

the reduced equation

$$n^3 + n^2 \frac{\alpha}{\beta} + nm - \left(\frac{\alpha}{\beta} + 1 + m - l\right) = 0.$$

The conditions for three equal roots, n_c, are

$$3n_c = -\alpha/\beta, \quad 3n_c^2 = m_c = \alpha^2/3\beta^2, \quad l_c = (1 + \alpha/3\beta)^2.$$

If we now take $\alpha = -\beta$, with $\beta = 1$, we find

$$I_c = \tfrac{1}{3}I_0, \quad H_c = \tfrac{1}{3}M_0 a_0^2 f, \quad kT_c = \tfrac{8}{27}M_0^2 a_0^2 f,$$

or, taken per unit volume,

$$T_c = \frac{8}{27}\frac{I_0}{R} \cdot M_0 a_0^2 f.$$

* W. Peddie, *Proc. R.S.E.* **47**, 165 (1927).

Now Ashworth, in connection with his discussion of the thermomagnetic equation of state*, as based on the analogy of van der Waals' equation, finds†

$$T_c = \tfrac{8}{27} I_0 b,$$

b where b differs very little in numerical value from a small integer in the cases of iron, nickel, cobalt, and Heussler's alloy. But $M_0 a_0{}^2 f$ is the maximum value of the internal field, i.e. the value at zero absolute temperature; so that, on this view, Ashworth's result would appear to indicate that, in these substances, the maximum values of the internal fields are small integral multiples of the gas constant taken per unit volume. This points to fields of the order of 10^7 gauss, if there be equipartition amongst the thermal and the magnetic freedoms. But, if the latter be sufficiently shielded, the fields may be of ordinary magnitude.

* J. R. Ashworth, *Phil. Mag.* **30**, Nov. 1915; **33**, April 1917.
† J. R. Ashworth, *Nature*, Sept. 12, p. 397 (1925).

THE ATOMIC MOMENTS OF IRON COBALT AND NICKEL AS DETERMINED FROM THE MAGNETIC SATURATION OF THE FERRO-COBALTS AND NICKEL-COBALTS*

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ABSTRACT. The atomic moments of iron, cobalt and nickel have been deduced from saturation data relating to 25 ferrocobalts and 9 nickel-cobalts at ordinary temperatures and to their temperature-coefficients down to the temperature of liquid air. It is found that each of these moments may have several different values. The atomic moments of the alloys, as functions of atomic composition, exhibit discontinuities which are connected with changes in crystal structure.

§ 1. THE FERROCOBALTS

Preuss† has found that alloys whose composition is in the neighbourhood of Fe_2Co are more highly magnetizable than pure iron, their intensity of magnetization per unit volume at saturation being greater by about 10 per cent. This result has been confirmed and even exceeded by our own observations. For alloys containing from 35 per cent. to 45 per cent. of cobalt the saturation intensity per unit volume is practically constant, and is 12·4 per cent. greater than that of iron. The alloy with 28 per cent. of cobalt gives a practically equal difference, namely 12 per cent., in spite of the considerably lower proportion of cobalt.

The absolute saturation n, shown in the upper full-line curve in Fig. 1 as a function of atomic composition, may be defined as the average number of magnetons per atom of alloy, i.e. the specific intensity at saturation, at the absolute zero of temperature, divided by the number of gram-atoms of iron and cobalt per unit mass and by the magneton number 1125·6. n

The absolute saturation follows a law less simple than the two linear relations between Fe and Fe_2Co and between Fe_2Co and Co which were given by Preuss. In order that the absolute magnetization $\sigma_{0, \infty}$ may be derived from the specific magnetization $\sigma_{T, H}$ which obtains in a field H at absolute temperature T, a double extrapolation for $T = 0$ and $H = \infty$ must be effected. The formulae employed for this purpose are σ, T, H

$$\sigma_{T, H} = \sigma_{T, \infty} (1 - a/T),$$

and
$$\sigma_{T, \infty} = \sigma_{0, \infty} (1 - AT^2),$$

where a and A are constants. The experimental data concerned in these two formulae are shown by the two lower broken-line curves in Fig. 1. The extrapolated a, A

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* Translated by the Editor.   † *Zurich thesis* (1912).

differences, which amount to less than 0·2 per cent., do not give rise to appreciable uncertainty.

The properties of the ferrocobalts are complicated by the fact that they crystallize in three different crystal lattices. At low temperatures for alloys containing from 0 per cent. to 78 per cent. of Co the stable structure is the centred cube (α-ferrocobalt), for those containing from 78 to 95 per cent. it is the face-centred cube (γ-ferrocobalt), and for those containing from 95 to 100 per cent. it is a hexagonal lattice of maximum density (H-ferrocobalt)*.
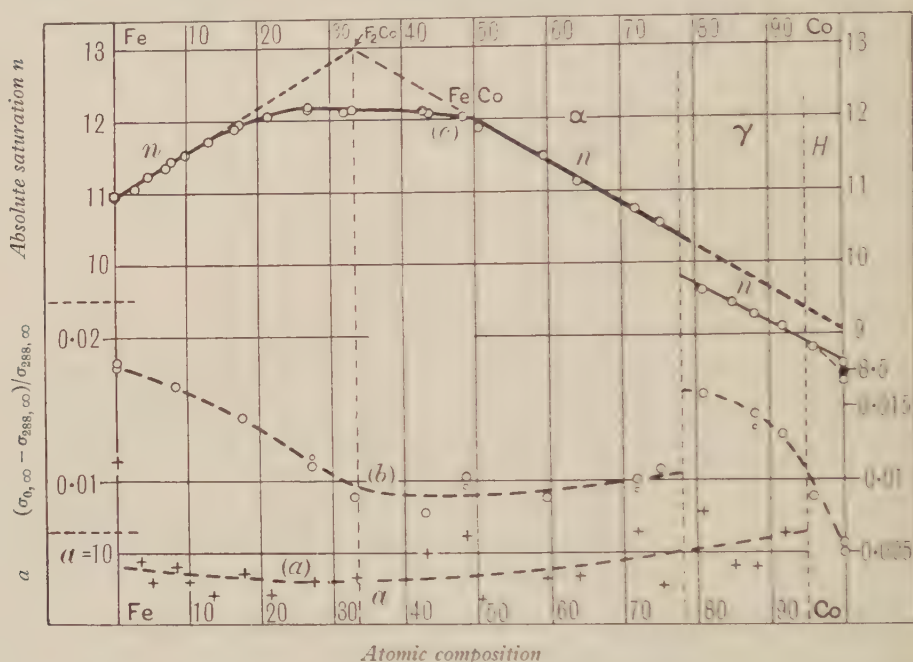
Fig. 1. Ferrocobalts: (a) the temperature-coefficient a; (b) the quantity $(\sigma_{0,\infty} - \sigma_{288,\infty})/\sigma_{288,\infty}$, where the first suffix of σ denotes T and the second H; (c) the absolute saturation n.

When an alloy is produced by the mixture without modification of two metals of fixed atomic moment, the mean moment varies linearly with the composition. A non-linear variation indicates a condition in which at least three moments are concerned; for instance, it may arise from the presence of one constituent of fixed atomic moment and another occurring in variable proportions in two different states characterized by different moments.

There are in the α-ferrocobalts two regions in which the moment varies in a strictly linear manner, namely those in which there is from 0 per cent. to 13 per cent. and from 50 to 78 per cent. of cobalt. They are separated by a region, from 13 to 50 per cent., in which the variation is non-linear at high saturation.

The two straight lines meet at a point whose abscissa is 33·3 per cent., corresponding to Fe₂Co, while its ordinate represents 13 magnetons; their gradients are

* Hakar Masumoto, *Tohoku Sci. Rep.* **15**, 449 (1926).

equal and opposite. The first straight line begins at the known iron-point at 11 magnetons and, if we assume that the other constituent with fixed moment is cobalt itself and not a compound, we find by extrapolation a form of cobalt with 17 magnetons.

The second straight line reaches the abscissa corresponding to 100 per cent. of cobalt at the point where $n = 9$, with a degree of accuracy limited by the difficulties of analysis and by the fact that resort must be had to extrapolation beyond the α region. Cobalt, in α-ferrocobalts rich in that element, has therefore an atomic moment of 9 magnetons. If the straight line be produced as far as the iron ordinate, a form of iron with 15 magnetons is found.

In the γ region a further straight line may be observed which, when produced to the cobalt ordinate, gives us another moment that is less than 9 and may be estimated at 8·67. If this straight line be produced backwards to the iron ordinate, a moment of 14 magnetons is obtained for γ-iron. This result is confirmed by the fact that in the case of the ferro-nickels a linear relation observed by Peschard between Ni and Fe_2Ni_3 gives this same form of iron with 14 magnetons.

Pure cobalt in the H condition is magnetically hard on account of its anisotropy, and the observed values of the intensity of magnetization are too low. We have found in this case that $n = 8·37$. On the other hand, tempered cobalt at temperatures above 470°, where the γ condition is stable, is a mixture of H- and γ-cobalts. Since γ-cobalt has a higher moment we obtain in this way a value of 8·60, which is too high. The interpretation of these facts has been yielded by a result obtained by Seiji Kaya* in a study of the magnetization of single crystals of H-cobalt. It is possible to deduce from his measurements the atomic moment of 8·5 magnetons in the direction of the hexagonal axis, which is an axis of high susceptibility. We shall find later a precise confirmation of this result in the investigation of the nickel-cobalts.

§ 2. THE NICKEL-COBALTS†

A preliminary study of the nickel-cobalts‡ indicated a linear variation of atomic moment with composition in samples containing from 0 per cent. to 70 per cent. of cobalt; the graph begins at the point where $n = 3$ and is directed towards the point where $n = 9$. From 70 per cent. onwards the alloys are very hard magnetically, and saturation could not be attained.

We now know the cause of this hardness. It is connected with the lower degree of symmetry which characterizes the alloys rich in cobalt; those containing from 68 to 100 per cent. of cobalt are stable at low temperatures in a hexagonal crystal lattice of maximum density (H-nickel-cobalts), while the alloys containing less cobalt crystallize in a face-centred cubic lattice (γ-nickel-cobalts)§.

* Seiji Kaya, *Tohoku Sci. Rep.* **17**, 1158 (1928).

† The measurements on nickel-cobalts were made by Francis Birch.

‡ P. Weiss and O. Bloch, *Comptes Rendus*, **156**, 941 (1911); O. Bloch, *Zurich thesis* (1912).

§ Hakar Masumoto, *Tohoku Sci. Rep.* **15**, 449 (1926); T. Kasé, *ibid.* **16**, 491 (1927).

As in the case of the ferrocobalts the data, Fig. 2, relating to the approximation
to magnetic saturation, as a function of the applied field H and the absolute tempera-
ture T, are represented by the two lower broken-line curves. The difference between
the saturation intensity at absolute zero and that at ordinary temperatures, ex-
pressed as a fraction of the latter quantity, varies smoothly throughout the interval.
The coefficient of magnetic hardness varies smoothly in the γ region and reaches,
in the H-nickel-cobalts, very high values of the order of 200, not shown in the
figure. The extrapolations at $T = 0$ and $H = \infty$ can be made for the whole series of
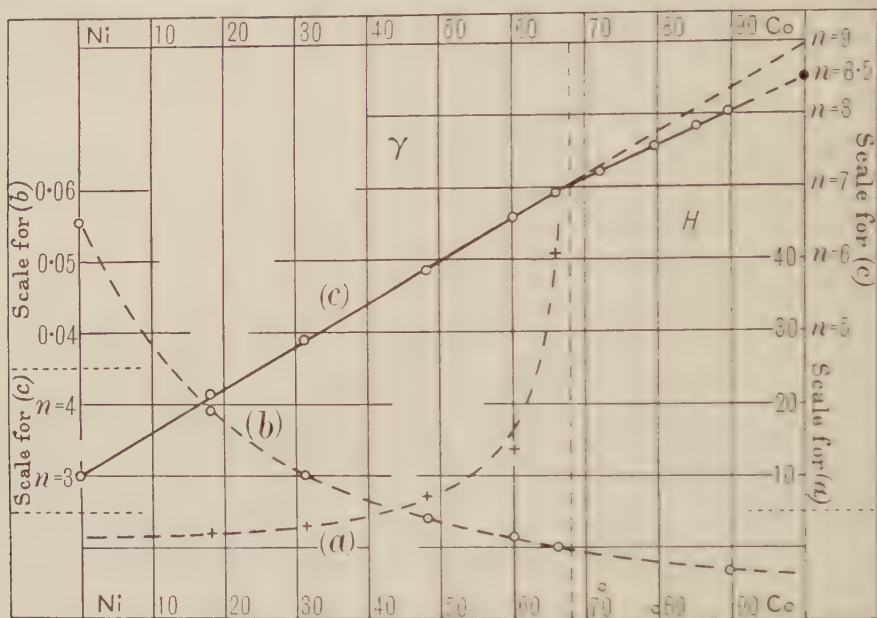
Fig. 2. Nickel-cobalts: (a) the temperature-coefficient a; (b) the quantity $(\sigma_{0,\infty} - \sigma_{288,\infty})/\sigma_{288,\infty}$, where
the first suffix of σ denotes T and the second H; (c) the absolute saturation n.

alloys without the introduction of appreciable uncertainty. Only pure cobalt is too
hard magnetically for the magnetization at $H = \infty$ to be obtainable.

The absolute saturation in magnetons is represented by the upper full-line
curve in Fig. 2. It comprises two regions of linear variation in the regions γ
and H. The first passes through the known point $n = 3$ for nickel, and points
towards the integer $n = 9$ for cobalt. The second gives $n = 8\cdot5$ exactly for cobalt.
This is precisely the moment which, in discussing the ferrocobalts, we have
deduced from the measurements made by Seiji Kaya on a single crystal of H-cobalt
in the direction of high susceptibility. When produced backwards as far as the
nickel ordinate, this straight line gives a moment in the neighbourhood of 4, though
the precision is not great on account of the distant extrapolation.

§ 3. CONCLUSIONS

To sum up, we may say that this investigation of the ferrocobalts and nickel-cobalts indicates that, like the ions of the iron group, the atoms of iron, nickel and cobalt are capable of assuming several different atomic moments. The change of moment is not always associated with a change in the crystal lattice.

Thus, in alloys belonging to different ranges of atomic composition, α-iron has been found with moments of 11 and 15 magnetons, and γ-iron with a moment of 14 magnetons. When cobalt occurs in a ferrocobalt crystallized in centred cubes it must be regarded as α-cobalt, a form which is unknown for pure cobalt. Each of the linear regions in the moment-curves of the α-ferrocobalts gives, therefore, a moment of α-cobalt. In this way moments of 17 and 9 magnetons have been found.

It is remarkable that cobalt with a face-centred cubic lattice, i.e. γ-cobalt, has not the same moment when the metal with which it is alloyed is iron as when this is nickel. In the first case the moment is 8·67, in the second it is 9. Even if the value 8·67, which is the less precisely known, were not rigorously exact, the fact would remain that the two moments are different.

In H-cobalt and H-nickel-cobalts the moment of cobalt is 8·5 magnetons.

From the measurements on the γ-nickel-cobalts the moment of nickel has been found to be 3, the value known long previously for the pure metal. The moment of nickel in the H-nickel-cobalts has been determined as 4, though with an accuracy which is not high on account of the distant extrapolation. Even if the latter numerical value be regarded as doubtful, it is nevertheless certain that nickel has two different moments in the nickel-cobalts.

Out of the ten moments which we have determined by means of these alloys, eight are integral multiples of the experimental magneton, 1125·6, and two are mixed numbers containing simple rational fractions, namely $8\frac{2}{3}$ and $8\frac{1}{2}$. It is known that the experimental magneton is, to within a few thousandths, one-fifth of a Bohr magneton. The atomic moments deduced from saturation data at low temperatures are, therefore, like those derived by spectrum-analysis, rational fractions of the natural unit furnished by the quantum theory. The general occurrence of the sub-multiple $\frac{1}{5}$ remains to be explained.

The study of ferrocobalts and nickel-cobalts brings out a further interesting fact. At the transition from the α-ferrocobalts to the γ-ferrocobalts, at a composition of 78 per cent., the atomic moment exhibits a marked discontinuity. On the other hand, the variation appears to be continuous at the transition from the γ-ferro-cobalts to the H-ferrocobalts, and certainly is so at the transition from the γ-nickel-cobalts to the H-nickel-cobalts. These facts may be compared with the change from 8 to 12 in the number of neighbouring atoms on transition from the centred cube (α) to the face-centred cube (γ), and the absence of change in the number (12) of neighbouring atoms on transition from the face-centred cube to the hexagonal lattice. Further, if the atoms are comparable to spheres of the same radius, the density increases on transition from the centred cube to the face-centred cube and remains unchanged on transition from the face-centred cube to the hexagonal lattice.

A NEW RELATION BETWEEN MAGNETIC AND ELECTRIC PHENOMENA*

By Prof. WALTHER GERLACH, München

ABSTRACT. The paper describes investigations of (*a*) the variation with temperature of the temperature-coefficient of the resistance of nickel, this quantity being related to the specific heat; (*b*) the resistance-changes produced in nickel by the application of a longitudinal magnetic field; and (*c*) an e.m.f. which arises when a ferromagnetic body, placed in a magnetic field, is subjected to a temperature gradient.

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§1. INTRODUCTION

THE relations between electrical and ferromagnetic properties have again obtained special significance, since the theoretical treatment of the two problems is relevant to the same fundamental treatment by wave mechanics of the electrons in metal. Three questions of this kind have lately been studied more closely in my Institute: (*a*) the change of the resistance with temperature; (*b*) the change of the resistance through a homogeneous longitudinal magnetic field; (*c*) a newly discovered thermogalvanometric effect: a potential difference which arises through the longitudinal magnetic field in a ferromagnetic conductor in the presence of a temperature difference.

Special attention was given to the fact that all measurements must be carried out on the same wire, and in each electrical measurement the magnetization curve of the same wire was also taken. Experiments have so far been carried out on nickel.

§2. THE CHANGE OF RESISTANCE WITH TEMPERATURE

In this connection a very large number of measurements are already available. The established fact is: at temperatures above the Curie point nickel behaves like any other metallic conductor, in that the temperature-coefficient of the resistance is constant for a greater range of temperature. Immediately below the Curie point the resistance very quickly falls with falling temperature. Fig. 1 represents such a curve. The measurements so far made are not sufficient for the exact determination of the true temperature-coefficient of the resistance, above all in the neighbourhood of the Curie point.

The true temperature-coefficient as a function of the temperature was determined by a sensitive differential method, and the resistance-change was measured with temperature-increments of only a few degrees (about 1–3° C.) between 0° C. and 400° C. As a result a curve of the temperature-coefficient as a function of the temperature was obtained, as shown in Fig. 2 (full line). The course of this curve

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* Translated by H. C. Booth, National Physical Laboratory.

agrees everywhere very closely with the known curve as given by P. Weiss for the true specific heat in nickel as a function of the temperature (dotted curve in Fig. 2).
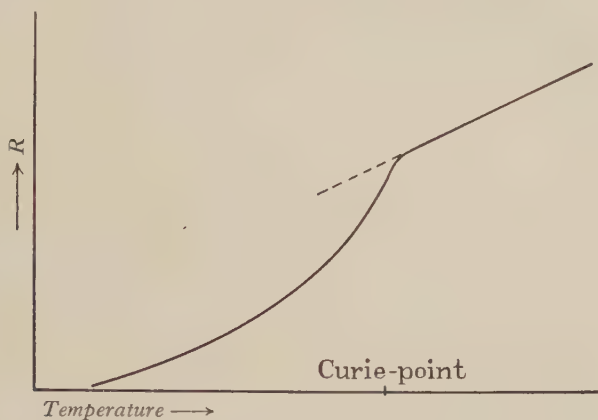
Fig. 1. Resistance as a function of temperature.
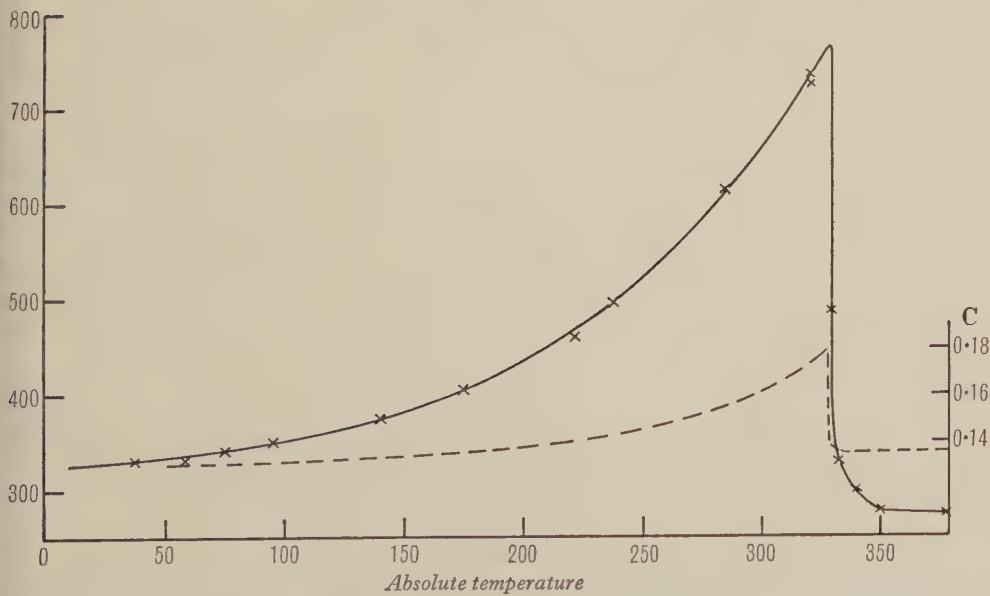
Fig. 2. × × Temperature-coefficient as a function of the temperature. --- Specific heat.

The physical significance of the relation found for dR/dT as a function of the absolute temperature T is seen from the following considerations. The resistance can be dimensionally represented in accordance with the simple electron theory by $R = A.E$. Here A is a quantity which contains the free wave-length, the charge

R
T

A, E

28

and the velocity of the conduction of electrons. We therefore assume that the well-known Dorfman* experiment indicates that the magnetic energy represents an increment in the energy of the conducting electrons. For a ferromagnetic body, therefore, E is to be regarded as consisting of two parts, one, E_0, independent of the magnetization and the other, E_m, dependent upon it.

E_0
E_m

The differential of R with respect to the temperature T therefore gives in the quantity $-A \cdot dE_m \, dT$ a quantity which varies extremely quickly for changes of temperature in the neighbourhood of the Curie point. If A and E_0 are to be regarded as dependent on temperature, then this temperature-dependence is single-valued. The course of the curve $dR \, dT - f(T)$ is therefore actually given by $dE \, dT$. The direct connection with this specific heat becomes clear because $dE \, dT$, according to Weiss, is equal to $\frac{1}{2}N \cdot d\sigma^2 \, d\tau$, where N is a constant of the internal field, σ the spontaneous magnetization, and this is equal to the magnetic part of the true specific heat.

N, σ, τ

The quantitative evaluation of the investigation is not yet finished.

§3. RESISTANCE-CHANGE OF NICKEL IN LONGITUDINAL FIELD

(*a*) *At temperatures far below the Curie point.* The resistance-change in the longitudinal field as a function of the magnetizing field-strength and of the temperature is especially well-known through numerous English investigations. We have made the arrangement such that we are able to measure the magnetization at the same time as the resistance-change. The percentage resistance-change from 0·1 to
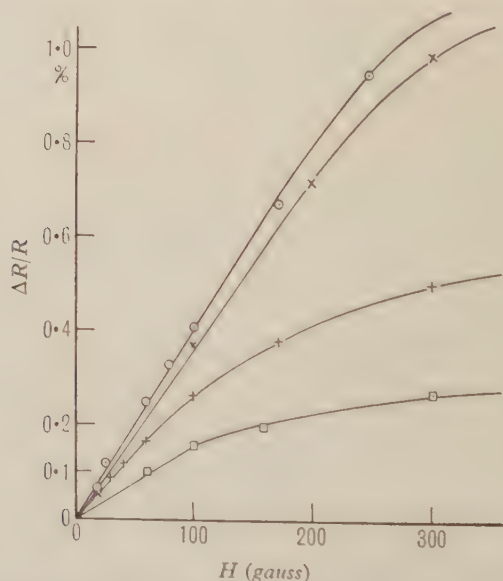
Fig. 3. Magnetic resistance-changes as a function of the field-strength. (The lower curves correspond to the higher temperature.)

* J. Dorfman and R. Jaanus, *Zeit. für Phys.* **54**, 277 (1929).

ι per cent. can be measured with an accuracy of a few per cent. There was no uniform connection of the magnetization with the magnetic resistance-change. In weak fields the magnetization increases much more quickly than the resistance-change. The resistance-change is here noticeably proportional to the external field-strength (Fig. 3). On the other hand the resistance-change of the magnetization is linear from about the beginning of the knee of the magnetization curve up to
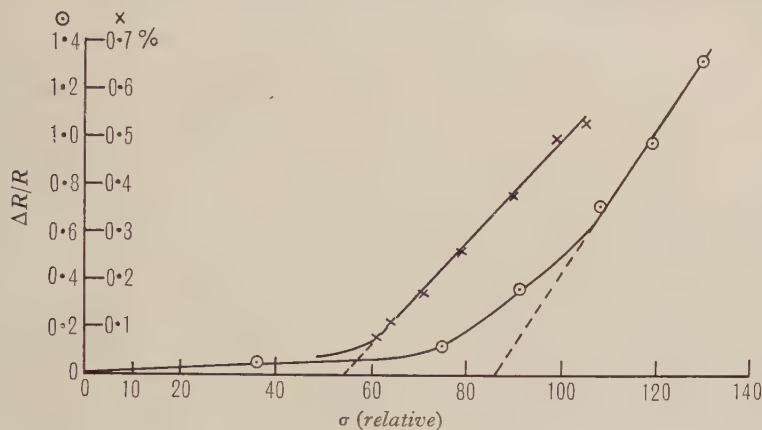
Fig. 4. Magnetic resistance-change as a function of the magnetization.
⊙ Temperature at about 290° K.
× ,, ,, ,, 470° K.
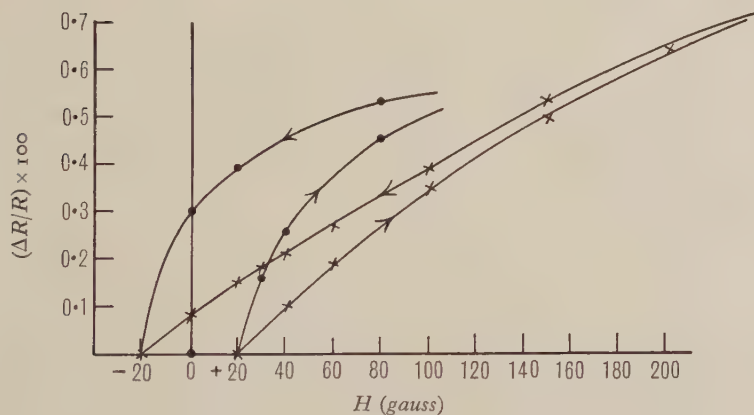
Fig. 5. Hysteresis: ··· magnetization, × × resistance-change.

saturation. Fig. 4 gives as an example the percentage change of resistance dR/R as a function of the field and of the magnetization for about 20° C. and 200° C. If the straight parts are extrapolated till the line cuts the σ_0 axis, one obtains at various temperatures the value of σ_0 whose product by the absolute temperature, $\sigma_0 T$, is a constant.

The change of resistance shows a hysteresis which is much smaller than the hysteresis of magnetization. Fig. 5 gives an example of this.

28–2

The linear relation between change of resistance and magnetization above the knee, and relative to small resistance-changes in the case of weak magnetization, shows that only a certain part of the ferromagnetic magnetization phenomena are connected with a change in the resistance. I would like to put forward the hypothesis that here a distinction between reversible and irreversible magnetization phenomena is shown, which in R. Forrer's work plays a great rôle in the course of the magnetization curves. According to this it is to be assumed that the irreversible phenomena are without influence on the electrical resistance, and therefore on the phenomena which exist in a rotation of Forrer's magnetic elementary bodies. The reversible phenomena of the disposition of the magnetic axes of the elementary body leads to a change of resistance. This indeed is illuminating for only the latter part implies "magnetic" change of structure, whilst the first part indicates only a change of the elementary bodies as such without modification of the structure. Since the dependence of the reversible phenomena on the strength of the field is not absolutely independent of the irreversible phenomena (on account of the internal field), it is obvious why a small hysteresis of the resistance is still present.

(*b*) *At temperatures near the Curie point.* Especially interesting relations present themselves in the neighbourhood of the Curie point, for here the increase of resistance changes into a decrease in resistance. I believe that we can also understand this effect in connection with the magnetic properties in the neighbourhood of the Curie point*. Here, as a result of an external field, a true magnetization is created. Therefore through an external magnetic field an internal magnetic condition is created which, without an external field, is only present at low temperatures. According to Fig. 1 the resistance below the Curie point falls off sharply with increasing temperature: one may therefore expect a decrease of the resistance through magnetization. It is also to be remarked that this diminution of resistance exhibits no saturation point, but a steady increase with the external field.

The relations in the case of transverse magnetization we have not considered; they are complicated through the Lorentz force, and therefore we cannot expect any simple connection with the magnetization.

§4. A NEW THERMOMAGNETIC EFFECT

If a fall of temperature takes place in a ferromagnetic body, then an electromotive force appears between its ends in a homogeneous magnetic field whose lines of force run parallel to the direction of the fall of temperature. This electromotive force shows about the same degree of dependence on the magnetization as the change of resistance.

T_1
T_2
If a piece of nickel wire is kept at a temperature T_1 of about $20°$ C. (see Fig. 6), and if the temperature T_2 of the other end is raised, then a magnetic field parallel to the lines of temperature-fall creates an electromotive force which (cf. Fig. 7 *A*) steadily increases and reaches an approximately constant value as soon as T_2 exceeds the Curie point. If T_1 be now increased while T_2 is kept at any given

* W. Gerlach, *Zeit. für Phys.* **59**, 847 (1930).
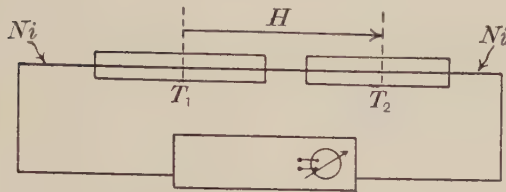
Fig. 6. Diagram of experimental arrangements.
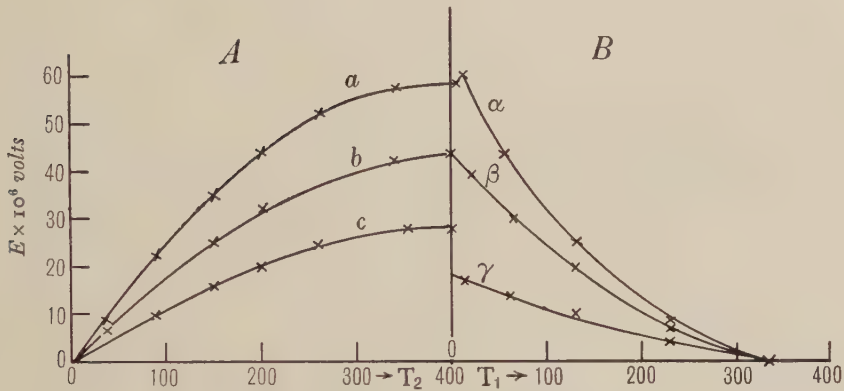
Fig. 7. E as a function of the temperature difference.

$A: T_1 = 15°$ C.; a 416 gauss; b 208 gauss; c 104 gauss.
$B: T_2 >$ Curie point; α 520 gauss; β 208 gauss; γ 62 gauss.
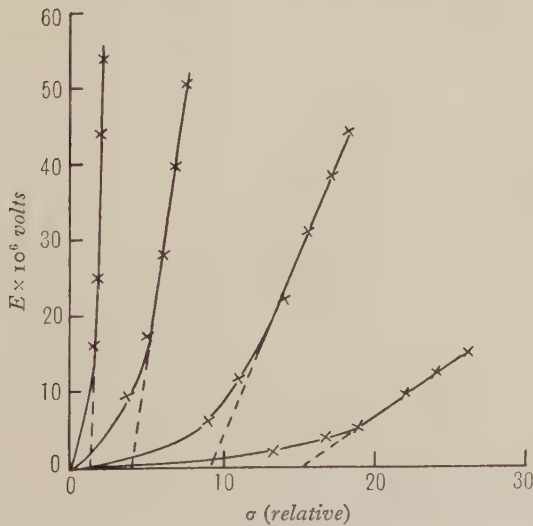
Fig. 8. E as a function of the magnetization.

temperature above the Curie point, then the electromotive force created by the magnetic field decreases and becomes zero as soon as T_1 is equal to the temperature of the Curie point. The electromotive force remains constant while the field effect remains constant. It is independent of the course of the temperature fall and only dependent on the temperature difference.

This effect is obviously connected in the same way with the magnetization as is the change of resistance at constant temperature: compare the course of the curves of field-strength of $\Delta R\,R$ and E in Fig. 3 and Fig. 7 A, but particularly
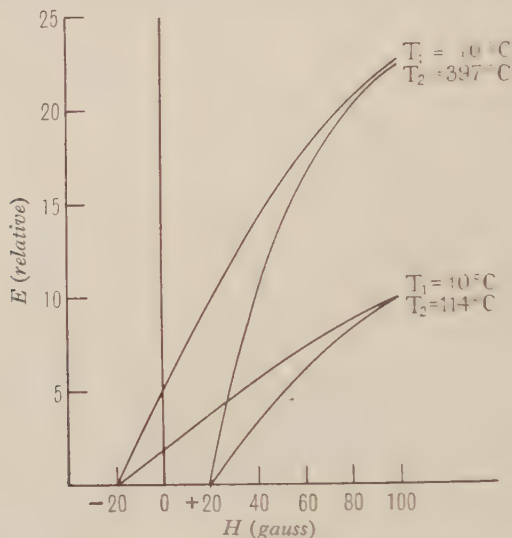
Fig. 9. Hysteresis of the longitudinal electromotive force E.

a dependence of the two magnetic effects on magnetization. In Fig. 8 the values of Fig. 7 as dependent on the magnetization have been transferred: here also at high values of σ we have a linear law connecting the effect with magnetization. Also the small hysteresis of the new effect perfectly corresponds to the hysteresis of resistance-change (compare Fig. 9).

We may consequently assume that this effect is also connected with the reversible part of the ferromagnetic magnetization. So far, the results of my investigation, which I have carried out in conjunction with Messrs Schneiderhan and Broili, show a very close connection between the electrical and magnetic phenomena; we hope that they will contribute to a further explanation of ferromagnetism.

METHODS OF EXPERIMENTING IN STRONG MAGNETIC FIELDS

By P. KAPITZA, Ph.D., F.R.S.

ABSTRACT. The paper describes the design of apparatus for the production of intense magnetic fields of brief duration, and indicates some of the experiments which were made with such fields.

THE model of the atom as now given in modern physics is essentially of an electrodynamic character, that is, it consists of moving charged bodies. All the properties of a given atom are due to the number and way in which the electrons move round the nucleus. By disturbing this motion by any means we shall alter most of the properties of the atom such as, for instance, its spectra, coherence forces, magnetic moment, etc.

The magnetic field is the most efficient disturbing agent of the motion of the electrons, and it is of the greatest interest and importance to study the disturbances produced by magnetic fields in individual atoms and in atomic aggregates such as crystals. For instance, it is well known that the results which were obtained from the study of the Zeeman Effect led to the classification and understanding of the spectra of atoms. There are quite definite indications that the most interesting region of the magnetic disturbance of the atom is reached when the strength of the magnetic field attains the same value as the field in the atom produced by the moving electrons. If an atom is exposed to such magnetic fields the motion of the electrons will be altered very markedly, as the coupling energy between the electrons will be of the same order as the perturbation produced by the field. However, if we begin to consider the magnitude of these magnetic fields we find that even for the most loosely bound electrons it should be of the order of 1,000,000 gauss. Ordinary methods of obtaining magnetic fields, such as the use of electromagnets, limit the field to a value not much higher than 50,000 gauss. This limit is due to the fact that iron gets saturated. The magnetic field produced by electromagnets increases very slowly in strength with increasing size of the magnet. The largest magnet ever built is that of Prof. Cotton, which is a most wonderful engineering construction of enormous size. The diameter of the iron cores reaches 1 metre and a man can easily stand between the pole pieces, whilst a special power station is required to supply the required current. Even this huge electromagnet cannot produce a field much stronger than 60,000 gauss in a space sufficiently large to make experiments.

The importance of strong magnetic fields led the author to attempt to develop a method of obtaining them by making the time of duration very short. As we shall see later, by means of the sacrifice of the length of time of existence the magnetic

fields can be made very much stronger, and a large region of important physical investigation can be covered.

The general idea of the method is as follows: If you take a coil and pass a current through it, then the magnetic field produced in the coil will be proportional to the current, but the increasing of the current is stopped by a natural limit which is due to the heat produced in the coil by the current. To reduce this heating two methods can be used; the first is to cool the coil to a very low temperature, when the resistance is reduced considerably, and even in certain metals to zero, when the metal begins to be supra-conducting. A difficulty in this case will be that the magnetic field produced by the coil will destroy the supra-conducting state and also very rapidly increase the resistance to a value very close to that at room temperature. The second method is to cool the coil by carrying away the heat: but still experiments and calculations show that even with the most efficient cooling of the coil it is difficult to get a field of more than 50,000 or 60,000 gauss.

The principal idea of the author's method is to produce the magnetic field for such a short time that the coil has practically no time to overheat. We have chosen this time to be of the order of 0·01 sec. For instance, if we want to produce a field of 1,000,000 gauss in a very efficiently designed coil with an inside diameter of 1 cm., a power of 50,000 kilowatts is required; in one second the coil will be heated to 10,000° C., and to keep the coil cool by means of water-cooling is practically impossible. If, however, we allow the current to pass for only 0·01 sec. the coil will only increase in temperature by about 1000° C., a quite permissible temperature rise.

There are a number of experimental difficulties which still lie before us in obtaining and applying this strong magnetic field of short duration: the first one is the large amount of power necessary. It is evident that it is an easier and cheaper proposition to obtain the huge amount of power for a short time than to use it continuously. We have found two methods suitable for our experiments. Our first method was to use accumulator batteries; these specially made accumulators had a very small capacity and a small internal resistance, and were rather rigidly built. By charging these accumulators for a few minutes and discharging them in 0·01 sec., we could easily obtain impulses of current up to 2000 or 3000 kw. All the preliminary experiments were carried out with these accumulators, and fields of 100,000 gauss were produced, but further increase in magnetic field was difficult as it was found rather difficult to break sufficiently suddenly the continuous currents of several thousand ampères supplied by accumulators. Consequently, when it was decided to go to larger powers the accumulators were replaced by a special generator. The generator chosen was similar to a single-phase a.c. turbo-alternator. This type was found to be of great advantage for obtaining the strong impulses required in our experiments. Firstly, the armature can easily be made very strong, and as it makes a large number of revolutions it possesses a good amount of kinetic energy, so that when the generator is short-circuited for 0·01 sec. sufficient kinetic energy is available to be converted into electrical energy. Secondly, with an a.c. machine only a half wave of the current is used, and if a synchronously adjusted break is

adopted, it is possible to interrupt the current at a point very close to zero, and the problem of the interruption of the current is thus simplified.

It is well known in electrical engineering practice that this type of machine when short-circuited can deliver very large impulses of current, and usually these machines are made in such a way that large currents cannot be obtained by an accidental short-circuit, which might otherwise result in a serious accident. The machine which we used, however, was designed in the opposite way: it is actually built to give these large impulses on being short-circuited. This required a considerable

Fig. 1. Perspective view of apparatus.

revision in the design of the machine, and it was especially important to consider all the electrodynamical forces which occurred, since they might easily result in a mechanical breakdown of the windings. The machine actually constructed was of the size of the usual turbo-alternator for delivering 2000 kw. at continuous rating. On being short-circuited on the test bench it gave us 220,000 kw. When it is short-circuited on a coil of equal impedance only half of the power will be available: half of it will be lost in the machine and the other half will go to the coil. In this way the required 50,000 kw. are obtainable. The machine is shown in Fig. 1.

The only drawback of using such a machine is that the current never remains constant in the coil, and thus the magnetic field also will vary as is seen on oscillogram Fig. 2. However, it was found possible with a certain design of armature to give such a shape to the excitation magnetic field that the output

current wave had a flat top, and during several thousandths of a second the magnetic field actually remained constant as seen on oscillogram Fig. 3.

The second problem we had to face was the designing of a special switch to make and break the current synchronously with the current wave. This was rather a complicated engineering problem, as the current had to remain on for only 0·01 sec. and the time we had for making or breaking would be a few ten-thousandths of a second, during which time a copper plate had to separate by several millimetres from a brush. The acceleration required to move the copper plate 1 kg. in weight this distance is about 1000 times that of the gravity field, and the force required is over a ton. An exceptionally strong and carefully designed cam shaft mechanism is used for this apparatus. There are also several accessory parts used during the break of the current, such as condensers to avoid over tension during a too sudden break and an air blast in the gap of the switch to make the break more efficient. The switch is shown in Fig. 4.
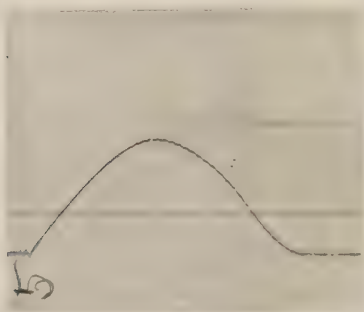
Fig. 2. Current in coil, with ordinary
excitation field.
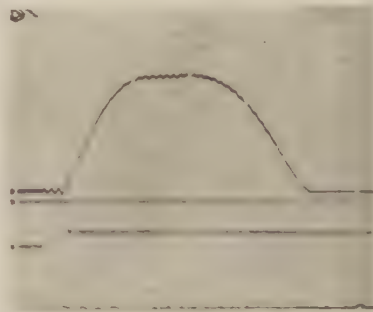
Fig. 3. Current in coil, with excitation
field designed to give flat-topped
wave-form.

The most difficult part of the whole apparatus to design was the coil itself, in which the magnetic field was produced. The density of the current in this coil reached a value of 500,000 amp. cm.^{2}, and the electrodynamical forces which try to enlarge the diameter of the coil are so large that at first most of our coils broke with a great explosion. It was necessary to work out a method to reinforce these coils with steel bands and to find a coil of such a shape that the electrodynamical forces together with the reaction forces of the reinforcement would be reduced to a uniform (hydrostatic) pressure on the copper. The force on the outside reinforcement of the coil which is actually now in use, and in which fields of over 300,000 gauss have been obtained, reached the value of 140 tons.

There are also a number of automatic devices and oscillographs which are used to record the currents in the coil, and thus to measure the magnetic fields. The experiments are carried out automatically by a number of timing devices which all act on the pressing of a single button. The apparatus is placed in a large hall at one end of which is the generator which is accelerated by a 80 h.p. electric motor to the required speed. The strong current is brought to the other end of the coil by means

of six thick cables. The distance between the machine and the coil has been chosen to be 20 metres for the following reason: when the machine is short-circuited the angular velocity of the armature, which weighs $2\frac{1}{2}$ tons, is reduced by 10 per cent. in 0·01 sec. This results in a strong couple which tends to turn the whole machine on its foundation and produces a shaking of the ground. It is evident that it would be very awkward, at any moment when an experiment was being made, for such an earthquake to disturb the apparatus, but by placing the machine far from the coil we bring it about that the earthquake actually arrives at the coil and measuring-apparatus after 0·01 sec., when the experiment is all over.

At present we have limited our experiment to field strengths up to 300,000 gauss, which can be reached in a volume of 3 cm.²
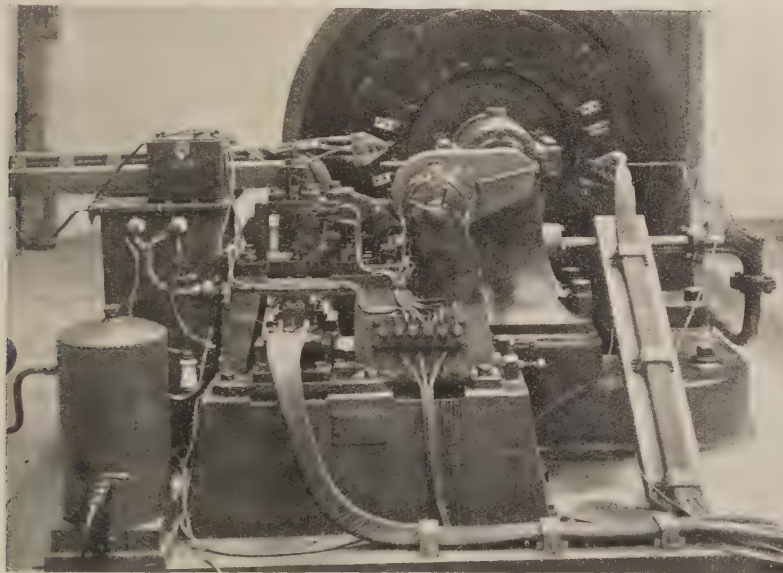
Fig. 4. Synchronous circuit-breaker.

We have now to consider the methods used to make experiments during 0·01 sec. It is evident that for the magnetic phenomena occurring in the atom this is a very long time, and all these phenomena have plenty of time to establish themselves. The difficulty is only in finding the methods of observing and measuring these phenomena. Actually, however, the loss in time in most experiments is compensated by gain in magnitude of the phenomena observed. For instance, in the study of the Zeeman Effect, the splitting of the lines was so large that an ordinary prism spectrograph could be used which had a large luminosity, and the time of exposure could be reduced to 0·01 sec. In ordinary weaker magnetic fields the time of exposure is sometimes several seconds, and échelons or gratings of large dispersive power, which have a much smaller luminosity, have to be used.

Up to the present, besides the Zeeman Effect, we have covered the following ground in magnetic research.

We have studied the change of resistance of various metals in magnetic fields. Usually the change of resistance amounts to a small fraction of 1 per cent., but in our case, with the strong magnetic fields, the resistance of most of the metals increased by 20 to 30 per cent.* and could be measured by an ordinary oscillograph, especially as for these short times large currents could be sent through the samples of metals without any danger of heating them. It was found in this case that in strong magnetic fields the law of change of resistance was different from that in weaker fields. In weaker fields the resistance increases proportionally to the square of the magnetic field; in stronger fields we found that a linear law obtained.

We have also measured the susceptibility of certain metals. For this purpose a special balance was made having a frequency of about 2000 to 3000 ∼. This balance was found sufficiently sensitive to measure the susceptibility of most of the substances, as in the case of strong magnetic fields the force which acts on a substance is about 100 times larger than usual, and amounts to several grams.

The magnitude of the force reached in strong fields can be illustrated by a simple experiment. If we place in the coil a Dewar flask filled with liquid air, and if a glass rod 3 mm. in diameter is placed in the flask, the liquid oxygen in liquid air, owing to its strong paramagnetism, is pulled into the Dewar flask when the field is on, and the glass rod is forced out, the force being sufficient to throw the glass rod to a height of about 7 or 8 metres. If the Dewar flask is filled with liquid oxygen only, the force of attraction due to a field of 300,000 gauss produces a pressure of several atmospheres and the Dewar flask breaks.

It was also found possible to make an apparatus to study the magnetostriction in metals during short periods. At present this phenomenon is only known for ferromagnetic substances, but in strong magnetic fields we find it specially marked in bismuth and some other substances, such as tin and graphite, which have a crystal structure of a low symmetry. We found that in bismuth crystals in strong magnetic fields the crystal increases in length in the direction of the trigonal axis, but decreases perpendicularly to it.

The great advantage of studying these phenomena in 0·01 sec. is that the main disturbance, which comes from the temperature change, is avoided, as during this short space of time the temperature is practically constant.

A very good example of the use of short duration fields with good success was afforded by the application of these fields to the α-ray tracks obtained in a Wilson expansion chamber. In strong magnetic fields, owing to the charge of the α-particle, the track is bent, and by measurement of the curvature along the track it is possible to find out how a single α-particle loses its velocity when passing through a gas.

Thus we can see that the magnetic field of short duration has a wide scope of application and can be used in nearly every case where the experiment is possible in ordinary magnetic fields, but special apparatus and methods have to be devised for the work.

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* The resistance of bismuth increases in certain cases 2000 times.

MAGNETOSTRICTION AND CHANGE OF RESIST-ANCE IN SINGLE CRYSTALS OF IRON AND NICKEL

By W. L. WEBSTER, Ph.D.,

Clerk Maxwell Scholar of Cambridge University

ABSTRACT. The process of magnetization must consist of two parts, first the switching over of the magnetization in small elements of volume from one direction of easiest magnetization to another, and then a gradual change from this direction to that of the applied field. These two stages involve different distortions of the crystal, and an attempt has been made to associate with them the principal characteristics of the phenomena of resistance-change and magnetostriction.

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§ 1. INTRODUCTION

IN two recent papers Akulov* has shown, for a single crystal of iron magnetized to its saturation value, that it is possible to account for the longitudinal and transverse changes of length accompanying changes in the direction of the magnetization relative to the crystal structure by means of forces arising from magnetic dipoles. And further, he has shown that there is a direct proportionality between the energies involved in such length changes and the energies required to produce the corresponding change of direction of the magnetization, the latter forces being the quadrupole terms postulated by Mahajani†. This must mean simply that the dipole and quadrupole effects are both due to the deviation of the same carrier of magnetic moment from its normal position of minimum energy in the crystal.

Akulov made no attempt to deal with magnetostriction in the region below complete magnetic saturation. In this paper an attempt is made to give a qualitative explanation of the effects in this unsaturated state.

§ 2. THE DEMAGNETIZED STATE

In order to deal with the unsaturated state it is necessary to know the magnetic condition of the metal in an apparently unmagnetized state. The phenomenon of spontaneous magnetism‡ leaves little doubt that an apparently unmagnetized body at ordinary temperatures consists of small volumes magnetized in different directions but adding up to give a zero magnetic moment to the whole. The orientation of the direction of magnetization of these small volumes cannot be entirely at random, for the magnetic intensity in each must be very nearly the full intensity in equilibrium with the temperature, and the Mahajani molecular field must therefore orientate

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* N. Akulov, *Zeit. für Phys.* **25**, 389 (1928); **54**, 582 (1929).

† G. S. Mahajani, *Phil. Trans.* **228**, 63 (1929).

‡ P. Weiss and R. Forrer, *Ann. de Physique,* **5**, 153 (1926).

them, as far as is consistent with a continuity of flux, along the directions of easiest magnetization. For iron the magnetization will be along the three cubic axes, and as these must all be equivalent, it can be assumed that there are equal volumes magnetized along each. The details will admittedly not be perfectly sharp, but the actual condition must be a close approximation to that described above.

The reason for the existence of such a chaos lies in the demagnetizing force due to the external shape of the body under consideration. This is a purely accidental effect from the point of view of the mechanism of ferromagnetism, and one must imagine that, could it be eliminated, the normal state of a ferromagnetic body in the absence of any external field would be complete saturation parallel to one of the directions of easiest magnetization. The unmagnetized state is essentially unnatural to a perfect, uniform, ferromagnetic crystal from which all extraneous influences have been removed. For such a crystal the only important variable would be the direction of magnetization, as the magnetic intensity at ordinary temperatures can only be altered to a comparatively small extent and that only with very large magnetic fields. Akulov then, who calculates the change of length accompanying a change in the direction of magnetization from its normal (100) direction, and deals only with complete saturation, gives what must be considered a reasonable description of the phenomenon of magnetostriction in an ideal crystal at low temperatures. He finds naturally that the results are in satisfactory agreement with the measurements on single crystals at ordinary temperatures, where the intensity of spontaneous magnetization is still within 2 or 3 per cent. of its maximum value, and under such conditions (i.e. with an external field sufficiently large to annul the demagnetizing force completely) that a state of uniform saturation is produced.

For the purposes of the present paper, which will discuss effects occurring in the region where the bulk magnetization is not complete, it is necessary to decide what the actual state of the body will be for different degrees of magnetization. It will then be possible to investigate the effect of the departure from the ideal state on the various phenomena which occur in this region.

§ 3. THE PROCESS OF MAGNETIZATION

An unmagnetized crystal of iron has already been described as consisting of an aggregate of a large number of small elements, each fully magnetized, but so oriented at random along (100) directions that there is no resultant moment. When bulk magnetization is made to appear, by the application of a magnetic field, some of these small elements must change their direction of magnetization in order to produce the resultant moment. This change of orientation may take place in three different ways. The magnetization may be reversed; it may change from one cubic axis to another; or it may depart altogether from a cubic axis. The first two of these processes are essentially the same. They may require a small magnetic force to break up the stability of the circuits of magnetic flux which must exist in unmagnetized iron, but no further work, as the change of direction of magnetization from one cubic axis to another involves no net change in the energy of the crystal. The third process, however, brings into play the Mahajani molecular field and requires fields

of the order of several hundred gauss. There will however be no hysteresis loss as in a complete cycle of magnetization the Mahajani force produces no change of energy.

The coercive force and hysteresis loss that are observed even in single crystals probably correspond to the energy lost in breaking up the circuits of flux in the iron. It is found* that the more nearly perfect the crystal the smaller is its hysteresis loss, a result due possibly to the encouragement of larger circuits of flux with a corresponding decrease in their stability.
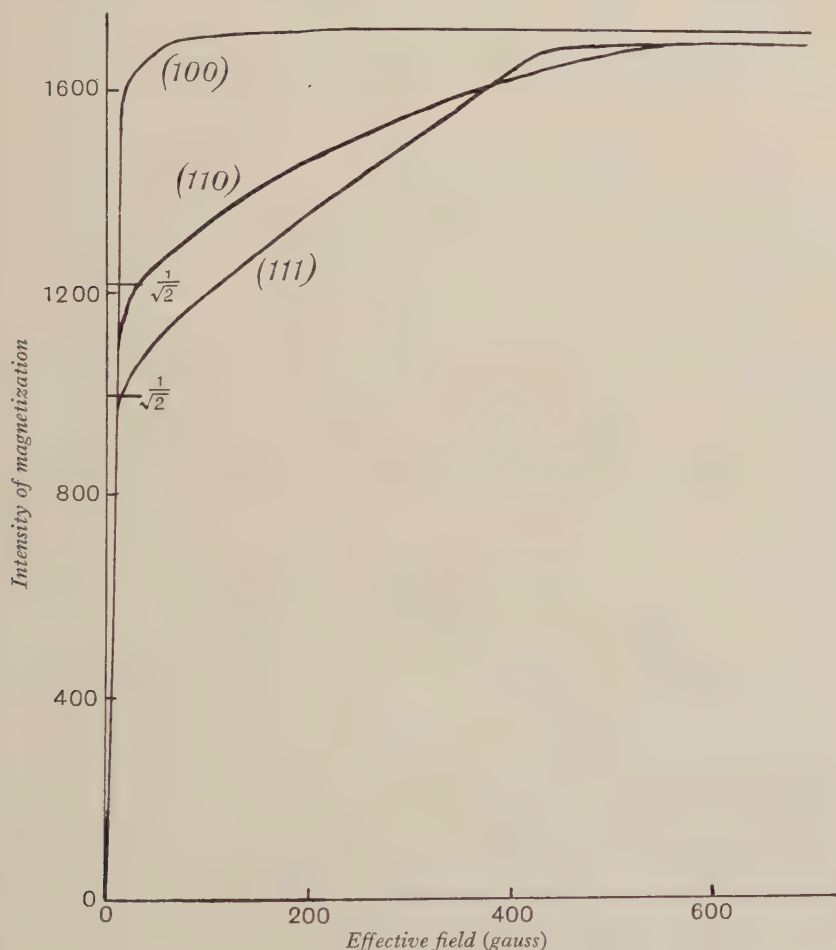
Fig. 1. Magnetization curves for iron.

There are, then, two fundamentally different ways in which bulk magnetization occurs. The first is a change from one cubic axis to another and takes place in fields of a few gauss†; the second is the departure from a cubic direction and occurs only

* K. Honda and S. Kaya, *Sc. Rep. Tohoku*, **15**, 728 (1926); W. Gerlach, *Zeit. für Phys.* **39**, 27 (1926).

† W. Gerlach, *Zeit. für Phys.* **38**, 828 (1926); K. Honda, H. Masumoto and S. Kaya, *Sc. Rep. Tohoku*, **17**, 118 (1928).

in comparatively large fields. These may be associated respectively with the initial rapid rise up to and the slow increase after the knee of the ordinary $I\,H$ curve. That this view is substantially correct is borne out by the fact that the Barkhausen effect, which is caused by the sudden change of magnetization in small but finite regions, occurs only during the initial part of the magnetization curve up to the knee. The second process, being more of the nature of an elastic deformation against the Mahajani force, must be continuous.
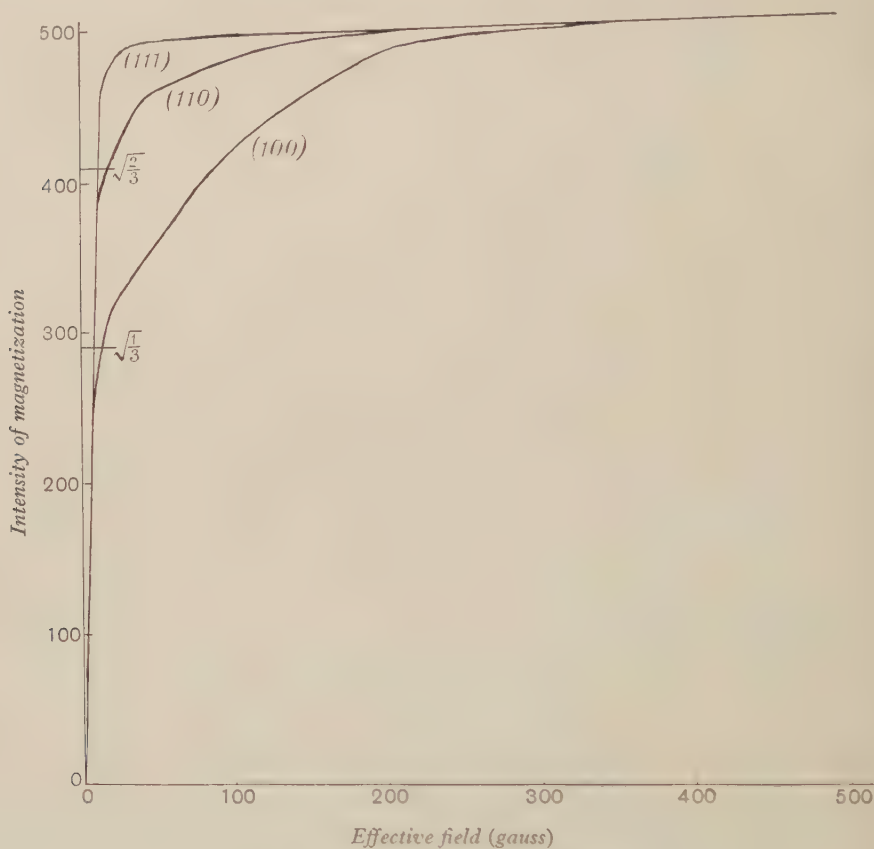
Fig. 2. Magnetization curves for nickel.

The value of the intensity of magnetization at which the break occurs will depend on the direction of the magnetic field relative to the crystal axes. When all the small elements in the crystal of iron have had their direction of magnetization changed to coincide as far as possible with the cubic directions nearest to the applied field, consistently with the condition that there should be no transverse magnetization, then any further increase of bulk magnetization must be produced by forced deviations from these cubic axes toward the direction of the applied field, and the knee will appear.

With an applied field along a cubic axis, the second process will not appear at all and the initial rapid rise should continue right up to complete saturation. For the (110) and (111) directions the transition should take place at intensities $\sqrt{\frac{1}{2}}$ and $\sqrt{\frac{1}{3}}$ of the saturation values.

In the case of nickel, where the (111) axis is that of easiest magnetization, this will be the direction for which there is no break in the I/H curve; while for the (110) and (100) axes the break will be at $\sqrt{\frac{2}{3}}$ and $\sqrt{\frac{1}{3}}$ of the saturation.

Experimental curves for these directions are given for iron* and nickel† in Figs. 1 and 2. In view of the difficulty of obtaining perfect single crystals of these metals sharply defined breaks cannot be expected, and the actual agreement is good.

§ 4. MAGNETOSTRICTION

With this picture of the process of magnetization it is possible to see what information may be gained about the phenomena of magnetostriction and magnetic change of resistance. The variation of length with intensity of magnetization is shown in Figs. 3 and 4‡ for the three simple directions in iron§ and nickel ‖. In the direction of easiest magnetization in each metal there is a gradual change of length, in one case an expansion and in the other a contraction, beginning with quite small intensities and gradually increasing as saturation is approached. In the direction farthest removed from the above there is no appreciable change of length till approximately half-saturation, when a contraction sets in and rapidly increases with the magnetization. In the intermediate direction there is obviously a compounding of these two effects.

The treatment of magnetostriction by Akulov is based on the distortion of the crystal lattice accompanying the rotation of the direction of magnetization away from the direction of easiest magnetization, and explains the difference in length of unit rods parallel to the different crystal directions when completely magnetized; but it does not account, for example, for the increase in length occurring when a rod of iron parallel to a cubic axis is magnetized.

The origin of this effect appears, however, in the course of Akulov's calculations, for he shows that a cube of iron supposed completely devoid of magnetization on an atomic scale becomes slightly distorted when it acquires its normal spontaneous magnetization. It becomes longer along the cubic axis of magnetization than along those at right angles. The magnitude is not calculated, but it must be at least of the same order as the deviational distortion. Demagnetized iron must therefore be

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* K. Honda and S. Kaya, *Sc. Rep. Tohoku*, **15**, 721 (1926).

† S. Kaya, *Sc. Rep. Tohoku*, **17**, 639 (1928).

‡ In the papers on the change of resistance and magnetostriction in nickel, quoted in the present paper, the intensities of magnetization were not given. They have been derived by the author from the effective magnetic fields by means of the magnetization curves of Kaya. As the same piece of nickel was not used in all three experiments there may be an appreciable error in the magnetization scale of Figs. 4 and 6 for values of the intensity up to the knee.

§ W. L. Webster, *Proc. R.S.* **109**, 570 (1925).

‖ Y. Mashiyama, *Sc. Rep. Tohoku*, **17**, 945 (1928).

considered as a mosaic made up of asymmetrical small volumes with their direction of distortion distributed equally along the three cubic axes. The resultant dimensions of a cube of demagnetized iron will be symmetrical but will be a complicated average between the two dimensions of the distorted magnetized cube. When magnetization takes place along a cubic axis there must be a longitudinal expansion, with a lateral contraction gradually increasing as the various small volumes become parallel.
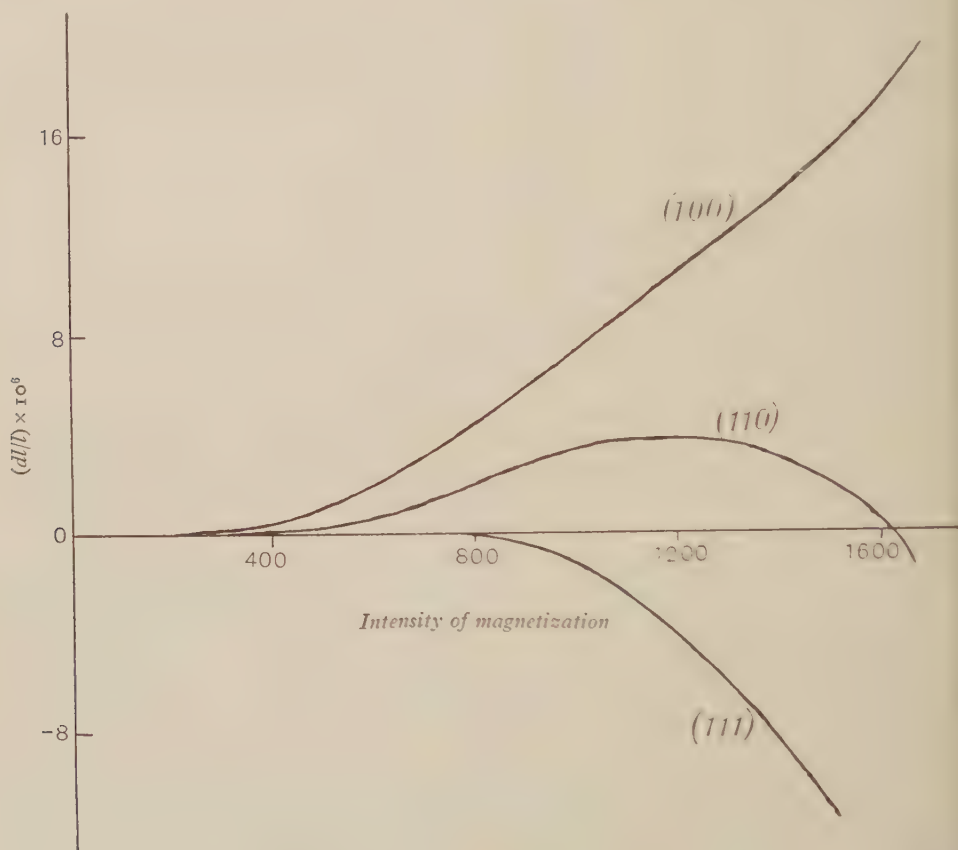
Fig. 3. Magnetostriction curves for iron.

For the (110) direction, as magnetization proceeds there is at first a concentration of the axes of distortion of the many small volumes along the two cubic axes nearest the direction of magnetization. There must therefore be an initial expansion continuing till the third axis has been completely abandoned. Further increase of magnetic intensity can only be produced by deviation from the cubic axes, and the corresponding contraction is sufficiently large to overwhelm the expansion due to increasing uniformity.

With the (111) axis, the three cubic directions are symmetrically disposed and there is no real increase in uniformity before the deviational distortion appears

consequently no initial expansion is observed. It may be pointed out that the contraction should set in for all directions at a point corresponding to the bend in the I/H curve, a result with which the experiments are in reasonable agreement.

For nickel the rôles of the (100) and (111) axes are interchanged, and the distortion produced by the increase of uniformity becomes a contraction. The two kinds of distortion can, however, still be separated. It is particularly noticeable that
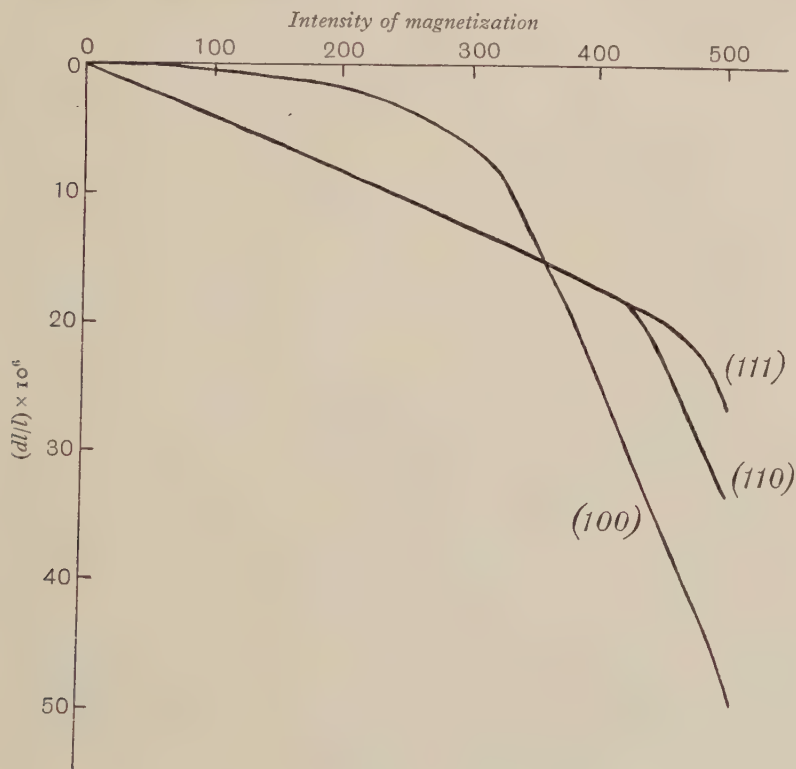
Fig. 4. Magnetostriction curves for nickel.

there is relatively little change in length for the (100) axis below intensities 300 c.g.s. units of magnetization. For the (110) direction, the effect of the deviation term is shown by the sharp increase in slope at 420 c.g.s. units, where from the I/H curve it is clear that the deviation is first brought into play.

§ 5. MAGNETORESISTANCE

In the light of the process of magnetization suggested above, it is possible to arrive at a qualitative explanation of the principal features of the magnetoresistance phenomenon, in so far as its appearance can be associated with a definite change of condition of the metal that is sufficiently likely to cause a change in the electrical properties. That such a connection between the magnetic and electrical properties

should exist is not surprising, since the work of Dorfman* on the Curie-point discontinuity of the Peltier effect has shown that in nickel the electrons responsible for the two properties are identical. Any disturbance of the magnetic condition of these electrons must react to some extent on the conductivity.

Experimental results for the longitudinal effect are shown for iron† and nickel‡ in Figs. 5 and 6. For iron the most remarkable fact is that, in spite of the enormous change of length for the (100) direction, there is practically no change of resistance. Rods were measured by the author which gave less than a fifth of the change shown in the figure. Further, it was pointed out that there is a remarkable parallelism between the change of resistance for the other directions and their contraction in magnetostriction. It is necessary then to conclude that the change of resistance is caused by the distortion of some sort of electron lattice by the pulling away of the magnetic axes of the electrons from their normal cubic direction against the Mahajani field. And also it is necessary to assume that a cube of iron magnetized along a cubic axis remains electrically isotropic in spite of its magnetostrictive distortion. These conclusions are borne out by the transverse§ magnetoresistance effect. In this case there is a small change of resistance roughly proportional to the applied field, which appears in all cases and is probably of the same nature as the change occurring in all metals, ferromagnetic or not. There is in addition a change depending for sign and magnitude on the directions of the current and magnetic field. This is the definitely crystalline effect, and it has the important property of vanishing whenever the magnetic field is parallel to a cubic axis, that is, whenever the magnetic condition of the iron is normal.

In the case of nickel there is an effect for the (100) and (110) directions exactly similar to that found for iron for the magnetically corresponding directions, but there is in addition a very large effect in the direction of easiest magnetization. This new change of resistance appears for both the longitudinal and transverse phenomena. From the appearance of the curve for the (111) direction, where the resistance change takes place in the region within 5 per cent. of saturation, this change might appear to be due to the acquisition, near the region of saturation, of uniformity and continuity in the magnetic condition of the crystal. Such an explanation would almost certainly involve a diminution of resistance rather than the increase actually shown, so that it is better to look elsewhere for the origin of this effect.

In the case of the transverse phenomenon, there is also a change of resistance with magnetization along the (111) axis, but in this case it is a decrease in resistance. This fact suggests that both the changes that were found for this direction are akin to the changes of length in the direction of easiest magnetization that occur in nickel and iron. That is, they are due to the electrical anisotropy of nickel magnetized along its magnetic axis. If this be so, then the appearance of the curve, which is not

* J. Dorfman, *Zeit. für Phys.* **54**, 277 (1929).
† W. L. Webster, *Proc. R.S.* **113**, 196 (1926).
‡ S. Kaya, *Sc. Rep. Tohoku*, **17**, 1027 (1928).
§ W. L. Webster, *Proc. R.S.* **114**, 611 (1927).
‖ W. Gerlach, *Zeit. für Phys.* **59**, 847 (1930).
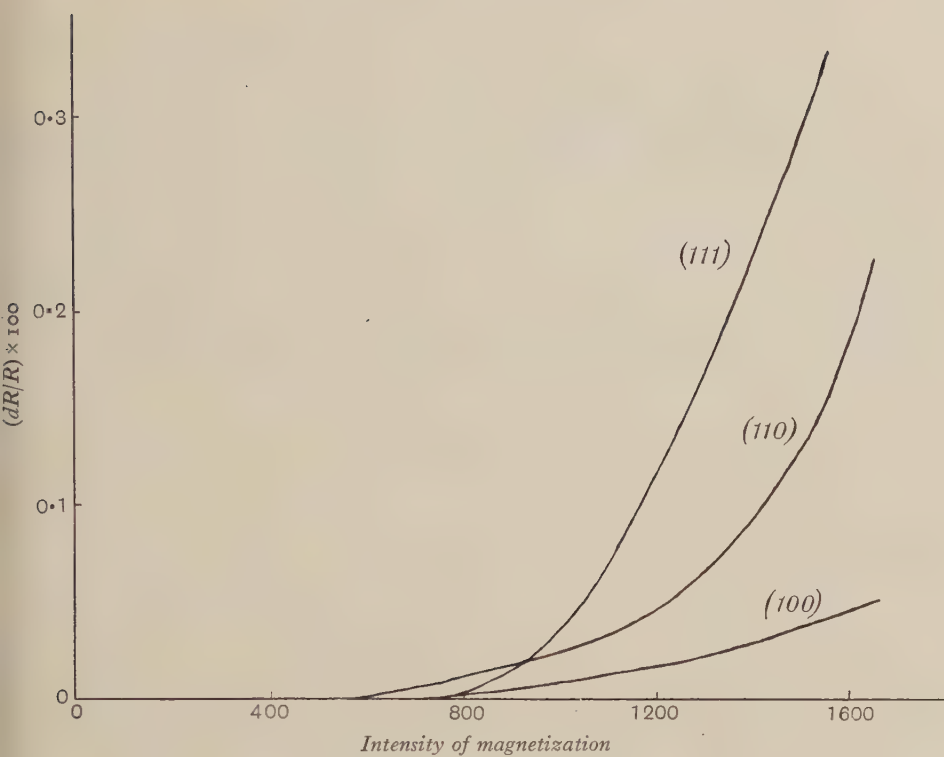
Fig. 5. Magnetoresistance curves for iron.
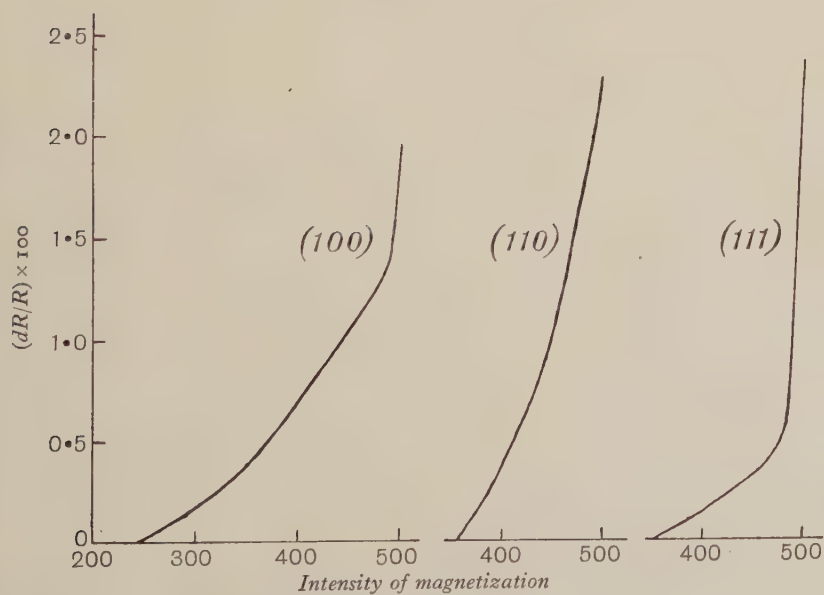
Fig. 6. Magnetoresistance curves for nickel.

at all what would be expected on such a view, must be due to comparatively large errors in the magnetization scale—errors particularly likely to occur for the direction of easiest magnetization.

The fact that the anisotropy occurs only in nickel and not in iron must be connected with the difference in the relation between the magnetic and crystallographic axes in the two metals. In the case of iron the two sets coincide and both have the simplest symmetry possible, whereas in nickel there are four magnetic axes which do not possess the simple symmetry of the crystal structure of this metal. There may be a further cause in that in iron there are approximately three magnetic electrons per atom, all of which have not yet been shown to have that relation to conductivity which was found for the single magnetic electron of nickel.

OBSERVATIONS ON THE SPECIFIC HEATS OF FERROMAGNETIC SUBSTANCES

By L. F. BATES, B.Sc., Ph.D.,

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ABSTRACT. Attention is drawn to the variation of the specific heats of ferromagnetic substances with temperature. In particular, the magnetic and thermal behaviour of a ferromagnetic compound of manganese and arsenic is described, and it is shown that, in this case, the maximum value of the specific heat coincides with the maximum value of dI_0^2/dT. The variation of the specific heats of ferromagnetic substances with temperature is then contrasted with that expected on the Weiss and on the Heisenberg theories of ferromagnetism.

THE specific heats of ferromagnetic substances have recently attracted considerable attention. It was shown by Weiss*, on the basis of his hypothesis of an internal field, that the specific heat of a ferromagnetic substance should exhibit characteristic changes in the neighbourhood of its magnetic critical point. Below this temperature the specific heat should be increased because it is necessary to supply heat to cause the substance to lose its intrinsic magnetism, i.e. the magnetism associated with it even when it is not exposed to the action of an external magnetic field. The increment, s, of specific heat which thus arises at any temperature is given by

$$s = -\frac{1}{2 \cdot J} \cdot \frac{1}{\rho} \cdot N \cdot \frac{dI_0^2}{dT} \text{ cal. per gm. per degree C.,}$$

where N is the constant of the internal field, I_0 the intensity of intrinsic magnetization per c.c., J the mechanical equivalent of heat, ρ the density of the substance and T the absolute temperature. The experiments made by Weiss and his collaborators† to test this theory were not in many respects conclusive and need not be elaborated here. The above formula leads us to expect that the specific heat of a ferromagnetic body should rise to a maximum at the magnetic critical temperature and should then decrease very rapidly, owing to the sudden disappearance of the magnetism at that temperature. Now, the experiments of Sucksmith and Potter‡ on the behaviour of nickel showed that whilst there was a very close connection between the specific heat of a ferromagnetic substance and its magnetic properties, the connection was not precisely that predicted by the Weiss theory. They found, for example, in the case of pure nickel that the specific heat rose to a maximum as Weiss predicted, but the increased specific heat extended over a range of some $26°$ C. above the magnetic

* P. Weiss, *Journ. de Phys.* **7**, 249 (1908).
† P. Weiss, A. Piccard et A. Carrard, *Arch. Sci. Phys. Nat.* **42**, 378 (1916); **43**, 22, 113, 199 (1917).
‡ W. Sucksmith and H. H. Potter, *Proc. R.S.* A, **112**, 157 (1926).

critical temperature. Unfortunately, they were not able to determine whether the maximum value of the specific heat coincided with the maximum value of $dI_0^2 \, dT$ or not. In the case of manganese arsenide, a substance of chemical formula MnAs which possesses a convenient and sharply defined magnetic critical temperature in the neighbourhood of $45°$ C., the author* found that the specific heat rose to a sharp maximum at $42\cdot2°$ C. and then decreased rapidly. The form of the specific heat curve was similar to that which would be expected on the Weiss theory. The experiments on the magnetic properties were, however, not so favourable to the theory. The sharpness with which the magnetic critical point was defined meant that the curves of $dI_0 \, dT$ and $dI_0^2 \, dT$ were somewhat similar in shape to the specific-heat curve, but whereas the first curve exhibited a maximum at $42\cdot2$ C., the second exhibited a maximum at $41\cdot5°$ C. The difference, $0\cdot7°$ C., appeared to be far outside the limits of experimental error. It was therefore felt that the variation of specific heat with temperature was more nearly proportional to $dI \, dT$ than to $dI_0^2 \, dT$. At the same time it was felt that this conclusion was also justified on theoretical grounds, and the author concluded in agreement with previous workers that, although there was an intimate connection between the specific heat of the ferromagnetic substance and its magnetic properties, it was not that given by the Weiss theory.

The state of our knowledge of this subject could clearly not be regarded as satisfactory, and it was hoped that additional information might be obtainable by an examination of the magnetocaloric phenomena exhibited by manganese arsenide. When a ferromagnetic substance is suddenly inserted into a magnetic field, thermodynamical reasoning† shows that if the process is reversible, an adiabatic rise ΔT in temperature results, according to the equation

$$\Delta T = -\frac{T}{(s)_H} \cdot \frac{\partial I_0}{\partial T} \cdot \Delta H,$$

H, s   where ΔH is the increase in field strength and $(s)_H$ the specific heat of the substance when the field is constant. For a positive increment of H, ΔT is positive, since $\partial I_0/\partial T$ is negative. A similar adiabatic fall in temperature should occur when the field is decreased suddenly. A preliminary calculation with the values of $(s)_H$ and dI_0/dT to hand for manganese arsenide showed that in the neighbourhood of $42°$ to $43°$ C. a field of even 3000 gauss when suddenly applied should produce a rise in temperature of about $1°$ C. Now manganese arsenide exhibits a temperature hysteresis. On losing its magnetism at $45°$ C. it does not become ferromagnetic again until it has been cooled to below $34°$ C., and if it is heated, say, to $43°$ C. and then cooled, its magnetic properties remain practically constant over several degrees. Hence, it would be expected that if the substance were suddenly placed in a magnetic field its temperature would be raised and its magnetic properties more or less permanently changed. In other words, its magnetic properties after it has been

* L. F. Bates, *Proc. R.S.* A, **117**, 680 (1927).

† P. Weiss et A. Piccard, *Comptes Rendus*, **166**, 352 (1918); P. Weiss et G. Foex, *Le Magnétisme*, p. 148 (1926).

placed in and then withdrawn from a magnetic field, should correspond to those at a temperature higher than that which it initially possessed. It might be thought that, on this view, introduction of the substance into a magnetic field would result in a rise in temperature, because of the rapid change of magnetization with increase in temperature, whilst withdrawal from an equal field would result in no appreciable fall of temperature, since the variation of magnetization with a small decrease in temperature is very small. Reference to the expression given above reminds us, however, that the specific heat must also be taken into account, and in the first case the specific heat is larger than in the second, so that we cannot say much about the relative changes of temperature, except that if the Weiss theory of the specific heat is correct the rise in temperature should be greater than the fall. These remarks are intended to apply, of course, only to those temperature regions where the hysteresis is observed.

It seemed, therefore, pertinent to enquire whether an effect of this kind could have entered into the magnetic experiments which the author carried out when testing the validity of the Weiss theory. Since the magnitude of the effect was then unknown and the need for eliminating it was not realized, the experiments were actually carried out in such a manner that disturbances due to the effect must certainly have arisen. In these earlier experiments, the arsenide was placed in a thin copper tube 6·3 cm. long, an induction helix being wound upon the whole length of the tube. The latter was mounted, in series with a compensating helix wound upon a similar tube, between the rounded pole-pieces of an electromagnet. A field of 1960 gauss was usually employed for the induction measurements. The specimen and compensating helix were maintained at suitable temperatures by means of a well-stirred and electrically heated oil bath. The magnetic induction of the specimen was determined from the deflection of a ballistic galvanometer when the specimen was rapidly rotated through 180°, or from the deflection when the magnetic field was switched on or off. In the second case the disturbances were clearly bound to occur. In the first case, since no particular reason was seen for maintaining the magnetic field except when observations were actually being recorded, the field was switched off between measurements. Moreover, even when a field was maintained constant during a complete set of observations, it was not uniform over a large area, and as the specimen rotated certain portions of it passed from a strong into a weaker field and then re-entered a field of the original strength. Here, again, there was the possibility that mere rotation of the specimen may have produced important changes. In fact, it was observed in regions where the magnetization changed rapidly that the galvanometer deflection produced by the first rotation was always markedly greater than those produced by later rotations. It was, of course, probable that this was in part due to the fact that the substance was in the form of a powder, and, of course, to ordinary hysteresis effects in which heat is generated. Therefore, prior to the recording of galvanometer deflections, such rotations were carried out many times, in the expectation that the substance would acquire a steadier state. In this case, too, the magnetic induction actually recorded would be characteristic of a temperature higher than that of the liquid in the surrounding bath. Hence the recorded values of dI_0^2/dT would exhibit a maximum at

a recorded temperature lower than that at which the maximum would occur in the absence of any magnetocaloric effect.

It was therefore concluded that the presence of the effect just described provided a possible explanation for most of the lack of agreement between the experimental results and the Weiss theory, and new experiments have been carried out to obtain the variation of I_0^2 with temperature in the absence of such disturbances. The arsenide was packed inside a small thin-walled glass tube of ellipsoidal shape on which an induction helix was wound. This, together with a compensating helix, was placed in a magnetic field produced by an electromagnet with flat pole-pieces,
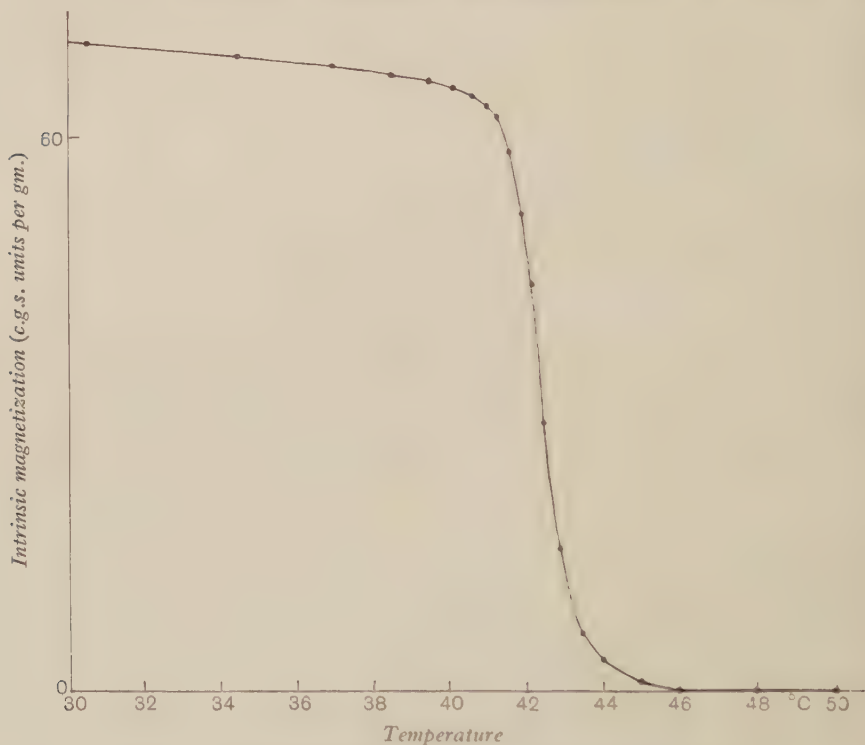
Fig. 1. Variation of intrinsic magnetization with temperature.

and the induction was measured by the deflection of a ballistic galvanometer when the specimen was suddenly rotated through 180°. The substance was therefore maintained in a constant field during the rotation, except for certain minor changes produced by the shape of the specimen. The substance was first cooled to 0° C., when the field was switched on; the field was then maintained constant throughout the whole series of observations. Even with these arrangements and procedure, distinct differences between the galvanometer deflections for first and subsequent rotations at a specified temperature were observed when the temperature was in the region where the magnetization changed rapidly. These were, of course, partly due to ordinary hysteresis heat changes. It was possible to compute, without

likelihood of appreciable error, the deflection which would result if all such disturbances were completely absent. To enable I_0 to be determined from the deflections obtained with a constant field, separate experiments were carried out with various fields, to obtain the relations between the deflection and I_0. Actually, with the fields at the author's disposal, which, unfortunately, did not exceed 4000 gauss,
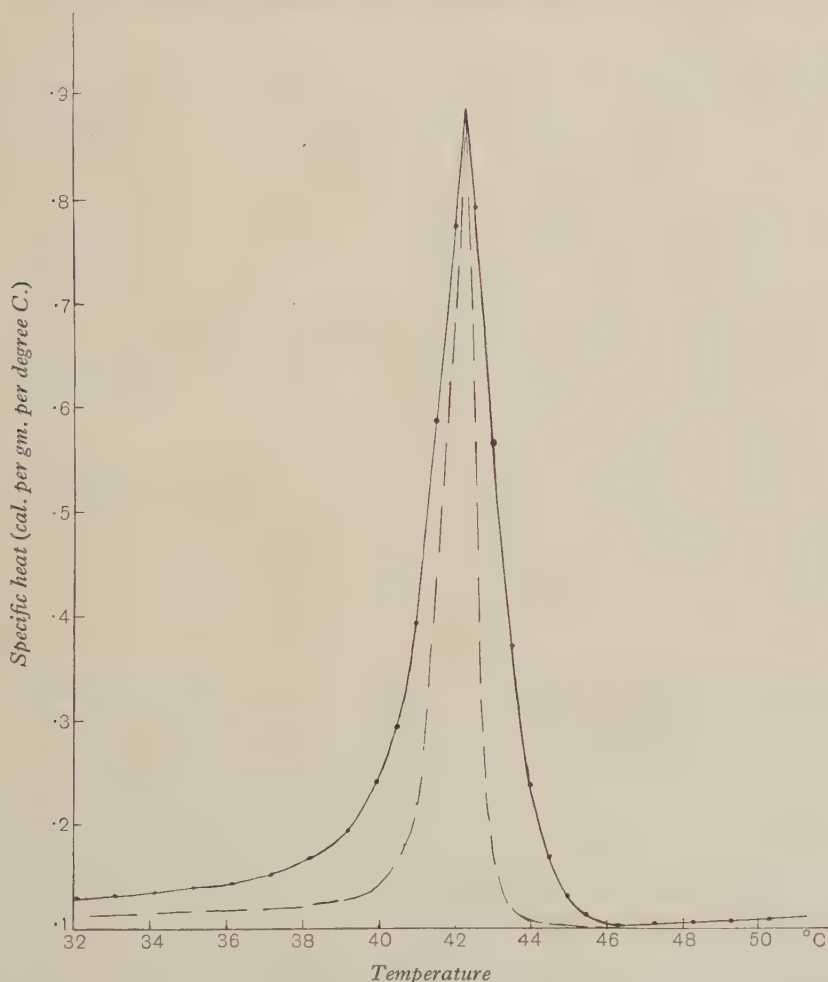
Fig. 2. Variation of specific heat (full line) and of dI_0^2/dT (broken line) with temperature.

it was found that the values of I_0 over the range which is important in this investigation could be found by multiplying the deflections obtained with a single field by a constant numerical factor. The variation of I_0 with temperature as thus determined is shown in Fig. 1.

In Fig. 2 are plotted the values of the specific heat obtained in an earlier research, the continuous curve being the curve of specific heat. The broken curve represents the behaviour of dI_0^2/dT obtained from the curve of Fig. 1. It is quite clear that the

specific-heat maximum and that of $dI_0^2\, dT$ occur at the same temperature. The maximum values of course are not likely to be highly accurate in both cases. Reference should be made to the earlier paper for further details of the specific-heat determinations. There is, however, no doubt that the specific-heat curve differs in several particulars from the curve of $dI_0^2\, dT$*. It is clear that appreciable specific-heat changes occur before $dI_0^2\, dT$ has appreciably changed, and even if the marked changes in specific heat end concurrently with the disappearance of $dI_0^2\, dT$, there is no doubt that the specific heat is considerably greater just before the magnetization disappears than the Weiss theory would lead us to expect. For the sake of completeness we may add that the curve showing $dI_0\, dT$ against temperature gives a sharp maximum at a slightly higher temperature than $dI_0^2\, dT$, and falls less sharply after passing through the maximum.

We may calculate the maximum value which the specific heat should possess on the Weiss theory. The value of N, recently obtained† from susceptibility measurements, may be taken to be 3390, ρ to be 6·02 gm. per cm.³, and $\tfrac{1}{2}dI_0^2\, dT$ at 42·2 C. to be 1560 c.g.s. units per gm., so that the additional specific heat at 42·2 C. is found to be 0·67 cal. per gm. per degree. Hence the total specific heat at this temperature should be 0·77, whilst the maximum measured specific heat is 0·8. The close agreement is rather striking and is probably accidental.

We will now turn to the treatment of the specific heats of ferromagnetic substances as developed by R. H. Fowler and P. Kapitza‡ along the lines of Heisenberg's theory of ferromagnetism. Unfortunately the theory is not sufficiently developed to tell us the actual shape of the specific-heat curve of a ferromagnetic substance in the neighbourhood of its magnetic critical point, but it predicts a step-down in the specific heat at this point of approximately 3 cal. per gm. mol. per degree. The step-down given by the Weiss theory and by experiment in the case of manganese arsenide is of the order of 100 cal. per gm. mol. per degree. Now Fowler and Kapitza's expression for the specific heat contains two factors. One of these is $d\xi^2\, dT$, where ξ is the ratio of the intrinsic magnetization, I_0, to the limiting magnetization, I_∞, which the substance may acquire at very low temperatures or under extremely high fields. At the magnetic critical point T, this factor is taken as equal to 3 T. The experimental data for manganese arsenide show that this is not far wrong. The discrepancy between the experimental and theoretical results must therefore be attributed to the remaining factor, which depends on the interaction integrals of two similar interacting systems each containing one magnetizable electron, calculated from the perturbation theory. The obvious inference is that the magnetism of the arsenide is due to interacting systems each containing more than one magnetizable electron. Fowler and Kapitza suggest that a theory to take account of more than one such electron will differ from that already developed by them only in complication of detail, and will give a value for the step-down of two or three times

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* For convenience in comparison the maxima and minima of the two curves have been arranged to have equal values.

† L. F. Bates, *Phil. Mag.* **8**, 714 (1929).

‡ R. H. Fowler and P. Kapitza, *Proc. R.S.* A, **129**, 1 (1929).

hat mentioned above. It would appear that some more detailed discussion of the more complicated problem is necessary, as the step-down in the case of the arsenide is at least thirty times the value predicted by the theory so far developed. The numerical values obtained by Sucksmith and Potter for the specific heat of nickel are, as we shall see, in rough agreement with Fowler and Kapitza's prediction, but it must be pointed out that their theory is no more successful than the Weiss theory in explaining why the step-down in the case of nickel extends over such a wide temperature range, or why the specific heat continues to change when the magnetism has disappeared. The careful experiments of Sucksmith and Potter therefore force us to conclude that any theory is incomplete which gives a simple term depending only on the magnetization in the expression for the specific heat. We might get
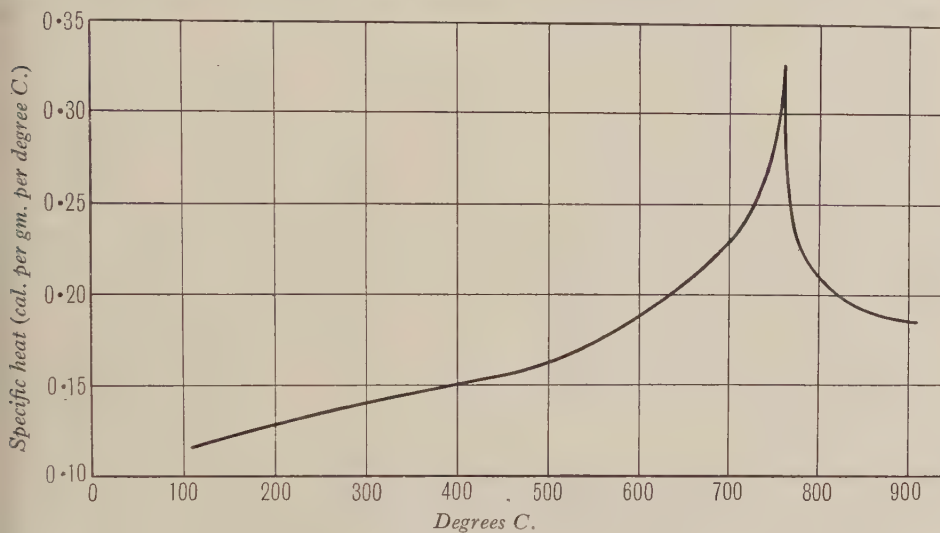
Fig. 3. Klinkhardt's specific-heat/temperature curve for pure iron.

over the difficulty to some extent by taking into consideration the fact that whilst the specific-heat changes continue there is a very small change in the magnetization still in progress. The actual step-down for nickel, calculated from the curves of specific heat of nickel given by Sucksmith and Potter, varies from about 0·9 to about 1·34 cal. per gm. mol. per degree, depending on the specimen of nickel. The corresponding value given by Weiss and his collaborators is 1·67, whilst Klinkhardt, whose work is referred to below, gives 1·47. Now, Fowler and Kapitza remark that they would expect the step-down to be rather less than 3, so that the above numbers may be taken to indicate that the magnetic behaviour of nickel is due to the interaction of systems each containing one magnetizable electron.

In the case of iron, Weiss, Piccard and Carrard give 6·8 cal. per mol. per degree for the step-down. An interesting series of measurements on iron was carried out by Klinkhardt*, who mounted his specimen on quartz hooks in an electric furnace

* H. Klinkhardt, *Ann. der Phys.* **84**, 167 (1927).

and placed it inside a highly evacuated vessel. The furnace served to raise the specimen to any required initial temperature, and when the furnace conditions were such that the temperature of the specimen was rising slowly and regularly, additional heat was supplied by electron bombardment. This additional energy was supplied for about $\frac{1}{2}$ to 1 minute, when the bombardment was stopped, and the rise of temperature, amounting to from 3 to 5° C., was measured with a thermocouple. His curve, obtained for chemically pure iron—apparently not of the highest degree of purity—is reproduced in Fig. 3. The A_2 point was found at 760° C., which is rather low. The step-down indicated in this case is rather more than 8 cal. per gram-mol. per degree C., and it extends over a range of some 150° C. Unfortunately Klinkhardt does not give any magnetic data, but there is no doubt that his work, too, shows that the specific-heat changes persist far above the temperature at which ferromagnetism has disappeared, and that the specific-heat curves are essentially continuous. In fact, the use of the term "step-down" is a little unfortunate, as we tend to restrict its use to changes which are essentially discontinuous.

Another feature which requires explanation is the large minimum value of the specific heat above the magnetic critical point. In the case of iron and nickel we always find minimum values which give atomic heats much greater than the limiting values for other metals. This is particularly emphasized by Klinkhardt's measurements on the specific heat of iron. His minimum value for the specific heat of $Fe\beta$ is approximately 10·3 cal. per gram-atom per degree C., and his value for $Fe\gamma$ is 8·6.

Finally, then, we see that a satisfactory theory must explain firstly the continuous change in specific heat as the substance passes through its magnetic critical point; secondly, the existence of pronounced changes of specific heat at temperatures far beyond those at which the external magnetic properties disappear; thirdly, the magnitude of the change in the case of manganese arsenide; fourthly, the coincidence of the maximum specific heat with the maximum of dI_s^2/dT; and, lastly, the high value of the minimum specific heat above the magnetic critical point. I hope that I have succeeded in showing exactly how far these requirements are met by current theories.

SOME RELATIONSHIPS AMONG THE FERROMAGNETIC CONSTANTS

By J. R. ASHWORTH, D.Sc.

ABSTRACT. Attention is directed in this note to a ferromagnetic equation written after the model of Van der Waals' equation of state for fluids. It is shown that the ratios of the absolute critical temperatures to the maximum intensities of magnetization are nearly the consecutive numbers 2, 3, 4, 5, 6 for iron, cobalt, nickel, Heusler's alloy and magnetite respectively. A formula for the discontinuity of the specific heat at the critical temperature is derived which agrees well with experimental determinations, and a simple formula is given connecting this discontinuity with the true specific heat at the critical temperature. Also certain simple relations among the constants of ferromagnetism disclose themselves which show that the ferromagnetic properties of iron and nickel are to one another as those of cobalt and magnetite.

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IN his paper on magnetism Curie regarded magnetism as analogous to fluids and threw out the suggestion that there may be critical constants for magnetism as for fluids. If this suggestion is developed by writing an equation to ferromagnetism on the model of Van der Waals' equation to fluids as follows,

$$(H + a' I^2) (1/I - 1/I_0) = R' T,$$

where

H is the magnetic field strength,

I is the intensity of magnetization,

I_0 is the maximum intensity of magnetization,

$a' I_0^2$ is the maximum intrinsic field,

R' is the reciprocal of Curie's constant in volume units, and

T is the absolute temperature,

then certain simple relationships amongst ferromagnetic quantities disclose themselves. Some of these will be set down here with as much brevity as possible.

(1) The above equation gives for the critical temperature θ,

$$\frac{\theta}{I_0} = \frac{8}{27} \times \frac{a'}{R'} \qquad \ldots\ldots(1).$$

When numerical values are applied a'/R' has simple integral values to a close approximation as shown in Table 1.

The numbers in the last column progress by uniform steps of nearly 0·3.

(2) *Specific heat and ferromagnetism.* If ΔC is the discontinuity at the critical temperature, then

$$n . \Delta C . \theta = a' I_0^3 \qquad \ldots\ldots(2),$$

or if ΔC be in heat and mass units, then

$$n . \Delta C . \theta J \rho = a' I_0^3 \qquad \ldots\ldots(3),$$

where J is the mechanical equivalent of heat, ρ the density and n a simple number. Thus

$$\Delta C = a' I_0^3/n J \rho \qquad \qquad \ldots\ldots(4).$$

Table 1.

	a'/R'	Percentage difference from whole numbers	$\theta \; I_0$
Iron	1·97	+ ·5	$0·58 = \dfrac{1058^\circ}{1817}$
Cobalt	3·20	− 6·0	$0·95 = \dfrac{1349}{1422}$
Nickel	4·02	− 0·6	$1·20 = \dfrac{661^\circ}{552}$
Heusler alloy	5·09	− 1·7	$1·50 = \dfrac{633^\circ}{420}$
Magnetite	6·03	− 0·5	$1·79 = \dfrac{853^\circ}{477}$

From equation (1) we can substitute for $a' I_0^3$ its equivalent $\frac{27}{8} R' I_0^2 \theta$, and we get

$$\Delta C = \tfrac{27}{8} R' I_0^2 \, n J \rho \qquad \qquad \ldots\ldots(5);$$

the truth of this equation is shown in Table 2.

Table 2.

	n	$\dfrac{27}{8}\dfrac{R'I_0^2}{nJ\rho}$	By experiment	Percentage difference
Iron	1	0·120	0·120	0
Cobalt	1	0·114	0·098	16
Nickel	2	0·029	0·029	0
Heusler alloy	2	0·037	0·036	2
Magnetite	3 2	0·079	0·079	0

It can be shown experimentally that

$$R' I_0^2 = R \, an \qquad \qquad \ldots\ldots(6),$$

where R is the gas constant and $83·15 \quad 10^6$, a is the atomic weight, and an the molecular weight.

Substituting for $R' I_0^2$ in the formula for the discontinuity, we get

$$\Delta C = \tfrac{27}{8} R/J a n^2 \qquad \qquad \ldots\ldots(7),$$

and as $\frac{27}{8} R/J = 6·683$, we have

$$\Delta C = 6·683/a n^2 \qquad \qquad \ldots\ldots(8).$$

In the formula given by Prof. Weiss and amended by H. A. Lorentz the numeral is 4·97 instead of 6·683. The results to which the Weiss-Lorentz formula leads are compared with theory in Table 3.

Table 3.

	Atoms in molecule	Weiss-Lorentz formula	By experiment
Iron	1	0·089	0·120
Nickel	3	0·282	0·285
Magnetite	$\frac{1}{3}$ Fe$_3$O$_4$	0·064	0·079

Table 3 shows a considerable difference between theory and experiment for iron and magnetite.

(3) There is a simple relation between the true specific heat at the critical temperature, C_θ, and the discontinuity ΔC. It is C_θ

$$C_\theta = \tfrac{5}{2}n.\Delta C \qquad \text{......(9).}$$

This formula is probably a reduction of the more general formula

$$maC_\theta = 5na.\Delta C \qquad \text{......(10),}$$

ma and na being molecular weights below and above the critical temperature. The verification of the first formula is given in Table 4.

Table 4.

	n	$\tfrac{5}{2}n.\Delta C$	By experiment	Difference per cent.
Iron	1	0·300	0·309	− 3
Cobalt	1	0·245	0·270	− 9
Nickel	2	0·145	0·154	− 7
Heusler alloy	2	0·180	0·179	+ 0·5
Magnetite	3/2	0·296	0·299	− 1

By using the equations given above and substituting we arrive at a formula for the true specific heat at the critical temperature. It is

$$C_\theta - 16\cdot79/na \qquad \text{......(11).}$$

For the same ferromagnetic the numerical values of n are the same in all the formulae where it appears. Throughout, theory and experiment agree least satisfactorily in the case of cobalt, for which the numerical data are probably less precise than with other ferromagnetics.

The values of C_θ and ΔC are mainly taken from the work of Prof. Weiss and his collaborators. The reduced formulae for C_θ and ΔC apply to the ferromagnetic elements, where a has a definite value, and need a modification for Heusler alloy and magnetite.

(4) There are some interesting correspondences between the ratios of the constants of the ferromagnetics, quite apart from any theory, which deserve notice. They are set out in Table 5.

Table 5.

	a'	R'	I_0	θ	ΔC	Increase of specific heat $(C_\theta - C_{15})$
Iron / Nickel	Ratio 0·085	0·17	3·3	1·6	4·0	4·0
Cobalt / Magnetite	Ratio 0·09	0·17	3·0	1·6	1·23	1·25

Some of these correspondences follow from the ratios of θ to I_0, set out in the first section. Evidently the ferromagnetics are closely allied to one another in their constants.

DISCUSSION ON MAGNETISM

OPEN DISCUSSION

Dr E. C. Stoner (communicated). Among the more general problems of magnetism there is one to which I would like to draw special attention, and that is the precise relation between para- and ferromagnetism.

In the Langevin-Weiss theory the relation between the ferromagnetic properties below, and the paramagnetic above, the Curie point is clear and simple for an ideal substance consisting of an aggregate of magnetic molecules whose interaction can be represented by the conception of the molecular field. The general character and simplicity of the relations are not essentially changed when the modifications necessitated by the older quantum theory are introduced, nor, I think, from the point of view here considered, by the present interpretation of the significance of the molecular field.

The typical normal paramagnetics are salts. The typical ferromagnetics are metals. Above the Curie point the three ferromagnetic elements do behave, over extended ranges of temperature, as normal paramagnetics (in obeying the Weiss law). There is no evidence of definite structural changes at the Curie point. The question then arises whether there are those simple relations between the magnetic characteristics below and those above which would be anticipated from theory.

The answer seems to be that there are not. Below the Curie point the magnetic properties can be accounted for by attributing the rôle of carrier to electron spin. Above, this does not seem to be possible. Too many electron spins per atom would be required. This suggests the conclusion that orbital moments may then also be involved. Even if this view is accepted, however, there remain many difficulties. There is one in the apparently unreasonable numbers of electron spins required per atom to account for the low-temperature saturation values; and another in that it is difficult to connect these numbers simply with the magnetic moments per atom deduced from the susceptibility measurements above the Curie point.

The possibility of a substance being ferromagnetic depends on the sign of the interchange interaction integral; experimentally the Curie temperature is positive. There is thus a possibility of a series of substances passing from those that are ferromagnetic at fairly high temperatures through those that are ferromagnetic at low temperatures to those that are purely paramagnetic; among these last may be some which have a large negative Curie temperature, so that they appear to possess an almost constant paramagnetism. The investigation of alloys in which a continuous change in the Curie temperature can be made promises to shed much light on the whole problem of paramagnetism.

I have brought forward very briefly these two points—the precise relation between the ferromagnetic and paramagnetic properties of a single substance,

and the relation between the magnetic properties of a series of metals and alloys—as perhaps deserving a fuller consideration and discussion than they have generally received.

Mr S. EVERSHED. Sir Alfred Ewing has reminded us of the long years that have gone by since the first publication of his theory of the magnetic control of the process of magnetization. I was from the first a disciple, and from that day to this the ideas at the root of the theory have been my constant guide in ferromagnetic research. Indeed, to anyone who finds himself in daily contact with ferromagnetism experimentally, Ewing's theory is indispensable. Experimental research cannot be effectively conducted without the aid of some working hypothesis, something to provide a foothold or a vantage point. A formula may sometimes serve the purpose, but speaking generally nothing is of greater assistance to the experimenter in his daily work than a mental picture of what is happening. And the more complex the phenomena the greater the need for guidance of that kind.

I know nothing in experimental physics more complicated, more confusing, than the actions of ferromagnetic bodies in the course of magnetization. In his opening remarks Sir Alfred Ewing refers to their behaviour as "an attractive tangle of conspicuous magnetic phenomena." It is an apt description, and if the researcher is to avoid losing himself in the tangle he must carry with him the mental picture which Sir Alfred gave us forty years ago. Up to that epoch various imaginary forces had been suggested to account for the extreme complexity disclosed by magnetization curves, and at a time when little or nothing was known of atomic structure the replacement of all those clumsy expedients by the conception of magnetic control was a stroke of genius.

After forty years the position is very different. Within the present century the structure of the atom has been disclosed to us, and we now see that inside the ferromagnetic body there are only three kinds of inter-molecular action: gravitational, electric, and magnetic. Of these, the first is feeble, and in any case gravitation cannot have an orienting effect. The second, electric force, is inoperative between neutral atoms. Magnetism is therefore the only possible origin for the powerful forces controlling the orientation of a mass of magnetic molecules; and since magnetism is created by the motion of electricity, the controlling field can only come from the planetary electrons. Ewing's fruitful idea, therefore, provides a true picture of the ferromagnetic mechanism so far as the broad outline is concerned. But much of the detail is still hidden from us. Hysteresis requires a system composed of more or less self-contained groups of magnetic molecules, or possibly something equivalent in the atomic structure, and at the present time this part of the mechanism can only be vaguely perceived. Nevertheless, seen or unseen, the machinery must be there, for every magnetization curve, every curve of recoil declares its existence.

Hard as it is to see in our mind's eye what is going on inside a ferromagnetic body during the progress of orientation, it is harder still to provide any alternative in the way of a mathematical framework for the phenomena. In some respect

his is a drawback. To minds with a mathematical bent it may even appear to be
defect in the picture drawn by Ewing. But I rather think the defect lies in our
hental equipment; mathematics by all means, but not at the expense of imagination.
n putting imagination first and mathematics second, I am conscious of attempting
o swim against the prevailing stream of thought, for in recent years there has been
n amazing extension of the field of purely mathematical research, an extension so
ruitful that it tempts physicists to believe that Nature in all her infinite variety
heeds nothing but mathematical clothing. Indeed the ultra-modern physicist
ppears strangely reluctant to contemplate Nature unclad. Clothing she must
have. At the very least she must wear a matrix, with here and there a tensor to
hold the queer garment together.

Such is the present tendency; but I am convinced that in a subject like ferro-
magnetism nothing can take the place of mental imagery. We shall never behold
reality, but we can at least strive to catch sight of an image of it, however fleeting
and imperfect. That, after all, was the method of Faraday. He had no mathematical
knowledge. But he had an unclouded imagination, and with the aid of those images
which formed so readily in his mind he found his way through the tangle of
phenomena to splendid discoveries.

Mr S. EVERSHED (communicated subsequently). In the papers of Mr F. C.
Powell and Dr Bates, the energy absorbed by a ferromagnetic substance when the
change takes place from the magnetizable to the non-magnetizable state is con-
sidered, and Mr Powell compares observed values with those computed on the
basis of Heisenberg's theory. I shall not comment on the theory, beyond remarking
that it needs further development if it is to come into line with the confusing
but well-ascertained facts of ferromagnetism. Sooner or later every hypothesis
bumps into some awkward fact, and whether Heisenberg's theory will survive a
shock of that kind remains to be seen.

But in making numerical comparisons between theory and experiment it
is well to be sure of the facts, and I think the authors of these two papers have
been a little unfortunate in their choice of experimental data. To begin with, the
assumption is made that the magnetic change occurs at a critical temperature, the
so-called Curie point. No doubt an individual atom of alpha-iron has a definite
temperature at which it will transform itself into an atom of the beta kind, provided
it is not influenced by what its neighbours are doing. But observed values are
necessarily derived from experiments on test-pieces containing millions of atoms,
and in such circumstances the loss of magnetism does not occur at a critical
temperature. The change of state from alpha to beta takes place progressively
over a fairly well-defined range of temperature, a range of about 40° C. in the
case of pure iron. Those are the normal conditions of experiment, and I do not
know where the Curie point is to be found. I only know it as it appears in
equations.

The gradual loss of magnetism as the temperature is raised was first traced by

Hopkinson*, and expressed by a curve connecting magnetic state with temperature. Since his time other workers have obtained similar curves, but owing to the difficulties of experiment few of the published curves are free from ambiguity. So far, the most accurate curves of loss and recovery of magnetism appear to be those obtained by Prof. Honda† and in the Research Laboratory at Acton Lane Works, the latter having been observed for me by Mr Finnis‡. In both cases the magnetometer method was employed and every precaution was taken to ensure freedom from the many sources of error involved in experiments at high temperatures.

In pure iron the curve of loss of magnetism is coincident with the curve of recovery, and the true form of such curves having been determined with certainty it is easy to distinguish between good and bad observation. For example, curve 11 in Fig. 1 in Mr Powell's paper is obviously wrong. A well-marked feature of the true curve of loss of magnetism is the very gradual approach to the zero line, and the abrupt ending of curve 11 is quite impossible. On the other hand, in Dr Bates' paper, the curve in Fig. 1 for a specimen of manganese arsenide is on the face of it a correct representation of the loss of magnetism under the conditions of the experiment. To determine the temperature at which the magnetic molecules began, one after the other, to pass into the non-magnetic state, it would have been necessary to reduce the initial magnetization to a somewhat lower value, and in that case the final approach to zero would have been rather more prolonged.

The earliest attempt to measure the energy associated with the magnetic change was made by Hopkinson and is described in the paper already referred to. He computed the energy by the method which Dr Bates refers to as the "step-down." That is to say, Hopkinson assumed that the vertical distance between the portions of the temperature-energy curve above and below the allotropic change was a measure of the energy absorbed by the transformation. Now if this change occurs at a truly critical temperature this simple way of determining the energy is correct, but not otherwise. Here, in short, is a trap for the unwary, and judging from the published data for the energy absorbed in the change from alpha- to beta-iron (the magnetic change) it seems that everyone, including Hopkinson, has fallen into the trap. Certainly I fell into it myself, but when I discovered where I was I got out again, and set to work to compute the energy correctly. Since the magnetic change occupied a range of temperature I resorted to a step-by-step integration from one end of the range to the other. For each element in the summation it was necessary to know the specific heat of alpha-iron, and that of beta. It was also necessary to know, for every temperature within the range, the proportion of alpha molecules to beta molecules. For the specific heats I used the values obtained by Wüst and his co-workers§, and it would be hard to find any

* John Hopkinson, "Magnetic properties of iron at high temperatures," *Proc. R. S. A*, p. 464 (1889).
† Honda and Murakami, *Science Reports, Tohoku University*, **6**, No. 5 (1918).
‡ S. Evershed, *Journ. I.E.E.* **63**, 725 (1925).
§ Wüst, Meuthen, und Durrer, *Die Temperatur-Warmeinhaltskurven der technischen wichtigen Metalle. Forschungsarbeiten auf dem Gebeite des Ingenieurwesens*, No. 204 (1918).

experimental determinations of the kind conducted with greater skill and precision. The proportion of alpha to beta was obtained directly from a curve expressing loss of magnetism as a percentage of the initial value of the flux density.

The result of this somewhat laborious calculation of the energy absorbed in the transformation from alpha to beta is included in the accompanying table, where the energy, it should be noted, is given in small calories per gram of iron.

Table giving the items of allotropic energy in pure iron.

Temperature °C.	Allotropic change	Energy absorbed cal./gm.
700⎫ 770⎬ 810⎭	Precursor effect of unknown origin Alpha gradually changes to beta (loss of magnetism)	4·5 1·4
919	Beta suddenly changes to gamma	6·7
1405	Gamma suddenly changes to delta	1·9
1528	The iron changes from solid to liquid	49·3

The temperature at which the change from alpha to beta takes place in pure iron is apt to vary by a few degrees, possibly as much as 10° either way. These variations, which appear to be due to the presence of gas dissolved in the metal, do not affect the extent of the range of temperature within which the change occurs. In ordinary iron, pure or of industrial quality, the range is always about 40° C., and although the same attention has not been paid to the other magnetic elements, enough is already known to establish the rule that each has its own characteristic range within which the magnetic change occurs.

The facts, which I have sketched as briefly as possible, all point one way. For a mass of magnetic molecules there is certainly no critical temperature; and in view of the wide, and apparently widening, gap between experimental knowledge of ferromagnetism and the theoretical work of continental physicists, it would be useful at this juncture to have a statement of what is meant by the Curie point, and in what way it is related to the range of temperature occupied by the magnetic change? As one who remembers Hopkinson and all he did for scientific progress I should like to suggest that the temperature range of a ferromagnetic substance should be called the "Hopkinson range," he having been the first to investigate it.

An ambiguity occurs in Mr Powell's paper on p. 398, where he gives, for iron, a calculated value for the observed energy, on the basis of Heisenberg's theory, namely 4·4 calories *per degree per gram-atom*. This result is then compared with an observed value 6·8. Similarly the calculated value for nickel is given as 3 calories *per degree per gram-atom*, and compared with an observed value 1·7. But these two observed values are clearly energy in calories *per gram of metal* and hence, unless there has been some slip in stating the result of calculation, the comparison between theory and experiment falls to the ground. No doubt Mr Powell will clear up this doubtful point before his paper appears in the *Proceedings*. Whatever

the event may be, I should prefer to see figures relating to atoms stated in terms of the actual weight of the atom, since the weight of the hydrogen atom is now known with some precision. Surely the time has come to abandon the clumsy gram-atom, the unit of antiquated chemistry.

Mr R. H. Fowler. I want first to comment on some of the difficulties pointed out by Dr Bates. First of all about the step in the specific heat for Fe which may be as much as 8 cal. gm.-mol. I do not think there is any real difficulty in this. Admittedly one cannot work a theory for iron with one magnetizable electron per atom. But there is no reason why there should not be three per atom, and then very likely the theory will work all right, though the exact details are still lacking in spite of Heisenberg's extensions of his original paper*. Of course the enormous step for MgAs is quite outside the possible range of any existing theory. It is extremely interesting and I must congratulate Dr Bates on finding it.

Then again I think Dr Bates said that the specific heats above the Curie point were too high, being about 10 cal. gm. instead of 6 cal. gm. But as he said that the step in Fe and Ni was not too far out, presumably therefore the specific heat is too high in this sense, both above and below the Curie point. I see nothing strange in this. Such extra specific heats are well known in other substances and probably have nothing whatever to do with magnetism.

I should like to supplement Mr Powell's remarks on his determination of $\partial J_0/\partial a$, especially with regard to the sign $(-)$ for Fe. This is most unexpected as it means that the interaction integral, which increases very rapidly as the distance diminishes at large distances, has passed its maximum and is decreasing again as we put the atoms still closer together. This suggests that an approximate theory, using only nearest neighbours, is in some danger of overlooking terms almost as large arising from next nearest neighbours. This point requires further examination.

In conclusion, I should like to remind you that a real theory of metals should be able to answer the question why an assembly of, say, normal Cu atoms with a suitable given energy-content is a metal and not, for example, a gas. As yet we have only made the very first beginning of an attack on this—the real problem of the theory of metals. This beginning has been made by Slater in his recent work in the *Physical Review* and extended by Block for use in the theory of ferromagnetism. I think this work marks the taking of a new step forward of first class importance.

Prof. A. M. Tyndall said that the results obtained by Mr Sucksmith for the gyromagnetic ratio represent work whose difficulty is far greater than his paper indicates, particularly as regards the extreme standard of purity required in the specimens, which must not be handled, for instance.

Dr G. Temple (communicated, and presented by Mr G. B. Sutherland). In his paper contributed to this discussion Prof. Allen has drawn attention to the quantization of the effective magnetic induction in the rigid rotator. I prove below that this

* See the contribution by Mr F. C. Powell.

esult also applies to the hydrogen atom, as was suspected by Prof. Allen. When he principal quantum number, n, is equal to 1, 2 or 3 the result may be obtained y direct integration of the expression for the current density, and these calcula- ions have been actually carried out by Mr G. B. Sutherland of Trinity College. But for higher values of n this method becomes extremely laborious. Fortunately he integrations may be evaded by the device given below, which applies in the most general case.

The effective magnetic induction is defined by analogy with the induction of current flowing in a closed linear circuit. If j is the strength of the current and L the self-induction of the circuit, the current energy is

$$E = \tfrac{1}{2}Lj^2,$$

nd the magnetic induction is

$$N_m = Lj = 2E/j.$$

Prof. Allen takes the same relation, $N_m = 2E/j$, to define the effective magnetic nduction in an atom possessing a magnetic moment. In this case E is the total atomic energy in a particular quantum state and j is the total current crossing a emi-infinite plane bounded by the magnetic axis of the atom. The quantization of N_m is expressed by the equation

$$N_m = nh/e,$$

which may be proved as follows.

With the usual notation for the hydrogen atom, the current velocity at the point (r, θ, ϕ) is

$$v = \frac{h}{2\pi m_0} \cdot \frac{m}{r \sin \theta},$$

nd is in the direction of increasing ϕ. The element of area in a plane passing through he magnetic axis $(\theta = 0)$ is $r\,d\theta\,dr$. Hence the total current crossing the half of his plane on one side of the axis is given in e.s.u. by

$$j = \int_0^\infty \int_0^\pi \rho v \cdot r\,d\theta\,dr,$$

where ρ is the charge density. Since $r^2 \sin \theta\,d\theta\,d\phi\,dr$ is the element $d\tau$ of volume,

$$j = \frac{1}{2\pi} \int \frac{v}{2 \sin \theta} \cdot \rho\,d\tau = \frac{hm}{4\pi^2 m_0} \int \frac{\rho}{r^2 \sin^2 \theta}\,d\tau,$$

the integral being taken over all space.

To evaluate this integral we note that the wave equation,

$$\frac{1}{r^2} \frac{\partial}{\partial r}\left(r^2 \frac{\partial \psi}{\partial r}\right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta}\left(\sin \theta \frac{\partial \psi}{\partial \theta}\right) + \frac{m^2}{r^2 \sin^2 \theta}\psi + \frac{8\pi^2 m_0}{h^2}\left(E + \frac{Ze^2}{r}\right)\psi = 0,$$

nvolves the expression $(r \sin \theta)^{-2}$ for which we require the mean value. Now Schrödinger's solution of this equation shows that the eigen values of

$$\lambda = (8\pi^2 m_0/h^2)E$$

re of the form

$$\lambda_n = -\frac{1}{a^2 n^2},$$

where $(n - m)$ is a positive integer and a, the radius of Bohr's "ground orbit," is $h^2/(4\pi^2 m_0 e^2 Z)$. This result persists even if m is not integral. Thus, if we replace m by $(m + c)$, where c is a small constant, n is replaced by $(n - c)$, and λ_n by

$$- \frac{1}{a^2 (n + c)^2} = - \frac{1}{a^2 n^2} + \frac{2c}{a^2 n^3} + \ldots$$

The new wave operator differs from the old by the addition of the perturbation term

$$\frac{2mc + c^2}{r^2 \sin^2 \theta}, \text{ which } = \frac{2mc}{r^2 \sin^2 \theta} + \ldots$$

Now the ordinary perturbation theory indicates that the change in λ_n is equal to the integral

$$\int \frac{2mc}{r^2 \sin^2 \theta} \cdot \psi^* \psi \cdot d\tau,$$

and we know that $\rho = - e \psi^* \psi$.

Hence

$$m \int \frac{\rho}{r^2 \sin^2 \theta} d\tau = \frac{- e}{a^2 n^3},$$

and

$$j = \frac{- eh}{4\pi^2 m_0 a^2 n^3}$$

$$- \frac{eh \lambda_n}{4\pi^2 m_0 n} = \frac{2e E_n}{nh}.$$

Therefore

$$N_m = 2E/j = nh/e.$$

Dr L. G. CARPENTER. With regard to the statement of Dr Bates that a satisfactory theory of the specific heat of ferromagnetic substances must explain, amongst other things, "the large minimum value of the specific heat above the magnetic critical point," it should be remembered that the property of possessing an atomic heat at constant volume which exceeds $3R$ is not confined to ferromagnetics. It occurs to some extent in all metals at high temperatures, and in the case of tungsten, for instance, the atomic heat at constant volume is of the order of 7 or 8 calories at 1200° abs. Incidentally Klinkhardt's value of the atomic heat of iron above the Curie point, to which Dr Bates refers, is the atomic heat at constant pressure. The atomic heat at constant volume will be of the order of 1 calorie less than this.

On the whole, therefore, as Dr Fowler has already pointed out, it does not seem necessary to demand of the theory of the specific heat of ferromagnetic substances that it shall explain the large minimum value of the specific heat above the magnetic critical point.

Dr J. R. ASHWORTH. Prof. Gerlach's results on the correspondence between the change of electrical resistivity (σ) of nickel with temperature and the variation of the true specific heat as a function of the temperature (t) up to and above the magnetic critical temperature are similar to my own published in the *Philosophical*

Magazine*. Experiments described there showed that the ratio of $d\sigma/dt$ at air temperature and at the critical temperature, for both nickel and iron, is closely the ratio of the true and specific heat at air temperature and the critical temperature. The conclusion then reached was that "there seems little doubt that the large and increasing change of resistivity up to the critical temperature is due to the increase of the specific heat; and as this in its turn depends on the mutual magnetic forces between the molecules we have an explanation why ferromagnetic substances exhibit an abnormally large change of resistivity with rise of temperature." It was further shown that corresponding states hold at least for nickel and iron approximately in regard to the change of resistivity as well as the change of thermo-electric power with temperature. No doubt the large change of the specific heat with rise of temperature in ferromagnetics is responsible for most of the abnormally large changes in other properties with rise of temperature which ferromagnetics exhibit up to the critical temperature.

The e.m.f. which Prof. Gerlach finds between a magnetized part of a piece of nickel below and an unmagnetized part above the critical temperature seems to be analogous to the e.m.f. found between a magnetized electrode of nickel or iron and an unmagnetized electrode of the same metal when they are in a dilute electrolyte. Hurmuzescu† and Paillot‡ made measurements of this e.m.f. and found it to be of the order 10^{-6} volts in fields of 10,000 gauss. This is of the same order of magnitude as that found by Prof. Gerlach.

The theory of this effect§ shows that an e.m.f. of this magnitude would arise if the magnetized electrode possessed a field of the order of the intrinsic field of a magnet, namely, about 10^7 gauss.

Dr H. R. Lang referred to the magnetic researches of Baron von Reichenbach, the earlier results of which were published in 1845‖. Surprising phenomena had been obtained with magnets, such as a glow of light surrounding the poles. This effect he connected with the aurora borealis. An English translation of the work was published in 1850 by Taylor, Walton, and Maberly.

Prof. B. W. Holman. I also have read the work of Baron von Reichenbach with considerable interest. Although that writer's physics was far more original than classical, his work deserves more attention than it has received, because he was a mineralogist of eminence, and because his work was a lengthy record of alleged experimental results attained by him. It was not a record of mathematical exercises. It might interest some members to know that a worker at Faraday House has systematically investigated the extraordinary phenomena recorded by von Reichenbach—in particular the rendering of galena and quartz responsive to an ordinary permanent magnet. I have been shown a piece of transparent mineral which was apparently quartz (I was not allowed to touch it), and responded to a permanent

* *Phil. Mag.* **43**, 401 (1922). † D. Hurmuzescu, *J. de Phys.* **4**, 118 (1895).
‡ R. Paillot, *Comptes Rendus*, **131**, 1194–5 (1900).
§ J. R. Ashworth, *Mem. Manchester Lit. and Phil. Soc.* No. 11 (1914).
‖ Liebig's *Annalen* (March and May, 1845).

magnet and had pronounced phosphorescence. The phosphorescence was so strong that it could be observed readily when the piece of alleged quartz was placed on the shelf of a cupboard in the laboratory and the doors partially closed. Another point of interest in connection with this work of Baron von Reichenbach was that he selected the minerals —galena, blende, chalcopyrite, etc.—in that order, as those which were most readily rendered "magnetic." It will be noted that the minerals he gives (including others not mentioned here) and the order in which he gives them agree with the order of suitability of such minerals for crystal receiving-sets in wireless.

The complete neglect of this rather fantastic work is paralleled in the complete neglect of the very remarkable and by no means fantastic experimental work of Mr W. M. Mordey on the behaviour of paramagnetic substances in two-phase alternating fields. Mr Mordey demonstrated many of these phenomena before the Royal Society and the Royal Institution a good many years ago. The fact that an electromagnet can be made to repel magnetic substances —and not only to repel them but to repel them with a considerable and easily-measurable force which is not at all proportionate to the paramagnetic constant of the material repelled— is a remarkable fact. Moreover, that when a series of such magnets is placed below a dish or plate of glass, certain materials move rapidly in a horizontal direction, and move in a direction at 180 different from that demanded by electromagnetic theory, appears to be a result of fundamental interest—one which should have excited widespread investigation. As a matter of fact, these and many other peculiar and unpredicted quantitative results obtained by Mr Mordey have been completely ignored by nearly all writers on magnetism. In this connection it is interesting to note that Sir Alfred Ewing, in his paper on paramagnetism and hysteresis, ignores the years of careful work which Mr Mordey has put in on the hysteretic behaviour of metals and minerals in alternating current fields.

It is with a sense of deep disappointment that I have attended this discussion. It seems that fundamentally new results such as those of Mr Mordey and other workers in the field of mineral separation by magnetic methods have met with no attention. The majority of the papers appear to deal more with the elaboration of known phenomena, or the clothing of them in mathematical form. Dr Edmund Stoner, who has written a large book on *Magnetism and Atomic Structure*, and has also contributed to this discussion, likewise completely ignores the work of Mr Mordey, and of other people who have published fundamentally new results in this field, but have, like Mr Mordey, published facts and not mathematical hypotheses.

I would refer those who may, after these remarks, be tempted to acquaint themselves with the experimental findings obtained by Mr Mordey, to his lecture before the Royal Institution in 1924 and to his paper before the South African Institution of Electrical Engineers in 1930.

I hope that my remarks will be taken as constructive, not destructive, criticism. I wish more attention devoted to the work which has been done by investigators in the field of the magnetic separation of minerals, a field of work in which many

novel and interesting results—some of them in direct contradiction to accepted magnetic theory—have been obtained, and are relied upon for the sometimes unscientific purpose of making sufficient profits to pay dividends on the investments concerned. I do not wish my remarks in any way to be taken as belittling the wonderful work which has been carried out by one or two of those who have contributed to this discussion—foremost amongst these, of course, being Prof. Gerlach and Dr Kapitza.

Dr L. F. BATES. In reply to Mr R. H. Fowler and Dr Carpenter, I think that I must adhere to the criticism that a satisfactory theory of the specific heats of ferromagnetic substances must explain the large minimum value of the specific heat above the magnetic critical point. It is true that Klinkhardt's value of the atomic heat of iron above the magnetic critical point to which I referred is the atomic heat at constant pressure, and that the atomic heat at constant volume will be somewhat less. But when this deduction is made, the minimum atomic heat still remains very large, and I think it is well to remind ourselves of it.

In regard to Mr Evershed's remarks concerning the use of the term "Curie point," I agree that the term is often somewhat loosely employed. Throughout my contribution to the discussion, I have used the term "magnetic critical point." This has been defined as the temperature at which the value of dI_0^2/dT is a maximum, and also as the temperature at which the intrinsic magnetization disappears. In my opinion it is very desirable to adopt the former definition, and then many of the difficulties mentioned by Mr Evershed disappear.

Mr F. C. POWELL. Mr Evershed raises an important question when he points out that the transition from the ferromagnetic to the paramagnetic state does not occur at a critical temperature, but in a temperature range. There is, in fact, actually no temperature which possesses all the properties of the theoretical Curie point. Some of these theoretical properties may be briefly mentioned. Below the Curie point T_c the substance possesses a spontaneous magnetization σ (certainly on the micro-scale, but not necessarily in bulk) which tends to zero as the temperature T approaches T_c, in such a way that $d\sigma^2/dT$ approaches a finite value. Above the Curie point the substance is paramagnetic. One would expect to find discontinuities in the specific heat and the coefficient of thermal expansion arising from the discontinuity in $d\sigma^2/dT$.

T_c, σ

T

The experimental magnetization/temperature curves obtained for iron and nickel show that σ tends to zero in the required way as the temperature is raised, except for very small values of σ, where the curve flattens out. This flattening out may be taken as a first indication of a critical range of temperature in which the normal ferromagnetic properties are replaced by normal paramagnetic properties, in a way which is theoretically inexplicable at present. It should be noted that $d\sigma^2/dT$ and hence the specific heat and coefficient of thermal expansion do not change discontinuously, but rise up sharply to maxima (or descend sharply to minima) just below the critical range. The existence of this critical range shows that some factor has been overlooked by the theory, but it is small and its effect is marked only

in the neighbourhood of the critical temperature. One would not be justified in applying the theory within the critical range, where the factor is important, but presumably the theory may be applied outside that range without serious error.

In calculating the change of specific heat at the Curie point one compares the specific heat where $d\sigma^2 dT$ is zero (i.e. just above the Curie point) with the specific heat where $\mid d\sigma^2 dT \mid$ is greatest (i.e. just below the Curie point); if, owing to the presence of some extra factor, the Curie point is replaced by a critical range, the calculated change of specific heat may be interpreted as the difference between specific heats just above and just below the critical range (provided the latter be not too large). Now this is precisely the experimentally observed quantity which is usually called the change of specific heat at the Curie point, although what is really meant is the difference between the specific heat in the paramagnetic state and the specific heat at the maximum which occurs just *below* the critical range (not *in* the critical range, since the maximum occurs where $\mid d\sigma^2 dT \mid$ is greatest).

It would thus appear that although the presence of some factor overlooked by the theory modifies considerably the transition from the ferromagnetic to the paramagnetic state, the calculated change of specific heat at the Curie point may justly be compared with the experimental values. The occurrence of the Curie point in the theory does not invalidate the theory completely, while the use of the term in describing the experimental results is purely for convenience, and does not imply an unfortunate choice of data. A similar argument holds for any quantity which theoretically shows a discontinuity at the Curie point. It is not clear which temperature ought to be taken for the T_c which occurs in the formulae, but for most purposes it makes no practical difference which temperature in the critical range is selected. Mr Evershed suggests that I have made a slip in stating the experimental values of the specific-heat changes for iron and nickel, but they are, I believe, correctly given. The values were obtained by Weiss, Piccard, and Carrard*. I should like to draw attention to the more recent measurements of Umino†, which unfortunately I had previously overlooked. From these Honda‡ obtains the values for the specific-heat changes given in the following table:

Substance	ΔC_v cal. per degree per gram-atom
Iron	4·6
Nickel	2·1

These values are in much better agreement with the calculated values.

It should be emphasized that in obtaining the theoretical values which I gave, I neglected the Gaussian distribution of energy levels, a step which is justifiable only when s, the number of atoms whose interaction with a given atom must be taken into account, is large. But if, as Heisenberg supposed, interactions with nearest

* Weiss, Piccard, and Carrard, *Arch. Sci. Phys. Geneva*, **42**, 378 (1917); **43**, 22, 113, 119 (1917).
† S. Umino, *Sci. Rep. Tohoku*, **16**, 593, 1009.   ‡ K. Honda, *Zeit. für Phys.* **63**, 141 (1930).

eighbours only are important, somewhat lower values for ΔC_v are obtained, as
hown in the following table:

Substance	Number of electrons per atom	z	ΔC_v cal. per degree per gram-atom
Iron	3	8	4·1
		∞	4·4
Nickel	I	12	2·2
		∞	3·0

We should expect the experimental values to lie somewhere between the two limiting
alculated values. The above experimental values lie in both cases just outside
he theoretical range.

THE INDUCTION OF ELECTROMOTIVE FORCES IN A MOVING LIQUID BY A MAGNETIC FIELD, AND ITS APPLICATION TO AN INVESTIGATION OF THE FLOW OF LIQUIDS

By E. J. WILLIAMS, Ph.D.

ABSTRACT. A magnetic field induces electromotive forces in a moving liquid, and by investigation of the e.m.fs. produced by a known magnetic field it is possible to obtain information about the distribution of velocities in the liquid. Experiments on the flow through straight tubes show that potential differences of the order of 10^{-4} to 10^{-3} volts, set up by a magnetic field in a moving liquid consisting of an aqueous solution of copper sulphate, can be satisfactorily measured. Observations on the flow through curved tubes have also been made.

§ 1. INTRODUCTION

I T follows from Faraday's laws of electromagnetic induction that electromotive forces are induced in a liquid which is in motion in a magnetic field, the e.m.f. per cm. at any point being equal to the vector product of the magnetic force and the velocity. The velocity at any point may therefore be calculated from a knowledge of the induced e.m.f. and the field strength. This fact may be used for investigating the distribution of velocities in a moving liquid, for this distribution can be deduced from observations on the e.m.fs. induced at different points in the liquid by a known magnetic field.

This method of investigating the flow of liquids has the advantage of using the very simple and universally valid relation which exists between the velocity and the induced electromotive force. As the latter depends upon the direction of the velocity as well as its magnitude, the method differentiates between motions in different directions. It also has the advantage that the e.m.fs. are induced instantaneously, so that in the case of unsteady motion velocity-alternations of high frequency are as faithfully represented as those of low frequency[*].

The method involves the measurement of electrical potential differences, and its successful application depends upon the magnitudes of the e.m.fs. induced in actual cases and their relation to the magnitudes of any spurious potentials which may exist. For a velocity of 1 cm. sec. the gradient of e.m.f. induced by a field of 10,000 gauss is 10^{-4} volt cm., whilst a moderately good galvanometer is capable of registering 10^{-6} to 10^{-7} volts. The order of magnitude of the e.m.f. is thus

[*] The only limitation in this respect is due to the inductance of measuring apparatus.

ufficiently large to offer reasonable scope for accurate measurement. Experiments vhich have been made on the steady flow through a straight cylindrical tube for vhich the distribution of velocities is known show that induced potential differences f the order of 10^{-3} volts or greater, set up in a moving liquid consisting of an queous solution of copper sulphate, correspond satisfactorily to the theoretical .m.fs. required by Faraday's laws of induction, spurious effects being comparatively mall. In this connection it might also be pointed out that the potential differences lealt with in experiments on the effect of a magnetic field on the electrical resistance f liquid metals are due to e.m.fs. induced in a moving liquid by a magnetic field, nd in such experiments induced e.m.fs. as small as 10^{-6} to 10^{-7} volts have been neasured without hindrance from spurious effects*.

In applying the above method of investigating the flow of liquids it is in general ecessary to introduce "searching" electrodes into the liquid in order to determine he e.m.f., and these may disturb the flow. It is also necessary to use an electrically onducting liquid. These difficulties may be met by the use of fine wires as electrodes nd of mercury or aqueous solutions of salts as the moving liquid. In the present xperiments an aqueous solution of copper sulphate was used, whilst the lectrodes were made of copper. Under such conditions it was found that though o polarisation e.m.fs. are produced at the electrodes, contact potentials of ppreciable magnitude are set up. As was indicated above, the induced e.m.f. in hese experiments was of the order of 10^{-4} to 10^{-3} volts, and in comparison with his voltage the contact potentials were found to be reasonably constant and did not eriously affect the measurements. A more serious limitation to the present method rises from the fact that the quantity that can be measured directly by experiment s the potential difference between two points in a liquid and not the e.m.f., and hese are not equal when electric currents are induced in the liquid. Fortunately n a large number of cases which are of particular interest the induced currents are ither small or entirely absent and in such cases the quantitative validity of the nethod is not impaired. For cases in which the induced currents are expected to e large the method may still be used to give an accurate description of certain eatures of the motion, such as the frequency of velocity fluctuations in turbulent notion, even though the magnitude of the velocities cannot be accurately deter- nined. It is possible to proceed in certain special cases of this kind by assuming a elocity distribution and allowing for the effect of induced current in calculating he corresponding potential gradient. The calculated potential gradient may then e compared with the observed potential gradient. This procedure, which is lescribed more fully in § 4, is especially successful in the case of flow through traight pipes.

In those cases in which induced currents are produced, the Ampère forces etween the magnetic field and the induced currents may modify the original flow, nd this is another possible source of trouble from induced currents†. When the

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* See accompanying paper by the author, p. 479.

† This secondary effect gives rise to an additional resistance to the flow of the liquid, and the xtra work done is equal to the energy dissipated by the induced currents.

liquid is an aqueous solution the induced currents are so small, owing to the high resistivity of the liquid, that this secondary effect of the field is negligible*. Even for good conductors such as mercury the difficulty may be met by the use of low magnetic fields. The corresponding reduction in the magnitude of the induced e.m.f. is compensated by the fact that in the case of mercury it is possible to measure induced e.m.fs. of a much smaller magnitude than in the case of aqueous solutions, owing to the comparative absence of spurious effects arising from contact potentials.

A further discussion of the e.m.fs. and currents induced in various types of flow is given in §§ 2, 3 and 4. General equations for the induced currents are given in § 2, and these are applied to special cases in §§ 3 and 4.

An account of the present experiments is given in § 5. The experiments on the effect of a magnetic field on the electrical resistance of mercury, which also show the practicability of the measurement and interpretation of the e.m.fs. induced by a magnetic field in a moving liquid, are discussed in the subsequent paper.

§2. GENERAL DIFFERENTIAL EQUATIONS

u, v, w Let u, v, w be the components of the velocity of the liquid at x, y, z;

α, β, γ α, β, γ those of the magnetic force;

E_x, E_y, E_z E_x, E_y, E_z those of the induced e.m.f. per cm.;

I_x, I_y, I_z I_x, I_y, I_z those of the electric current per sq. cm.;

X, Y, Z X, Y, Z those of the electrostatic potential gradient†;

V V the electrostatic potential; and

σ σ the electrical conductivity of the liquid.

Then, from Faraday's laws of induction,

$$E_x = \beta w - \gamma v,$$
$$E_y = \gamma u - \alpha w,$$
$$E_z = \alpha v - \beta u \qquad\qquad \ldots\ldots(1)‡.$$

From Ohm's law the electric current is given by

$$I_x = \sigma\,(E_x + X),$$
$$I_y = \sigma\,(E_y + Y),$$
$$I_z = \sigma\,(E_z + Z) \qquad\qquad \ldots\ldots(2).$$

From (1) and (2)

$$\frac{\partial I_x}{\partial y} - \frac{\partial I_y}{\partial x} = \sigma \left(\frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x} \right) + \sigma \left(\frac{\partial X}{\partial y} - \frac{\partial Y}{\partial x} \right).$$

* The order of magnitude of the ratio of the Ampère forces to the mechanical forces is represented by $(H^2/\rho\eta) \times 10^{-9}$, where ρ is the resistivity expressed in ohm-cms., and η is the viscosity.

† Due to space and surface charges, set up by the induced e.m.f.

‡ The currents induced in the liquid produce a secondary magnetic field. This field is, however, negligible in comparison with the primary field and in all actual cases α, β and γ may be taken as the components of the primary field.

as the electric potential is single valued $\partial X/\partial y - \partial Y/\partial x = 0$, so that

$$\frac{\partial I_x}{\partial y} - \frac{\partial I_y}{\partial x} = \sigma \left(\frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x} \right) = \sigma \left\{ \frac{\partial}{\partial y} (\beta w - \gamma v) - \frac{\partial}{\partial x} (\gamma u - \alpha w) \right\}.$$

The conditions of continuity of velocity and magnetic force are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0; \qquad \frac{\partial \alpha}{\partial x} + \frac{\partial \beta}{\partial y} + \frac{\partial \gamma}{\partial z} = 0,$$

and hence

$$\frac{\partial I_x}{\partial y} - \frac{\partial I_y}{\partial x} = \sigma \left\{ \left(\alpha \frac{\partial}{\partial x} + \beta \frac{\partial}{\partial y} + \gamma \frac{\partial}{\partial z} \right) w - \left(u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} + w \frac{\partial}{\partial z} \right) \gamma \right\}.$$

Similarly

$$\frac{\partial I_y}{\partial z} - \frac{\partial I_z}{\partial y} = \sigma \left\{ \left(\alpha \frac{\partial}{\partial x} + \beta \frac{\partial}{\partial y} + \gamma \frac{\partial}{\partial z} \right) u - \left(u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} + w \frac{\partial}{\partial z} \right) \alpha \right\},$$

$$\frac{\partial I_z}{\partial x} - \frac{\partial I_x}{\partial z} = \sigma \left\{ \left(\alpha \frac{\partial}{\partial x} + \beta \frac{\partial}{\partial y} + \gamma \frac{\partial}{\partial z} \right) v - \left(u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} + w \frac{\partial}{\partial z} \right) \beta \right\}$$

$$\ldots\ldots(3).$$

In the case of steady motion the current also obeys the condition of continuity so that

$$\frac{\partial I_x}{\partial x} + \frac{\partial I_y}{\partial y} + \frac{\partial I_z}{\partial z} = 0 \qquad\qquad \ldots\ldots(4)^*.$$

The potential difference between any two points is given by

$$V_A - V_B = \int_A^B X\,dx + Y\,dy + Z\,dz \qquad \ldots\ldots(5), \qquad V_A, V_B$$

where the electric force is given in terms of the induced current and e.m.f. by equation (2).

§3. TWO-DIMENSIONAL FLOW

Let us consider two-dimensional flow in the xy plane in which case $w = 0$. We shall also suppose that the magnetic field is uniform and parallel to the z axis. Under such conditions the expressions on the right-hand side of (3) vanish so that no electric current is induced in the liquid†. The induced e.m.f. sets up space and surface charges and the electrostatic field due to these charges exactly neutralizes the e.m.f. The electromotive force between any two points is therefore equal but opposite in direction to the potential difference between the points, and the components of velocity are given by

$$v = - E_x/\gamma = X/\gamma,$$
$$u = E_y/\gamma = - Y/\gamma \qquad\qquad \ldots\ldots(6),$$

* The electrical capacity of the liquid is so small that even for unsteady motion this condition is obeyed to all intents and purposes.

† This may readily be seen from the fact that the total amount of liquid which flows out of a closed circuit in the xy plane is zero. For if u_n be the velocity normal to an element ds of the circuit the integral of the e.m.f. round the circuit $= \oint E_s ds = \oint \gamma u_n ds = \gamma \oint u_n ds = 0$.

the components X and Y of potential gradient being measurable by means of an external circuit.

Thus in the case of two-dimensional flow no complications arise from the effect of induced currents. This case includes several types of flow which are of particular interest, such as the motion past an infinite cylinder or aerofoil.

The expressions on the right-hand side of (3) also vanish, even if w is not zero, provided $\partial w/\partial z = 0$. This includes the case of flow through a pipe in which a circulation in the plane of the section of the pipe is superimposed upon the longitudinal motion. The circulation postulated by Dean* in the case of flow through a curved pipe may therefore be investigated without any uncertainties caused by induced currents.

§4. CALCULATION OF POTENTIAL GRADIENT FOR CERTAIN CASES IN WHICH INDUCED CURRENTS ARE PRODUCED

If induced currents are produced in the liquid then the potential gradient depends not only upon the e.m.f. but also upon the electric current. Whilst the e.m.f. at any point depends only upon the velocity and the field at that point the induced electric current theoretically depends on the velocity and field at all other points of the liquid. The potential gradient at a point is therefore not completely determined by the conditions prevailing at that point. In the case of turbulent motion it is, however, probable that the fluctuating part of the potential gradient is mainly dependent on the velocity fluctuations in the immediate neighbourhood of the point concerned, so that the existence of induced currents is not of much consequence. The average potential gradient in turbulent motion, or the steady potential gradient for stream-line motion, may on the other hand be appreciably dependent on the flow in parts of the liquid remote from the point investigated. In such cases it appears that the only way in which the method can be quantitatively applied is to assume a distribution of velocities and calculate the corresponding distribution of induced currents and potential gradient. A comparison of the calculated potential gradient with the observed potential gradient then indicates the accuracy of the distribution of velocities assumed. Such a procedure would, probably, be rather elaborate in most actual cases. We shall consider here the special case of flow through a straight tube and calculate the induced currents and potential gradient produced by a transverse uniform magnetic field. This is the case which has been dealt with in the experiments.

The distribution of velocity over the cross-section of a straight pipe of radius a has been the subject of much investigation. The distribution for stream-line flow is well established and is represented by

$$w = (2Q/\pi a^4)\,(a^2 - r^2) = k\,(a^2 - r^2) \qquad\qquad \ldots\ldots(7),$$

where w is the velocity parallel to, and at a distance r from, the axis of the pipe, and Q is the volume of liquid flowing through per second. For turbulent flow the

* *Phil. Mag.* **5**, 673 (1928).

velocity distribution is not so definitely established. The velocity (or more precisely the average longitudinal velocity) falls off much less rapidly with increasing value of r than in stream-line flow, and the velocity gradient lies between that represented by (7) and the zero gradient in a constant velocity distribution. A distribution of velocities in which w is proportional to $(a^4 - r^4)$, i.e.

$$w = (3Q/2\pi a^6)(a^4 - r^4) = k'(a^4 - r^4) \qquad \ldots\ldots(8), \qquad k'$$

gives a velocity gradient between these extremes, and though it may not represent the actual gradient in turbulent flow it can be combined with these extreme distributions to give other intermediate distributions. We shall now consider the distribution of potential gradient corresponding to the velocity distributions (7) and (8), produced by a transverse magnetic field.

Let us choose the x axis parallel to the magnetic field and the z axis parallel to the axis of the pipe. Then $\beta = \gamma = 0$, $u = v = 0$. The e.m.f. is confined to the xy plane (at right angles to the velocity) and is independent of z, in which case the equations (3) and (4) for the induced current reduce to

$$\partial I_x/\partial y - \partial I_y/\partial x = \sigma\alpha . \partial w/\partial x,$$
$$\partial I_x/\partial x + \partial I_y/\partial y = 0 \qquad \ldots\ldots(9),$$

where w is given by (7) or (8). The walls of the pipe may be assumed to be insulating so that the electric current normal to the walls is zero. From this boundary condition and equations (9) and (7) it is found by solving a Poisson equation that the induced current for stream-line flow is given by

$$I_r = (k\alpha\sigma/4)(a^2 - r^2)\sin\theta,$$
$$I_\theta = -(k\alpha\sigma/4)(a^2 - 3r^2)\cos\theta \qquad \ldots\ldots(10),$$

where I_r and I_θ are the radial and tangential components respectively. (θ is the angle the radius vector makes with the field, i.e. with the x axis.)

The electromotive force is equal to $\alpha \times w$ and is parallel to the y axis. Its components are therefore

$$E_r = k\alpha(a^2 - r^2)\sin\theta,$$
$$E_\theta = -k\alpha(a^2 - r^2)\cos\theta \qquad \ldots\ldots(11).$$

The components of potential gradient are therefore, by equation (2),

$$R = I_r/\sigma - E_r = -k\alpha(a^2 - r^2)\sin\theta = -(3Q\alpha/2\pi a^4)(a^2 - r^2)\sin\theta, \qquad R$$
$$\Theta = I_\theta/\sigma - E_\theta = k\alpha(3a^2 - r^2)\cos\theta = (Q\alpha/2\pi a^4)(3a^2 - r^2)\cos\theta \ (12). \qquad \Theta$$

R and Θ can be determined experimentally, and in view of the fact that the velocity distribution upon which (12) is based is well established, the observed potential gradient for steady flow should agree with (12). Observations on the potential gradient produced in this case are described in the next section.

The general nature of the distribution of induced current is represented in Fig. 1. It consists of two "circulations" symmetrical about the y axis, the current vanishing at two points on the x axis at distance $a\sqrt{3}$ from the origin. This

distribution is similar to the hydrodynamic circulation considered by Prof. Dean in the case of flow through a curved pipe provided the axis of curvature of the pipe in that case is chosen parallel to the magnetic field in the present case. Though the differential equations for the two cases are not identical there is a general analogy. The e.m.f. in the present case corresponds to the centrifugal force in the case of the hydrodynamical circulation produced in a curved pipe. Both are greater the greater the longitudinal velocity, so that in both cases the force near the axis is much greater than the force near the walls. This gives rise to the circulation.

The difference between the e.m.f. represented by (11) and the potential gradient represented by (12) is due to the potential gradient caused by induced currents and is
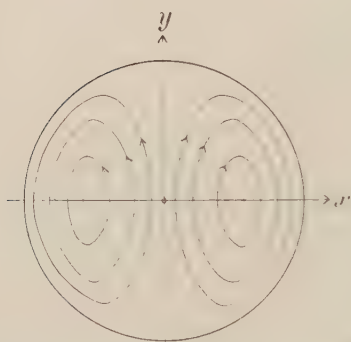
Fig. 1. Distribution of induced current in
liquid in a straight pipe.
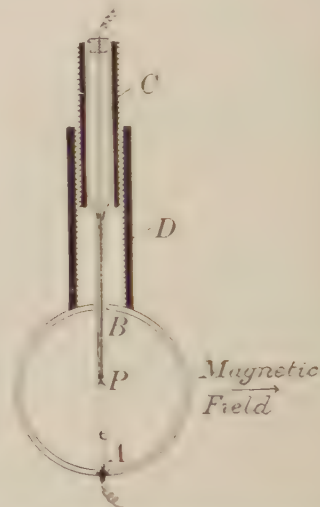
Fig. 2. Cross-section of tube showing electrodes.

about one-third of the total potential gradient. Neglect of the induced currents would therefore lead to appreciable error. As the velocity at the walls is zero, the e.m.f. vanishes at the walls, so that the tangential potential gradient at the walls, required by (12), is *all* due to the induced current.

The distribution of potential gradient corresponding to the velocity distribution for turbulent flow, equation (8), is given by

$$R = (Q\alpha/4\pi a^6) \{2r^4 \sin 5\theta - (7a^4 - 5r^4) \sin \theta\},$$
$$\Theta = (Q\alpha/4\pi a^6) \{- 2r^4 \cos 5\theta + (7a^4 - r^4) \cos \theta\} \quad ......(13).$$

Let us consider the potential gradient along a diameter perpendicular to the field (APB in Fig. 2). It can be shown from (12) and (13) that for both the velocity distributions (7) and (8) the potential difference between the ends of this diameter is given by

$$V_A - V_B = H\bar{w}d \quad ......(14\,a),$$

$\bar{w}$ being the mean velocity $Q\,\pi a^2$, d the diameter of the tube, and H the strength

of the magnetic field (replacing α). The potential gradient along the diameter at any point is also in both cases proportional to the velocity, w, at that point, so that

$$\partial V_p/\partial\epsilon \propto Hw \qquad \ldots\ldots(14\,b),$$

ϵ being the distance between A and P. Equations $(14\,a)$ and $(14\,b)$ together determine completely the potential gradient along the diameter AB in terms of the velocity gradient. In the case of a distribution of constant velocity across the tube there are evidently no induced currents produced and the potential gradient at any point is therefore equal in magnitude to the gradient of induced e.m.f. at that point, i.e. equal to Hw and in a direction parallel to AB. It follows that the potential gradient at any point on the diameter AB is in this case also given by equations (14). There is little doubt that for all practical purposes the potential gradient along AB is given by equations (14) for all velocity distributions in which the rate of falling off of the velocity from the axis to the walls is somewhere between that in steady flow and a constant velocity distribution*. Thus in the case of flow through straight pipes the effect of induced currents can be calculated and allowed for and the gradient of velocity can be deduced, by means of equations (14), from observations on the potential gradient along a diameter perpendicular to the field.

§ 5. EXPERIMENTS ON THE FLOW THROUGH A CYLINDRICAL TUBE

Fig. 2 represents the cross-section of the tube and the arrangement of electrodes used, the magnetic field being at right angles to the axis of the tube in the direction shown. The potential differences between the point A at the end of the diameter perpendicular to the field and a point P on this diameter at different distances from A were measured.

The diameter of the glass tube used was 1·075 cm. Two small holes 1 mm. in diameter were bored at A and B. A fixed copper electrode passing through the hole at A was sealed so that its end was flush with the inside walls of the tube. The electrode passing through B was fixed in a brass tube C which could be moved in the direction AB by a screw movement in another brass holder D, the latter being waxed to the walls of the tube. The copper electrode extending into the tube was encased in a tight-fitting glass capillary with thin walls. This served to keep the wire rigid as well as to insulate it up to its end. The external diameter of the capillary was 0·05 cm. and therefore caused little obstruction to the flow. Copper sulphate solution was used as the moving liquid. The strength of the

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* The reason for this result may be seen in a general way as follows. For the same mean velocity, $\bar{w}$, the integral of the gradient of e.m.f. along the diameter AB is greater the more rapidly the velocity falls off from the axis to the walls. It exceeds the value $H\bar{w}d$ for a constant velocity distribution by an amount depending on the steepness of this velocity gradient. However, the steeper this gradient the stronger are the induced currents, and the potential difference between A and B due to these must be subtracted from the integral of e.m.f. to give the resultant potential difference ($V_A - V_B$). The result expressed by $(14\,a)$ means that these effects cancel out leaving a resultant potential difference of $H\bar{w}d$ in all cases.

magnetic field used was about 10,000 gauss and in the experiments the change in the potential between A and P on reversal of the field was measured. This change in potential was measured by a potentiometer method. The arrangement is shown in Fig. 3. The resistance R was adjusted after the field had been reversed so that the current traversing the galvanometer G was the same as before the field was reversed, the initial current being partly due to contact potentials. The change in R then gives the potential difference between A and P due to the magnetic field*. In control experi-

Fig. 3. Potentiometer.

ments which were carried out, it was found that without the flow, or with the flow but with the electrodes nearly touching, no measurable potential difference was produced on application of the field. Of course any spurious potential gradient produced by the field that is scalar in nature leads to no error, provided, as in the present experiments, the effect of reversing the field is observed.
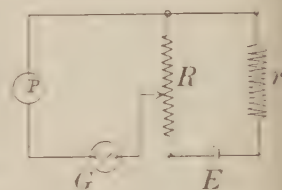
The critical mean velocity for the tube used was about 21 cm. sec., and observations were made on three velocities, one much less than the critical velocity, the other in the neighbourhood of it and the third much greater than the critical velocity. The observed values of $V_A - V_B$ in these cases and the values calculated from (14 a) are given in Table 1. They are seen to be in very satisfactory agreement.

Table 1. Potential difference ($V_A - V_B$) between the ends of a diameter perpendicular to the field. ($H = 9500$ gauss.)

Mean velocity $\bar{w}$ (cm./sec.)	$V_A - V_B$ (millivolts)	
	Calculated value $H\bar{w}d$	Observed value
11·2	1·15	1·18
20·0	2·04	2·02
66·0	6·73	6·81

The estimated probable error in the calculated value due to errors in the measurement of H, w and d is about 1 to 2 per cent. The observed values which are given are each the result of several observations differing amongst themselves, as the result of contact potential effects, by about 0·05 to 0·1 millivolt. The estimated probable error in the mean observed values is about 0·03 millivolt. The agreement between these and the calculated values given in the second column of the table shows that there are no unknown sources of error involved in the measurement of induced potential differences under the conditions of the present experiments†.

* The change in R was negligible compared with the resistance of the circuit containing the electrodes so that this change negligibly affected the current through the galvanometer due to contact potentials.

† The difference of 0·08 millivolt between the observed and calculated values for the highest velocity is only a difference of 1 per cent. and may easily be due to the error in estimating the theoretical value.

The potential differences between the fixed electrode at A and the movable electrode P for different positions of the latter on the diameter AB were also measured, and the results show how the velocity varies from the axis of the tube to the walls. The results for the velocity of $11 \cdot 2$ cm./sec. in the region of steady flow and of $66 \cdot 0$ cm./sec. in the region of turbulent flow are represented in Figs. 4
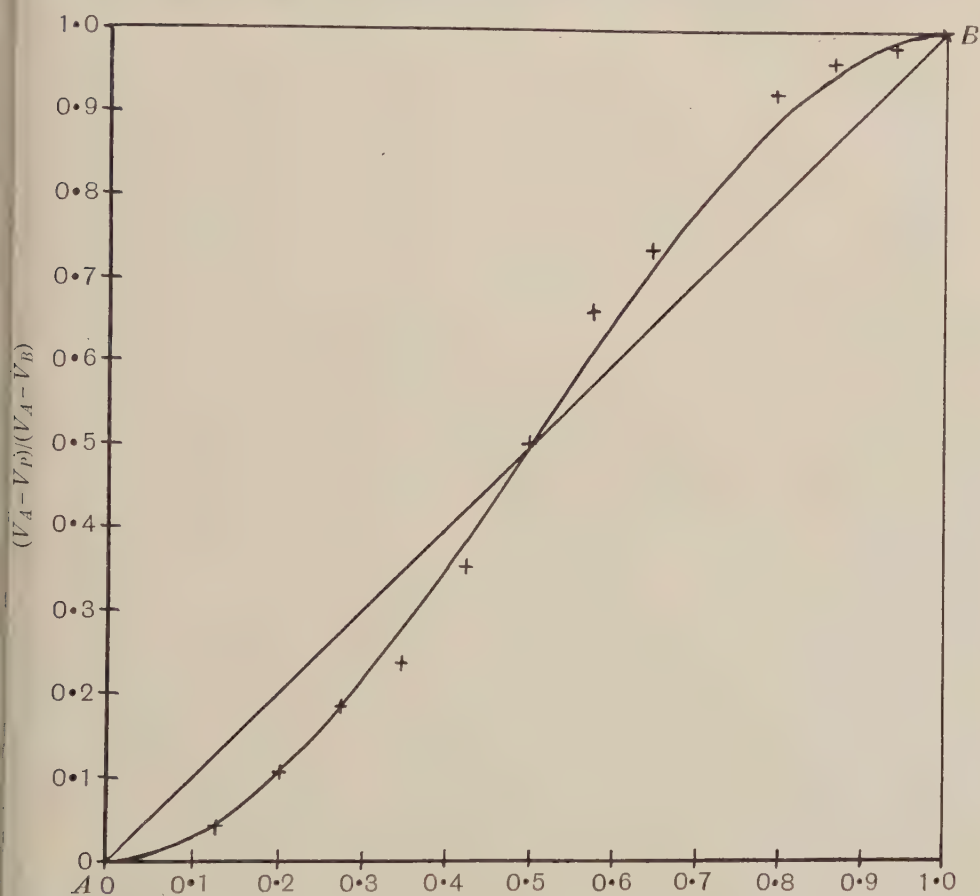
Fig. 4*. Variation of voltage along a diameter for steady flow.

× × observed values. Curved line AB: theoretical values. Straight line AB: values corresponding to a constant velocity across tube.

and 5 respectively. The ordinates represent $(V_A - V_P)/(V_A - V_B)$, and the abscissae the position of P in terms of ϵ/d. On these scales the positions of the observed points involve less error in the case of turbulent flow, since the potential differences measured in this case are on the average about six times greater than those measured in the other case.

The observed points in Fig. 4 clearly show the more rapid flow near the axis

* The observed value of $(V_A - V_B)$ was $1 \cdot 18$ mv. for Fig. 4 and $6 \cdot 81$ mv. for Fig. 5.

of the tube and an approach to a zero velocity near the walls. The theoretical value of $V_A - V_P$ in this case may be calculated from (12), or (14), and is given by

$$V_A - V_P = \int_{-a}^{\epsilon-a} (3QH\, 2\pi a^1)\, (a^2 - r^2)\, dr$$
$$= 3H\overline{w}d\, (1 - 2\epsilon/3d)\, \epsilon^2/d^2 = 3\, (V_A - V_B)\, (1 - 2\epsilon/3d)\, \epsilon^2/d^2 \,...(15).$$
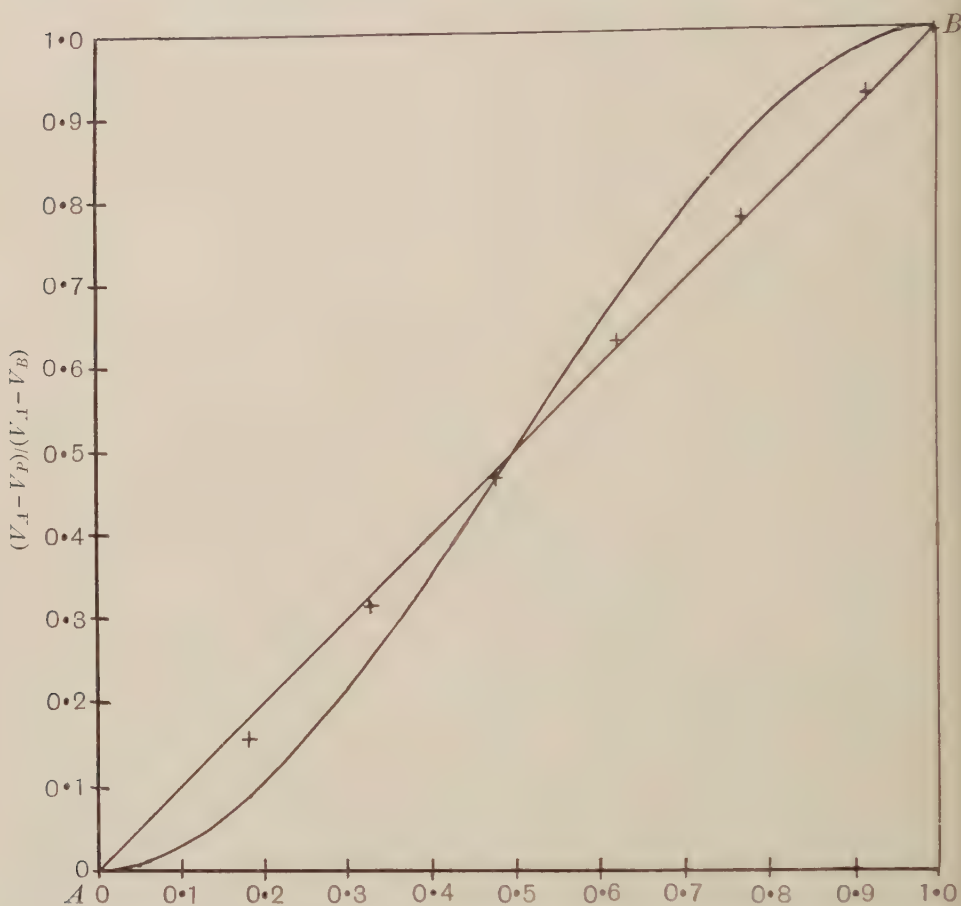
Fig. 5. Variation of voltage along a diameter for turbulent flow.

× × observed values. Curved line AB: theoretical values.
Straight line AB: values corresponding to a constant velocity across tube.

(15) is represented by the curved line AB in Fig. 4, and as it is based on the well established distribution of velocities in steady flow, the deviations of the observed points from this line show the accuracy with which the induced potential differences were measured. The root-mean-square deviation corresponds to about 0·02 milli-volt. This is somewhat less than the differences between the observed and theoretical values in Table 1. This is to be expected, since in Table 1 the absolute observed and theoretical values are compared whilst in Fig. 4 it is the values of the ratio

$V_A - V_P)/(V_A - V_B)$ that are dealt with, in which case errors in the measurement of the field and velocity do not come in. From the results in Table 1 and Fig. 4 we may conclude that potential differences induced in a moving liquid consisting of an aqueous solution of copper sulphate can be measured with the present arrangement with an accuracy of about one-thirtieth of a millivolt.

The results for the case of turbulent flow represented in Fig. 5 show that the velocity is nearly constant across the section of the tube. The straight line AB represents the values of $(V_A - V_P)/(V_A - V_B)$ for a constant velocity distribution, and it is seen that the observed points are much nearer this line than the curved line, AB, which corresponds to steady flow. It is of course to the gradient of $(V_A - V_P)$ that the velocity is proportional, and this cannot be measured with the same accuracy as $(V_A - V_P)$ itself. Still the results seem to indicate a bigger velocity near the walls than that found by F. E. Stanton in 1911 in experiments on the turbulent flow of air through straight pipes with smooth walls[*].

A few observations were made on the flow through a spiral with a view to observing the circulation in curved pipes deduced by W. R. Dean[†] and subsequently observed by G. I. Taylor[‡] in experiments using coloured bands. The coils of the spiral in the present experiments were nearly touching each other, the diameter of the coils being 4 cm. and that of the cross-section of the tube 0·7 cm. The spiral was placed in the magnetic field and measurements made on the potential along a diameter perpendicular to the field at a point on the spiral where the longitudinal flow was parallel to the field. In such a case the longitudinal flow produces no potential gradient so that any potential difference induced between the electrodes must be due to motion in the plane of cross-section of the tube[§]. The circulation in this plane, referred to above, arises from the effect of centrifugal force and its direction is therefore independent of the direction of the longitudinal flow. This circumstance was made use of to eliminate any effects due to the longitudinal flow which would be produced if, for instance, the latter were not adjusted exactly parallel to the field at the point investigated. The mean longitudinal velocity in the experiments was about 60 cm./sec., for which, according to an empirical relation due to C. M. White[||], the motion is steady. The observations gave no indication of motion in the plane of cross-section of the tube and the results show that under the conditions of these experiments the velocity, at any point, of the circulation described by Dean is probably less than $\frac{1}{2^{14}}$ of the longitudinal velocity at that point. In his experiments with coloured bands Taylor found that the ratio of the two velocities over a large range of longitudinal velocity was roughly equal to twice the ratio of the cross-sectional diameter to the diameter of the coils. In the present experiments this ratio is about $\frac{1}{3}$, and therefore from the results obtained for the velocities it appears that the relation mentioned by Taylor has no general applicability.

* *Proc. R.S.* **85**, 366 (1911).   † *loc. cit.*   ‡ *Proc. R.S.* **124**, 243 (1929).

§ Though induced currents are produced in other parts of the tube they vanish, by symmetry, in the section investigated.

|| *Proc. R.S.* **123**, 645 (1929).

The above observations on the flow through a spiral were carried out chiefly to demonstrate the applicability of the method to the determination of the component of velocity in any direction, even though this component may be much less than the resultant velocity. In the above case, for instance, we are able to set to the velocity in a certain direction an upper limit which is about 20 times less than the resultant velocity.

§6. ACKNOWLEDGMENTS

DISCUSSION

THE MOTION OF A LIQUID IN AN ENCLOSED SPACE

By E. J. WILLIAMS, Ph.D.

Communicated by Prof. W. L. Bragg, F.R.S., March 16, 1930.
Read and discussed, June 13, 1930.

ABSTRACT. The increase of resistance of a column of mercury in a magnetic field, found in experiments on this effect, is due to the internal motion of the liquid produced by the action of the Ampère forces between the magnetic field and the electric current traversing the mercury. The hydrodynamic significance of the results of such experiments is considered and the consistency of the results shows that e.m.fs. as small as 10^{-6} to 10^{-7} volts, induced by a magnetic field in moving mercury, can be accurately measured. This result shows the practicability of the method of investigating the flow of liquids proposed in the previous paper.

§ 1. INTRODUCTION

WHEN a liquid conductor carrying an electric current is placed in a magnetic field forced convection currents are in general produced by the action of the Ampère forces between the field and the electric current. If, for instance, the liquid is contained in a straight glass tube in a transverse magnetic field, motion is produced in the regions where the field falls off at the edge MM', Fig. 1, of the pole pieces of the magnet. The energy required to move the liquid is supplied by the battery which drives the electric current, and a back e.m.f. is set up in essentially the same manner as in an electric motor. The back e.m.f. is induced by the action of the magnetic field on a moving liquid and it is proportional to the field strength and the velocity of the liquid. Information about the motion of the liquid may therefore be obtained by observations on this induced e.m.f., and such experiments may be regarded as a special application of the method of investigating the flow of liquids described in the preceding paper. In the experiments under consideration here not only may the velocity be found by electrical measurements but as the motion is itself produced by electric forces the "driving" forces also may be measured electrically.

The back e.m.f. gives rise to an apparent increase in the electrical resistance of the conductor caused by the magnetic field and the author showed some time ago that this effect accounts for the results of nearly all experiments on the magneto-resistance of liquid metals*. The purpose of the present paper is to consider the hydrodynamical significance of such experiments. The results obtained show the practicability of the measurement and interpretation of e.m.fs. induced in a moving liquid by a magnetic field and therefore of the method of investigating the flow of

* *Phil. Mag.* **50**, 27 (1925).

liquids described in the previous paper. The type of flow investigated is that of a liquid in an enclosed space produced by external volume forces. The stream lines in such a case are essentially curved, and it is interesting to consider the results of the present analysis in relation to observations on the flow of liquids through straight and curved pipes.

The nature of the flow of a liquid through a straight pipe is well known. For small velocities the motion is stream-lined and the resistance is proportional to the velocity. For higher velocities the motion is turbulent and the resistance varies approximately as the square of the velocity. These facts are shown graphically in Fig. 2 (*a*), where the values of (resistance velocity) are plotted against the velocity. The transition from the first region to the second is marked by changes in the height as well as the slope of the curve. The recent experimental work of C. M. White*
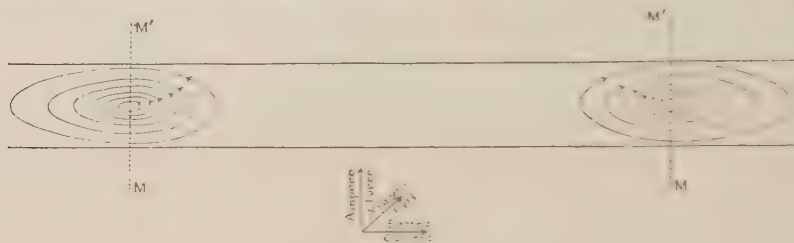
Fig. 1.

shows that for the case of flow through a curved pipe there exist three regions. His results are represented in Fig. 2 (*b*). For small velocities the motion is stream-lined and the resistance varies linearly with the velocity as in the case of straight pipes. This region is represented by the horizontal portion *AB* of the curve. As the velocity is increased above that corresponding to *B* the curve *gradually* leaves the straight line *ABB'*. The change is not marked by a discontinuity either in the height or slope of the curve and it is attributed to the gradual increase in the effect of centrifugal force which is essentially always present in a curved pipe. (C. P. Dean† has shown that this centrifugal force produces a stream-lined circulation in the plane of the cross-section of the tube.) Thus in the second region which begins at *B* the motion is still steady though the resistance is no longer proportional to the velocity. If the velocity is further increased a point *E* is reached where the stream-lined motion breaks down and turbulent motion sets in. This change corresponds to the transition which takes place in a straight tube but differs from it in that there is no discontinuity in the height of the curve. The interpretation of the three regions has been convincingly verified by G. I. Taylor by direct observation by means of coloured bands‡.

Since the motion of a liquid in an enclosed space, as in the case of a liquid conductor in a magnetic field, is essentially curvilinear, one would expect centrifugal force to produce the same general effect as in the case of flow through a curved pipe. The results of experiments on the apparent increase of electrical resistance of a

* *Proc. R.S.* **123**, 645 (1929).   † *Phil. Mag.* **5**, 673 (1928).
‡ *Proc. R.S.* **124**, 243 (1929).

iquid conductor due to a magnetic field show, however, that this is not the case. These experiments reveal only two regions as shown in Fig. 2 (*c*). As in all cases there is an initial region *AB* of small velocities for which the motion is stream-lined, the resistance being proportional to the velocity. In the second region *BG* the resistance varies approximately as (the velocity)$^{1\cdot75}$, and the motion is in all probability turbulent. The middle region for curved pipes, represented by *BE* in Fig. 3, is absent. We may suppose that turbulence sets in in these experiments before the effect of centrifugal forces becomes appreciable. However, in that case we should expect the transition to turbulence to resemble the transition observed in
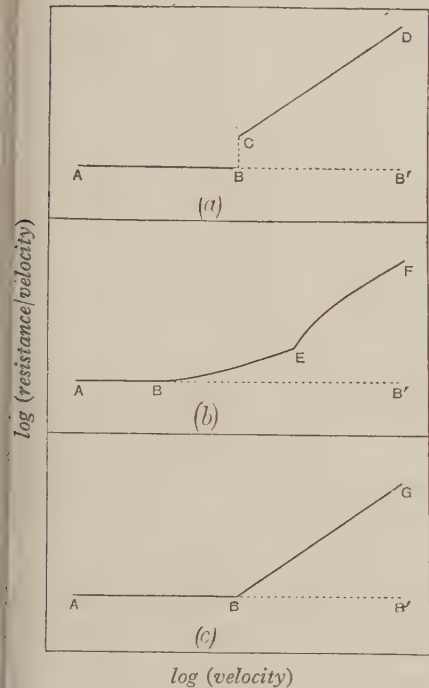
Fig. 2. Variation of resistance with velocity in (*a*) straight pipes, (*b*) curved pipes, (*c*) enclosed space.
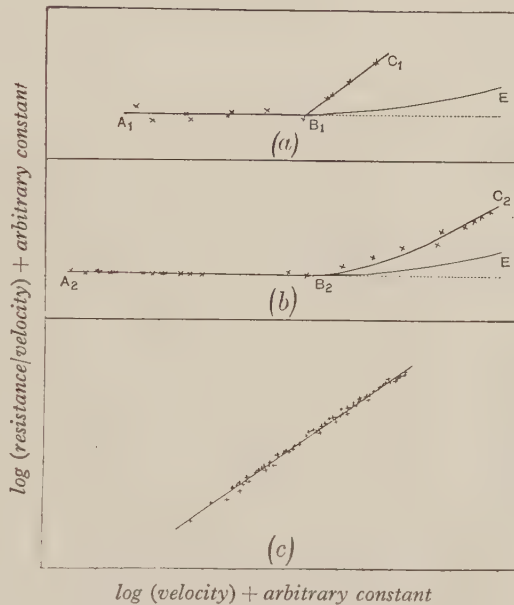
Fig. 3. Relation between resistance and velocity for mercury.

straight pipes and represented in Fig. 2 (*a*). This is not the case since the transition in the experiments concerned is not accompanied by a discontinuity in the height of the resistance/velocity curve, nor are there any velocities for which the motion is not stable.

§ 2. ANALYSIS OF THE EXPERIMENTS

In experiments on the effect of a magnetic field on the resistance of mercury the quantities measured are the dimensions of the liquid conductor, the strength H of the magnetic field, the electric current i traversing the conductor, and the increase δr in electrical resistance. In all the experiments that will be considered here the increase of the electrical resistance is wholly due to the motion of the mercury.

H

i

δr

The Ampère forces which maintain the motion of the mercury are proportional to the product Hi of the magnetic field and the electric current. When the motion produced reaches an average steady state these driving forces are balanced by the hydrodynamic resistance R so that

$$R \propto Hi \qquad \qquad \ldots\ldots(1)^*.$$

The induced back e.m.f. δe is proportional to the product of the magnetic field and the velocity v so that the latter is proportional to $\delta e \, H$. If δr is the apparent increase in electrical resistance δe is equal to $i . \delta r$ so that

$$v \propto i\delta r / H \qquad \qquad \ldots\ldots(2)^*.$$

The values of R/v and v which are the quantities represented in Fig. 2 are therefore proportional to $H^2/\delta r$ and $i\delta r/H$ respectively.

The values of $\log(H^2 \delta r)$ and $\log(i\delta r \, H)$ calculated from the results of experiments carried out by T. Jones† and P. Jones‡ are represented graphically in Fig. 3 at (a), (b) and (c). The following are specifications of the experiments concerned:

(a) Mercury contained in straight glass tube of diameter d equal to 0·96 mm. Magnetic field from 4,800 to 10,000 gauss. Induced e.m.f. from $5\cdot8 \times 10^{-7}$ to $1\cdot1 \times 10^{-5}$ volts.

(b) Mercury contained in bent glass tube for which $d = 1\cdot1$ mm., the tube being bent so as to cross the edge of the magnetic field several times. H from 3,200 to 10,000 gauss. Induced e.m.f. from $3\cdot6 \times 10^{-7}$ to 4×10^{-5} volts.

(c) Mercury contained in straight glass tube, $d = 5\cdot0$ mm. H from 2,000 to 10,000 gauss. Induced e.m.f. from $1\cdot1 \times 10^{-5}$ to 5×10^{-4} volts.

Curves (a) and (b) in Fig. 3 show the existence of two regions with different laws of variation of resistance with velocity. The portions A_1B_1 and A_2B_2 of the curves are horizontal so that for these low velocities we have

$$\text{Resistance} \propto \text{velocity} \qquad \qquad \ldots\ldots(3).$$

At B_1 and B_2 respectively this law breaks down. In (a) the slope of the curve changes suddenly at B_1 and for higher velocities the resistance is proportional to $v^{1\cdot8(\ldots1)}$. The departure from a linear law in the case of curved pipes is represented by the curve BE and it is seen to be much more gradual than that represented by BC. In the experiments represented at (b) the linear law breaks down at B_2, and though the change in slope is not as sudden as in the first case it is much more rapid than the change in the case of curved pipes. (The slope of the latter part of B_2C_2 corresponds to $R \propto v^{1\cdot6(\ldots1)}$.) The less rapid change in slope in (b) is not unexpected. In the corresponding experiments the tube used was bent so that the column of mercury crossed the "edge" of the magnetic field several times. As was pointed out in § 1 with reference to Fig. 1, the motion produced by the Ampère forces takes place in

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* Strictly speaking these expressions for R and v are valid only provided the stream lines of motion maintain a constant shape. This consideration affects very little the general nature of the results arrived at.

† T. Jones, *Phil. Mag.* **50**, 46 (1925).

‡ P. Jones and T. Jones, *Phil. Mag.* **2**, 176 (1926).

ie region where the field falls off so that in these experiments the motion takes place
about ten different places. The conditions obtaining are not exactly the same for
hese different places and they may therefore have slightly different critical veloci-
es. The transition for the resultant resistance/velocity curve is thus spread out.
n the case of the experiments represented at (a) the motion is produced at two
ymmetrical points so that the abruptness of the transition is not affected.

In the experiments represented at (c), the resistance is nowhere proportional to
he velocity. The points all lie within experimental error on a straight line, the slope
f which corresponds to

$$\text{resistance} \propto \text{velocity}^{1\cdot75\pm0\cdot03} \qquad \ldots\ldots(4).$$

his law is within experimental error the same as that which corresponds to the
lope of the parts of curves (a) and (b), after the transition point B. T. Jones carried
ut several experiments in addition to those represented at (c), and though the
onditions in these experiments vary considerably, comprising different shapes of
he boundary of the mercury and different distributions of field and current, the
aw of variation of resistance with velocity is in all cases in close agreement with
quation (4). It therefore appears that there are only two régimes for the motion of
liquid in an enclosed space corresponding to the two laws of resistance given by
quations (3) and (4). Rough calculations of the actual magnitude of the velocities
n some of the experiments for which the resistance obeys equation (4) give values
o high that it is inconceivable that the motion is stream-lined. (3) is only obeyed
when the conditions are conducive to steady motion, i.e., when the velocity and
diameter of the tube are small. The transition from the first régime, represented by
the horizontal portions of curves (a) and (b), to the second régime, represented by
the sloping portions of the curves, is therefore in all probability a transition from
stream-line motion to turbulent motion, the stream-line motion being such that the
effect of centrifugal forces is negligible. The transition takes place without a dis-
continuity in the resistance, in contrast with the transition which takes place in the
case of flow through straight pipes. If this difference is attributed to the curvi-
linear nature of the motion we must suppose that this has considerable influence
on the transition itself without appreciably affecting the stream-line motion for
velocities immediately preceding the transition point. When the stream-lines are
curved the centrifugal forces tend to make the resistance depend on a higher power
of the velocity than the first, and it is difficult to explain the absence of the middle
region BE, Fig. 2 (b), which exists in the case of flow through curved pipes.

The motion of mercury in an enclosed space of the type indicated in Fig. 1 is
thus of a different character from that of motion through a straight tube or curved
tube. In all cases there is an initial region of small velocities for which the resistance
is proportional to the velocity. In the motion under consideration this régime is
maintained until turbulence sets in, in contrast with the flow through curved pipes;
and the transition to turbulence takes place without a discontinuity in the resist-
ance/velocity curve in contrast with the flow through straight pipes.

§3. VARIATION OF THE RESISTANCE WITH THE VISCOSITY AND DENSITY OF THE MERCURY

In the case of steady motion the velocity produced by given driving forces is inversely proportional to the viscosity of the liquid and is independent of the density of the liquid. In turbulent motion, on the other hand, the density is a more important controlling factor than the viscosity. The exact relations may be found by applying the principle of dynamical similarity and it will be interesting to see if these agree with the observed dependence of the velocity on the viscosity and density. If we assume that the motion of the mercury depends only on the driving forces, the shape and size of the boundary of the liquid, its viscosity μ and its density ρ, then it follows from the above principle that the dimensionless quantities $(H^2/\delta r) \times (s/\mu)$ and $(i\delta r/H) \times (\rho\mu)$ are functions of one another, where s denotes the linear dimensions of the system*. It can be shown from this result that in the region for which the resistance is proportional to the velocity, and in that for which it is proportional to the (velocity)$^{1\cdot75}$, the velocities produced by given driving forces are respectively proportional to $(\mu^{-1}\rho^0)$ and $(\mu^{-1\cdot7}\rho^{-8\cdot7})$. T. Jones and P. Jones observed the values of δr at different temperatures, keeping the values of H and i constant. The corresponding changes in the velocity should therefore be given by the above expressions. The observed and calculated increases in the velocity due to increases in the temperature above room temperature are given in Tables 1 and 2. Table 1 refers to experiments carried out under conditions of steady motion, equation (3), and Table 2 to experiments carried out under conditions of turbulent motion, equation (4)†.

Table 1.

Tempera-ture t (° C.)	$\mu_{17} \div \mu_t$	ρ_{17}/ρ_t	(Velocity at $t°$) ÷ (velocity at 17°)	
			observed	calculated
100	1·30	1·015	1·30	1·30
180	1·51	1·030	1·51	1·51

Table 2.

Tempera-ture t (° C.)	$\mu_{17} \div \mu_t$	$\rho_{17} \div \rho_t$	(Velocity at $t°$) ÷ (velocity at 17°)	
			observed	calculated
100	1·30	1·015	1·055	1·045
280	1·58	1·048	1·119	1·090

* These results were used in a previous discussion, by the author, of the dependence of δr on h, i, etc., *Phil. Mag.* 50, 27 (1925).

† In these experiments the amount of liquid used amounts to only a small fraction of a cc. and this circumstance makes it easy to carry out experiments at different temperatures.

It is seen that the observed variation of the velocity with the viscosity and density of the liquid agrees very satisfactorily with the calculated variation. This means that all the factors which affect the motion of the mercury were taken into account in the application of the principle of similitude, when it was assumed that no slip takes place between the mercury and the glass and that there is no surface property involved. The verification of the results therefore shows that in the motion of mercury over glass no slip takes place either in steady motion or turbulent motion.

§4. MAGNITUDE OF INDUCED E.M.F.

The magnitude of the induced e.m.fs. measured in the experiments which have been considered ranges from about 4×10^{-7} volts to 5×10^{-4} volts, and even for the smallest voltages the measurements give consistent results. This is especially shown by the horizontal portion A_2B_2 in Fig. 3 (b), which corresponds to a range of voltage from $3\cdot6 \times 10^{-7}$ to about 6×10^{-6} volts. The points all lie within 1 or 2 per cent. on a straight line, which represents stream-line motion at low velocities. The experiments therefore show that, in the case of mercury, contact potentials and other sources of spurious effects do not hinder the measurement of induced e.m.fs. of the order of 10^{-7} volts or more.

§5. ACKNOWLEDGMENT

In conclusion I should like to thank Professor W. L. Bragg, F.R.S., for his kind interest in this work.

DISCUSSION OF THE PRECEDING TWO PAPERS

Dr W. Jevons: The effect described in the author's first paper was observed in the course of some experiments at sea in which I was privileged to take part in 1918*. Two series of experiments were made. In one series, pairs of electrodes were moored at various distances apart in the entrance to the River Dart, and connected by insulated cables to a recording millivoltmeter in a shore observation hut. Continuous records extending over prolonged periods clearly showed the existence of periodic electromotive forces in the sea having a period identical with that of the tide, and amplitudes which varied from a maximum at the time of spring tides to a minimum at neap. In the other series of experiments, two electrodes 100 yards apart were towed in tandem at distances of 60 and 160 yards from a ship, and connected by insulated cables to a millivoltmeter aboard. The direction and magnitude of the tide were roughly estimated by observation at a buoy, and then a quadrilateral course was steered, with sides respectively down, across, up and across

* F. B. Young, H. Gerrard and W. Jevons, "On electrical disturbances due to tides and waves," *Phil. Mag.* **60**, 149 (1920).

the tide. The millivoltmeter readings were of the same order on the down and up courses, but greater and less respectively on the cross courses, the difference between these latter readings being almost invariably of the expected sign and magnitude (about 2 mv. per 100 yards per knot of tide velocity). In all but the smoothest seas the e.m.f. time graphs showed short-period variations, which were most marked when wind and tide were opposite to one another and at right angles to the line of the electrodes; these were clearly the result of wave action. The experimental details and results are fully discussed in a paper published in 1920*. [Some of the records obtained in the two series of observations were shown in lantern slides at the meeting.]

Dr E. G. RICHARDSON: There are one or two points in these interesting papers on which I should like information. The electrodes are shown to give the integrated velocity gradient from the boundary to various points in the tube. This method, as in the ordinary capillary flow viscometer, requires one to assume a distribution of velocity across the tube and to test this against the observed flow. The hot wire possesses the advantage that it can be used to plot the velocity from point to point through the liquid. Is this possible with the present method?

The hot wire is liable to error when used in close proximity to the walls of the tube. Is there any correction required when the electrodes are close to the boundary?

The electromagnetic method certainly possesses the advantage of having no lag, and I suppose could be used to measure a fluctuating velocity with the aid of a string galvanometer. Has this been attempted? Dryden in Washington has used in connection with the hot wire an inductive circuit which compensates for the lag when rapidly fluctuating air currents are being measured.

AUTHOR'S reply: I am very interested to hear of the experiments by Dr Jevons on the e.m.fs. produced in the sea by the action of the earth's field on tidal motion. They represent the application on a large scale of the principle underlying the present method of investigating the flow of liquids.

I am glad that Dr Richardson, who has taken a large part in developing the hot wire method, is present. In reply to his first question I would say that the velocity from point to point in a liquid can be determined directly by the present method provided the conditions are such that no induced currents are produced by the magnetic field. The limitation set when induced currents are produced is emphasized in my papers, and in such cases the distribution of velocity can be determined only indirectly—by calculating the distributions of potential corresponding to different distributions of velocity and comparing them with the observed distribution of potential. In the case of flow through a straight pipe this procedure is very satis- factory, and the velocity at a point can for all practical purposes be connected with the potential gradient at that point and hence directly determined. It might be pointed out that the arrangement of electrodes used in my experiments is only incidental. The potential difference between the fixed electrode at the wall and a

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* F. B. Young, H. Gerrard and W. Jevons, "On electrical disturbances due to tides and waves," *Phil. Mag.* **60** 149 (1920).

ven position of the movable electrode gives of course only the integrated locity gradient, but by taking different positions of the latter the velocity at point can be determined from the slope of the curve connecting $(V_A - V_P)$ and ϵ. his could be obtained more directly by employing two movable electrodes kept a small constant distance apart, but for various experimental reasons this pro- dure was not adopted.

In the present method there is no reason to expect any special trouble in in- estigating the velocity close to the walls and no correction of the kind mentioned by r Richardson is necessary. In the present experiments, for instance, one electrode as kept flush with the walls but there was no indication of trouble due to this.

I have not used the method for investigating fluctuating flow. With the help f sufficient voltage amplification and a string galvanometer it could no doubt be sed for that purpose.

A SIMPLE METHOD OF SHOWING THE MODES OF VIBRATION OF A WIRE

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ABSTRACT. A method is described of showing the modes of vibration of a wire by heating it with an alternating electric current and varying its tension, the various modes being easily observable owing to the luminosity of the wire.

THE methods usually employed for showing the various ways in which a string or wire may be made to vibrate, such as Melde's experiment and the use of the monochord, are troublesome and tedious. An electrically driven tuning fork to one prong of which a thread is attached, provision being made for alteration of the tension by means of a tightening-peg, is satisfactory except when the thread is divided into only a few segments, in which case the applied tension is apt to overpower the fork. The method to be described is simpler to carry out than any of the foregoing and enables any number of vibrating segments between 20 and 1 to be obtained and readily seen.

In the method under notice regular impulses are imparted to a wire by means of an alternating electric current, which produces periodic changes in the length of the wire as its strength rises and falls. A thin wire, heated to a high temperature, is necessary in order that marked differences in the length of the wire may be produced during each cycle and a good amplitude obtained. A suitable wire is one made of an alloy of platinum and iridium, which may be obtained from dealers in various diameters of which the best for the experiment is 0·0014 in., or 0·0036 cm. This wire fuses with a current of 0·8 amp. at a temperature of about 1800° C. It has an average temperature-coefficient of 0·0009, and at the working current of 0·5 to 0·6 amp. has a resistance of 300–350 ohms per metre. A piece of this wire about 70 cm. long is fixed to terminals supported by two wooden stands or blocks and connected to a.c. mains through a rheostat of 400 ohms resistance capable of carrying 1 amp. This arrangement is suited to any supply voltage from 220 downwards. The rheostat is adjusted until the wire attains a white heat, when one of the stands is moved away gently until a slight tension is produced on the wire, which will then be seen to form into a large number of nodes and loops. The number of loops depends upon the temperature of the wire; with the procedure described it is usually about 18. The nodes, which are at rest, appear brighter than the loops, which are cooled by their movement through the air. If the stand be drawn slightly

rther away the number of loops may be reduced by one at a time until the wire reaks under the tension. To obtain the lesser number of segments the wire is hortened and the above procedure repeated. The fundamental form can be obtained on a length of 25 cm. with care, and readily on 15 cm. with an amplitude f 0·5 cm. or more, but there is a danger of the wire breaking if slightly overretched. In all cases fine adjustment can be made by movement of the slider of he rheostat. The experiment may be carried out with greater certainty if the wire e mounted between two sliding pieces on a bench, one of the sliders being movable y hand and provided with a clamping device, whilst the other is moved by a screw o as to give a fine adjustment of the tension.

A nichrom wire of the same diameter gives quite good results, but cannot be nade so hot as the iridio-platinum wire. At a safe working temperature the ibrating part of the nichrom wire is not so clearly seen as in the case of the wire ecommended.

The method described has the advantages of simplicity in the apparatus and nanipulation, and easy observation owing to the wire being self-luminous.

THE EFFECT OF PHOTOSENSITIZED MERCURY VAPOUR ON THE WALLS OF SILICA VACUUM TUBES

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ABSTRACT. An investigation is made into some phenomena occurring at the gas-solid interface in the well-known experiments in which mercury atoms absorb energy from the radiation λ 2537 and then transfer that energy to other gas molecules at collision. It is shown that a combination of both condensation and liberation of gas at the surface of the apparatus must be taken into account, if the effects of irradiation of a gas mixture are to be traced by any of the usual methods of observing pressure changes. In a severely outgassed tube of mercury and hydrogen, the initial pressure-fall due to adsorption of the products of dissociation reaches a limit when a monomolecular layer having a quarter of the surface density previously obtained by electrodeless discharges in glass tubes is completed. After the completion of this saturation, a liberation of gas from the silica begins. This liberation is shown to follow a law independent of the partial pressure of H_2 but dependent on that of Hg, and on the absorption of resonance radiation by the latter. This action on the solid surface is contrasted with the more usual experiments in which the final recipient of the energy of photosensitization is solely gaseous. Comparison may be made with other recent experiments in which atoms in high quantum states communicate their energy to solid surfaces, and also with the better-known phenomena in which gas is liberated from solids by the impact of multiply charged ions.

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§ 1. TRANSFORMATIONS OF THE ENERGY OF ABSORBED RESONANCE RADIATION

IN 1922 Franck and Cario [1] showed that when a mixture of Hg vapour and some foreign gas is irradiated with the Hg resonance line λ 2537, the energy absorbed by the Hg atoms may afterwards be transferred to the foreign gas by molecular encounters of the type called "collisions of the second kind." In terms of the quantum theory, an Hg atom is excited to the $2\,^3P_1$ level; if before the time for its spontaneous return to the original $1\,^1S$ level it happens to collide with a gas molecule, the energy difference between the levels is liberated and appears either as kinetic energy or as causing some disturbance, such as dissociation, of the gas molecule.

In the first experiments of Franck and Cario the foreign gas was hydrogen: the dissociation potential of this is about 4·4 volts, and the energy available in the excited Hg atom is, in the same units, about 4·9 volts. Accordingly the effect provided a convenient though indirect method of producing atomic hydrogen without the ionization which is unavoidable in an electric discharge. In contrast

to the very complex sequence of events in any discharge, we have here instead two stages: (i) the primary transformation of energy is the absorption of λ 2537, not by the H_2 which is transparent to it, but by the Hg which is extremely opaque to it; (ii) in a secondary transformation the activated or excited Hg, which we shall denote by Hg′, dissociates the H_2.

This type of process, said to be "photosensitized" by the presence of the Hg vapour, has since been extended by many physicists and chemists[2]; a large number of such reactions has been catalogued, though even in the simplest case of hydrogen it is doubtful whether the formation of an unstable hydride may not accompany the simple splitting of H_2 into atoms.

Now in many of these researches on mercury photosensitization, the progress and results of the energy transformations are inferred from the observed change in total gas pressure. Thus, various workers detected the formation of the atomic hydrogen in the irradiated mixture by the pressure-fall due to its well-known tendency to become adsorbed on the wall of the vessel; Franck and Cario first detected it by its reduction of metallic oxides, again measuring a pressure change. The interpretation of any such results of a photosensitized process introduces more than one complication, as seen for instance in the fact that Senftleben and Meyer both employed hot-wire gauges to trace the sequence of events, but each in an opposite way. The former observed the cooling of the wire due to the conductivity of H_1 formed in its vicinity, the latter observed the heating of the wire due to loss of pressure as the H_1 was adsorbed or entered into combination and was condensed. Meyer[3] pointed out that the consistency of either method would depend on the state of the walls, and that any heating of the wire by recombination of H_1 at its surface might also introduce a third factor in the measurement.

In view of these complications, and following an investigation I have made[4] on the laws governing loss of hydrogen pressure by adsorption, the work described in the present paper is designed to isolate and measure such of the pressure changes in a photosensitized mixture as can be shown to depend only on gas liberation and condensation at the surface of the silica vessel itself.

Accordingly it is shown here that the adsorption of products of the energy transformations exhibits a saturation maximum and a fatigue, in close agreement with that exhibited by the adsorption of hydrogen in electrodeless discharge tubes of glass. The surface density (number of particles per square centimetre) of the adsorbed layer on the silica is, however, only about a quarter of that on the glass; this fact suggests that a larger molecule than H_1 is included in the layer formed after photosensitization. When the saturation is completed, a reverse process is shown to set in, liberating far more gas than had been initially adsorbed. This process is not thermal, and it occurs at the same rate whatever the partial pressure of the hydrogen in the mixture, but it requires the presence of the Hg vapour and also its absorption of the resonance radiation. Further, the liberation ceases when a limit is set to the available supply of Hg vapour, which thus appears itself to be used up in attacking the silica vessel.

These experiments are mainly concerned with pressure changes on a small

scale, and are not likely to intrude as more than slight corrections into the usual photosensitization of gas mixtures at considerable pressures. In addition to such possible corrections, they serve to exhibit a new example of the transfer of energy of excitation of a neutral atom to a solid surface instead of to another gas molecule, the disruption of solid surfaces having previously been associated with the impact of charged ions.

§ 2. APPARATUS

A bulb of 135 cc., made of fused silica by the Thermal Syndicate, Ltd., is connected with the following apparatus: (a) a palladium valve for admission of pure H_2; (b) a vacuum system including pumps, drying materials, liquid air trap, and discharge tube; (c) a hot-wire pressure gauge of the Pirani type, with platinum filament connected to bridge and potentiometer, and suitably balanced and heat-insulated; (d) a mercury micromanometer of a modified optical-lever type, of which I have given details in the paper quoted [4]. This is capable of indicating pressure changes from 0·5 mm. to less than 10^{-3} mm. as rapidly as they occur.

The Pirani gauge is calibrated against the micromanometer for various potentials applied to its bridge, to give suitable sensitivity over various ranges between 10^{-4} mm. and 10^{-1} mm. total pressure. The micromanometer, in which a liquid Hg surface is necessarily exposed, will only measure the pressure of gas constituents additional to the Hg vapour. Accordingly, when the micromanometer is in the gas system the whole of the latter is filled with Hg vapour at the saturation pressure corresponding to room temperature. This pressure, of the order of 10^{-3} mm., is sufficient to enable the Franck and Cario process to be carried out if the measuring instruments are of a high sensitivity, though in many researches on the subject a reservoir of Hg has been warmed to provide a higher vapour pressure. In those parts of the present work where Hg vapour has to be excluded, the micromanometer has, of course, to be shut off, and the Pirani gauge alone to be used in conjunction with a liquid air trap. In calibration and check experiments when the two instruments are used together it is necessary to take into account the considerable time lag in the response of the hot wire to pressure variations.

The silica bulb can be baked in an electric furnace adapted to slide over it, and in certain cases it was made red-hot in a blow-pipe. For the principal experiments the bulb is exposed to a quartz mercury lamp, of whose spectrum the resonance line λ 2537 is the highest frequency that emerges with little absorption. In order that pressure-broadening of this line radiation may be avoided, the lamp is kept cool by means of a compressed-air blast. Heating of the reaction vessel itself is negligible when this precaution is taken.

§ 3. SATURATION MAXIMA OF THE ADSORPTION

Fig. 1 shows the fall of hydrogen pressure on irradiation of the SiO_2 tube of hydrogen and mercury with the cooled mercury arc. The pressure, being measured on the micromanometer, does not include the 10^{-3} mm. of Hg vapour which is also present. Before each determination the bulb was baked at about 400° C. for two

hours, the whole vacuum system having been tested to stand a cathode-ray vacuum for days. Pure dry H_2 was admitted, and the system was cut off and tested for the constant pressure shown in the initial horizontal portions of the graphs. Irradiation begins at the same point O on the time scale of each graph.

The curves may be compared with similar ones obtained with a glass tube and the same micromanometer, a high-frequency electrodeless ring discharge being used instead of the mercury photosensitization[4] as the source of activation of the hydrogen. Curves 1, 2 and 3 of Fig. 1 exhibit the character described in that previous investigation; namely, in each graph the initial rate of pressure-fall decreases until the curve becomes approximately parallel with the time axis, while in passing from one graph to the next the total amount of pressure-fall became less each time the experiment was repeated, although the tube was rebaked, rewashed with H_2, re-exhausted, and refilled before each repetition. Hence we have two fatigue phenomena, firstly the arrival at a steady state of pressure after each short period of excitation, and secondly the decrease in the difference between initial and final pressures each time the whole cycle of operations is carried out.

The first of these fatigue phenomena I refer to as saturation, implying that the surface has taken up as much gas as it can on that occasion, but not necessarily implying that there is no more space on the surface available for closer packing of an adsorbed layer. The numerical values on the graphs show, as in the previous work on glass, that this saturation is a property of the surface and not of shortage in further gas supply.

The capacity for maximum value of the pressure-fall is not easily regained after the second type of fatigue has reduced the adsorptive power to zero. It seems to be more completely restored by long atmospheric exposure than by heating *in vacuo* even to red heat. Thus, curve 4 was taken after the whole apparatus had been dismantled and rebuilt with the addition of the Pirani gauge chamber. An effect of the added volume of the latter is seen in the slower rate of pressure-fall. This curve was followed by another sequence of decreases in the saturation at successive adsorptions, as in the series of curves 1, 2, 3.

Langmuir was the first to interpret this loss of pressure which occurs in hydrogen when subjected to certain kinds of disturbance. He gave strong evidence for its being due to the dissociation into H_1, the atoms clinging to the walls of the apparatus and thus removing half their number of H_2 molecules from the gaseous phase. Langmuir's atomic hydrogen was derived from very hot tungsten filaments, and the adsorbable hydrogen of my previous paper[4] was derived from electron collisions in the ring discharge, in the total absence of metals. Although the ring discharge gives rise to other particles, charged and neutral, besides H_1, it was assumed there that the neutral atom is again the adsorbable product. On this assumption, and making use of the measured maximum pressure-fall and the dimensions of the apparatus, I have calculated the surface density of the adsorbed layer of monomolecular thickness in three glass tubes of very different sizes and shapes, obtaining respectively $5 \cdot 9 \times 10^{15}$, $3 \cdot 2 \times 10^{15}$ and $5 \cdot 3 \times 10^{15}$ atoms per cm.^{2}.

It has been assumed by all writers that the loss of pressure in a photosensitized

mixture of H_2 and Hg represents a similar adsorption of atoms, together with any pressure change due to reduction of oxides, etc. specially placed in the tube for observation. The structure of the layer formed has not hitherto been investigated. But in this determination of the density of the layer whose formation is inferred
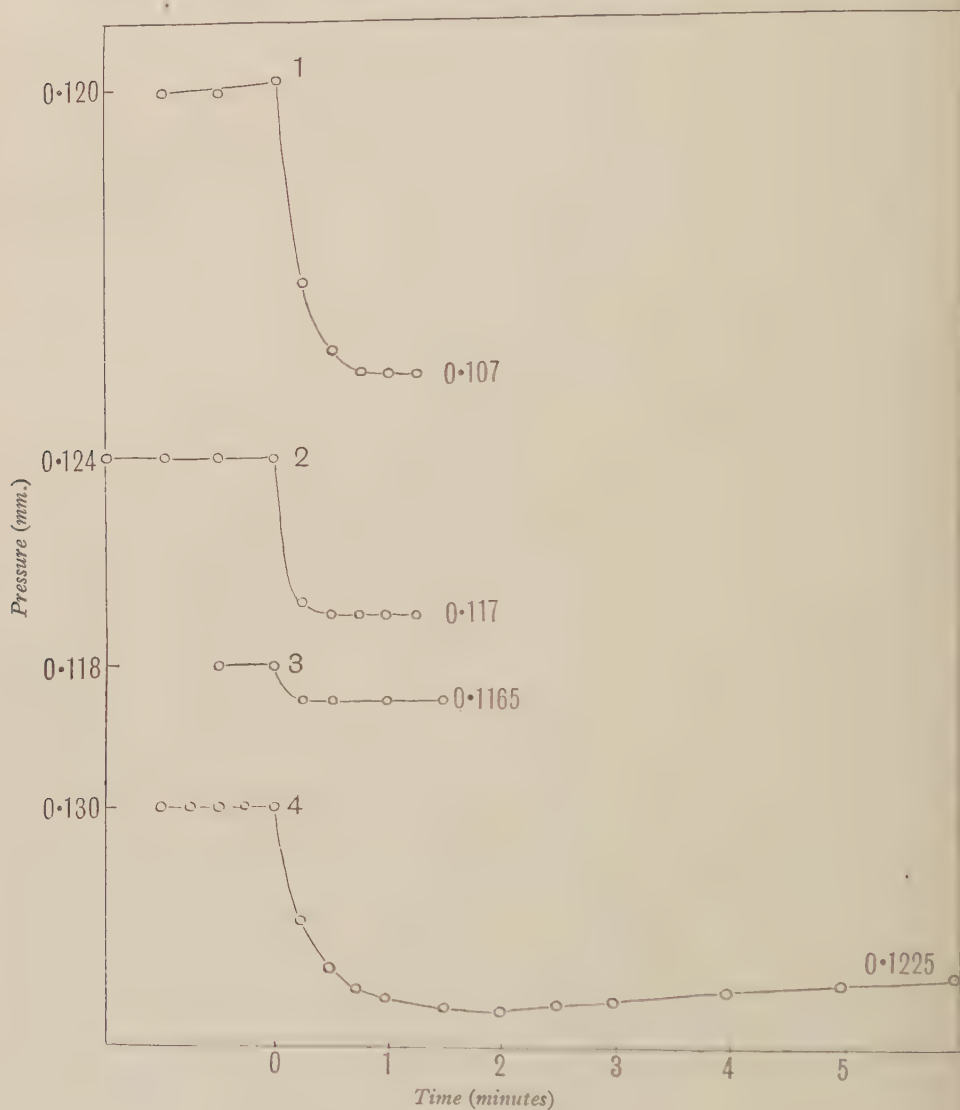
Fig. 1.

from Fig. 1, for comparison with the previous experiments, account must be taken of the ambiguity which is still not completely cleared up, as to the possible by-products of a photosensitized dissociation. Franck and Cario concluded that the principal reaction is

$$Hg' + H_2 \rightarrow Hg + H + H,$$

while not neglecting the further possibility that a diatomic molecule may be produced. On the basis of certain band spectra seen in mixtures of H_2 and Hg, Compton and Turner adopted the latter possibility, in the form

$$Hg' + H_2 \rightarrow HgH + H,$$

and it is not inconceivable, according to Mitchell and others, that H_2' plays some part in the process.

From their great instability it is highly probable that both HgH and H_1 would be strongly adsorbable in comparison with H_2, and either of the above two interpretations of the reaction would mean the disappearance from the gaseous phase of two particles for every H_2 molecule lost to the gas pressure. On this assumption I have calculated from the dimensions of the apparatus, and from curves 1 and 4 of Fig. 1, the density of the layer when saturated, obtaining respectively $1 \cdot 0 \times 10^{15}$ and $1 \cdot 2 \times 10^{15}$ particles per cm.². These figures are seen to be of the order of $\frac{1}{4}$ the surface density of the layer on glass in all the other experiments.

My experience shows that, to obtain the maximum possible saturation density, a surface must be rigorously outgassed to start with; hence, since the above values were not increased even by red-heating the silica, they suggest that the average adsorbed particle in the photosensitized experiments is of considerably larger diameter than the particle adsorbed in the discharge tube. As to the latter, Langmuir's hypothesis of a moderately closely packed monomolecular layer of atoms was well confirmed, since our maximum density, $5 \cdot 9 \times 10^{15}$ per cm.², is related to the density to which Bohr's H_1 orbits could theoretically be packed, $8 \cdot 9 \times 10^{15}$ per cm.², by just the degree of difference to be expected between a "structural" diameter and a "collision" diameter, or distance of nearest practicable approach of two non-combining atoms. If the present case of adsorption is also an example of the Langmuir structure, the collision diameter of any particles which share the monomolecular layer with these H_1 atoms must be of the order of at least 3 A.U. to give the density I have found. The two possible alternatives are H_2' and HgH. But since H_2' was present in the discharge-tube experiments, and since nevertheless the adsorbed layer there showed a packing so close as to be attributable to H_1 only, it is more probable that H_2' is not so strongly adsorbable, and that Compton's HgH, a necessarily large molecule, is here responsible for the lesser saturation maxima observed. For in the photosensitized experiments H_1 can only be formed at the expense of Hg', and hence the concentration of any activation products involving Hg is here comparable with that of H_1, whereas in the discharge tube such products could only contribute according to the initial partial pressure of Hg, which is small compared with that of H_2. The suggestion that some particle which includes an Hg atom can be adsorbed is also required by the measurements we proceed to describe, dealing with the dependence of the gas exchange at the surface on the partial pressure of Hg.

§4. THE LIBERATION OF GAS FROM THE SOLID SURFACE

In the previous experiments on adsorption in the electrodeless discharge the saturation was stable, i.e. the pressure graph having once become parallel with the time axis remained so, whether the discharge was maintained or not. But in the present photosensitized experiments a slight rise of pressure was observed to set in if the illumination was maintained. This is seen incipient in curve 4 of Fig. 1, and becomes more definite after the surface has been fatigued by successive adsorptions, Figs. 2 and 3.

Freedom from leak being guaranteed to a high degree of perfection, a rise in pressure can only be ascribed to one or more of the following causes: (i) heating by the warmth of the lamp; (ii) any gas reaction involving a dissociation (this would show dependence on the initial partial pressure of H_2); (iii) diffusion through the walls due to some effect of irradiation on the SiO_2 (this would be independent of the partial pressure of Hg); (iv) surface reactions: desorption of layer on the silica, decomposition of the silica, or liberation of gas from the interstices of the silica.

I base my conclusions on (iv), for the following reasons. Heating was eliminated as a cause by check experiments. (*a*) The bulb was replaced by a comparison tube of similar dimensions containing a thermometer. This was found to rise less than 2° in ten minutes of irradiation. (*b*) The bulb was heated in a flame until too hot to be touched, with only $\frac{1}{10}$ of the result seen in the pressure increases of Fig. 2, whereas during those experiments the air blast had kept the bulb quite cold to the touch. (*c*) Any tendency of the micromanometer to function as a thermometer would result in an effect proportional to the initial pressure, whereas the pressure-increases are found to be independent of this quantity. (*d*) The null effect in the absence of Hg, recorded below, supplies the upper limit of any thermal consequences.

To test whether any gas reaction of the very highly unstable H_1, for instance a dissociation of tap-grease vapour, could be responsible, the rise in pressure on irradiation was obtained at varying initial partial pressures of H_2. Fig. 2 shows that the effect of intermittent switching on and off of the radiation produces a pressure-rise which is approximately independent of the H_2 and consequently cannot represent a gas reaction. It even takes place when the H_2 is initially pumped out to below 10^{-3} mm. (curve 5), and in that instance the tube is filled to over 0·01 mm. as a result. That is to say, ten times the original content is liberated on irradiation. In this last curve, the impossibility of complete outgassing of the Hg micromanometer is shown in the slight steady increase of pressure which takes place at the highest evacuation, but superposed on this is a liberation of gas coinciding with irradiation as at the higher pressures. It may be remarked that curves 1 and 3 were the first and second, respectively, to be obtained, and exhibit the disappearing remnant of the initial adsorption, which becomes lost in the completed stage of fatigue.

The gas liberation being, then, independent of the partial pressure of H_2, its dependence on the partial pressure of Hg is investigated in the following manner, which also adds plausibility to the previous suggestion that Hg, in some form, is removed from the gas phase at the first irradiation.

So long as the micromanometer is in connection, Hg vapour is present to its saturation pressure of 10^{-3} mm., and any of it which may be removed is at once
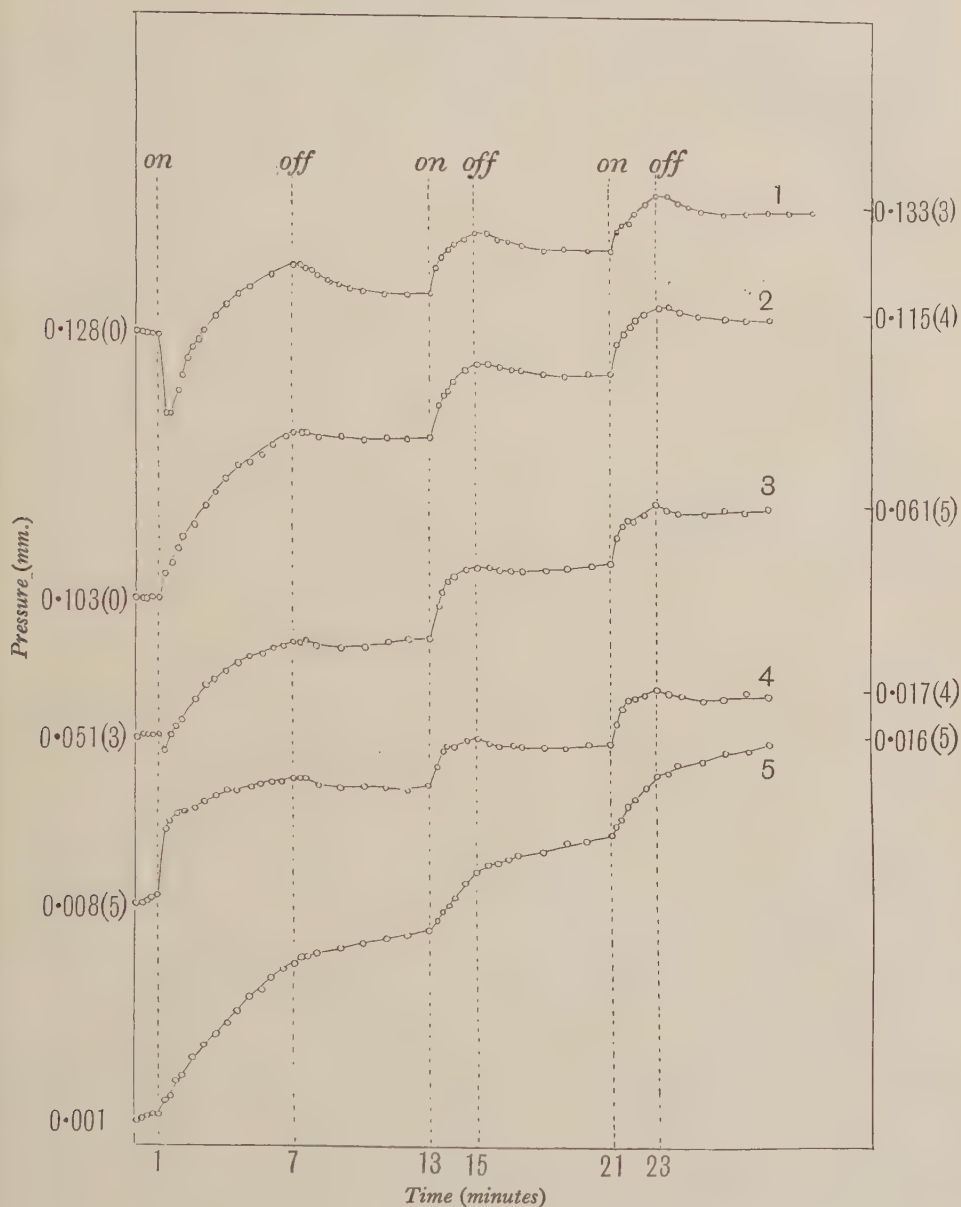
Fig. 2.

replenished from the liquid surface. Accordingly the Pirani gauge was constructed and added at this stage, and comparison was made between the liberation of gas under irradiation, (*a*) with the micromanometer tap open, i.e. with an unlimited

availability of fresh Hg vapour, and (b) with the tap closed, i.e. with an initial partial pressure of Hg vapour of 10^{-3} mm., but no further supply to be drawn upon. Fig. 3 shows pairs of curves of the pressure-rise on illumination under these two conditions. The effect reaches a limit in each case if the radiation is removed, but, when the radiation is maintained, it still reaches a limit if the Hg is not replenished, but increases indefinitely when a continuous supply is drawn from the micro-manometer.
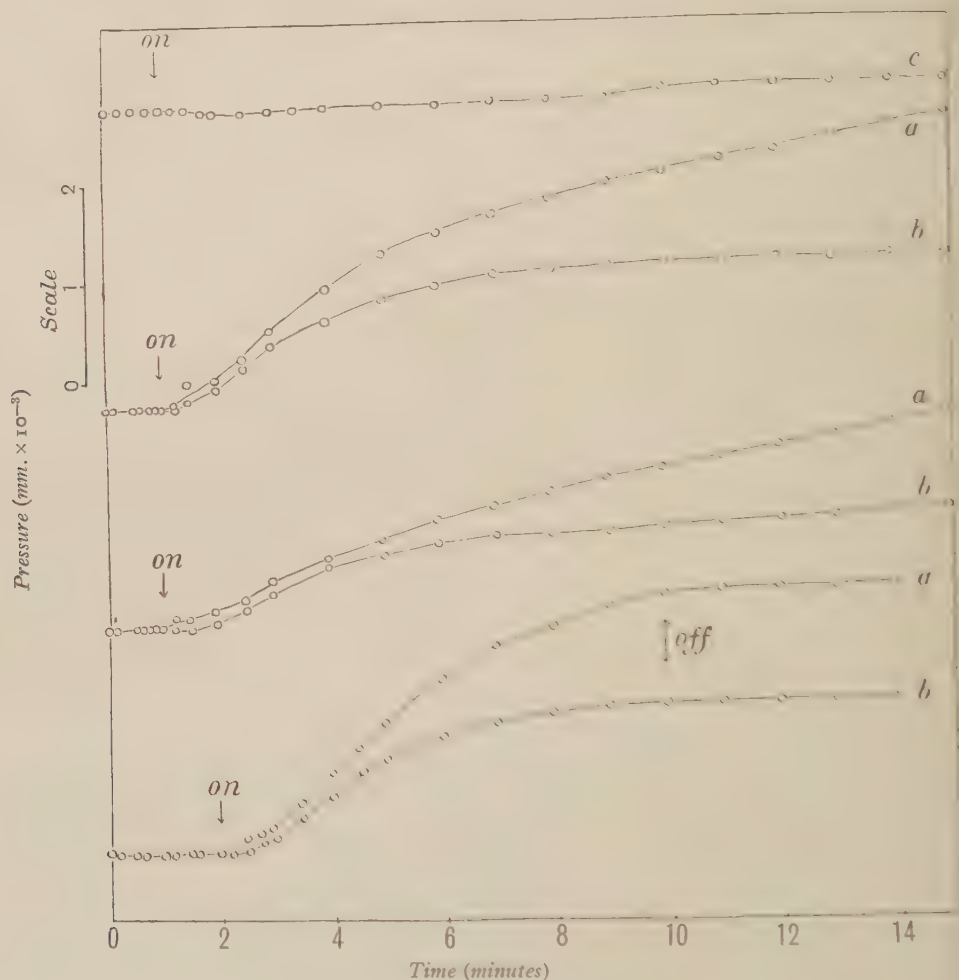
Fig. 3.

a, unlimited Hg.   b, limited Hg.   c, no Hg.

To carry this observation to its extreme, the micromanometer was kept shut off and Hg vapour was as far as possible removed from the whole system by six successive washings with H_2 over a liquid air trap. Repetition of the experiment then gave curve (c), showing a complete disappearance of the pressure-rise on

irradiation, and exhibiting only the slight changes which represent the sum of thermal and other corrections and observational error.

Hence we conclude the conditions governing the liberation of gas to be the following: (1) it occurs only when the bulb is irradiated; (2) it is not a thermal effect; (3) it is independent of the partial pressure of H_2; (4) it decreases and finally ceases when a limited quantity of Hg vapour is available; (5) it increases indefinitely if Hg vapour is replenished continuously; (6) it is non-existent in the absence of Hg vapour; (7) it only takes place when the surface has reached complete saturation with respect to adsorption of the original products of photosensitization.

It now becomes necessary to distinguish between the alternatives under the heading (iv) of p. 496 above, i.e. the possible interpretations which may be put upon a surface reaction of Hg'. We dismiss any decomposition of SiO_2 owing to its extreme chemical and thermal stability. In considering the possibility of liberation of the already adsorbed layer, we must remember that the pressure-rise involves a much greater volume of gas than had originally been removed from the mixture, and shows no sign of ceasing. Hence we conclude that Hg' causes a liberation of gases originally present in the interstices of the SiO_2 and not removed by baking. Silica is known to differ from ordinary glasses in trapping considerable quantities of air and CO in the process of its manufacture: it is to this that the opacity of the coarser grades is due. I proceed to indicate some points of interest in this conclusion.

§ 5. DISCUSSION OF RESULTS

A liberation of gases from solid surfaces and their underlying layers *in vacuo*, due to Hg vapour, is not a new phenomenon; it is known to occur in positive ray tubes, and I have previously discussed, in another paper[5], the mechanism which is there involved. But in all those cases it was the mechanical bombardment of the solid—a metal electrode—by multiply charged ions of Hg moving at high velocity, that was able to liberate the gas content of the solid. In the present case, on the other hand, no ions are present at all, and the liberation is consequent only on the excitation of the neutral Hg atom to the higher quantum state by absorption of radiation. Now the photosensitized reactions initiated by Franck and Cario provide many instances of the giving up of the energy of that state in collision with gas molecules, but here we have apparently the corresponding case where energy is given up in collision with the wall of the tube, and a disturbance of the structure results. Other recent cases of the giving up of energy of excitation of an atom to a solid are the experiments of Oliphant[6], who finds that metastable atoms of helium can cause the emission of electrons from a metal surface, and also the somewhat similar conclusions of Webb[7], of Messenger[8], and of Coulliette[9].

The most obscure part of the present experiments is the way in which the process of liberation depends upon saturation's having been reached in the adsorption of the primary products: there seems, from the graphs, to be something more than a masking of the former by the latter. § 3 of the present paper, in

comparison with the previous paper[4] on the adsorption of H_1, makes it appear probable that some at least of the constituents of the monomolecular layer are the unstable HgH molecules of Compton, and the disappearance of Hg vapour supports this view. Hg′ impinging on this adsorbed layer evidently causes not merely desorption of gas to an amount comparable with that already deposited, but also a mechanical disturbance of the underlying structure such as to allow an indefinitely continued liberation of the imprisoned gases which silica is known to contain. In noticing that the energy exchange between Hg′ and the solid is able to effect what the flame of a blow-pipe cannot, we must remember that the energy available in the Hg′ (112,000 calories per gram-molecule) is very great if it can only be localized when transferred. Since this energy is able to dissociate H_2, whose heat of dissociation is 10^5 cal., it should be equivalent to more than the heat of an oxyhydrogen flame, when it is applied to aggregates of not much more than molecular dimensions in, and immediately below, the vitreous surface.

§6. ACKNOWLEDGMENTS

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(9) *Phys. Rev.* **32**, 636 (1928).

DISCUSSION

Mr B. S. GOSSLING (communicated): As regards the preliminary outgassing of the silica vessel it is generally understood that the temperature at which the most copious evolution of gas occurs from vitreous substances is higher the more refractory the substance. For the softer gases 400 C. is ample, and for the harder 500° C., but for silica something like 1000 C. would be necessary. This supports the possibility which Mr Johnson infers that the evolved gas comes from the silica itself.

AUTHOR'S reply: I am grateful for Mr Gossling's hint, which will be valuable to me. While it makes more plausible the gain of gas from the solid, it also makes more remarkable the fact that this appears to be accomplished by mere contact of Hg′, to an extent far greater than even by the severe outgassing which Mr Gossling shows to be necessary. Actually I made the silica glow in more than one of the attempts to increase the saturation maxima, with no appreciable further liberation of gas.

A SENSITIVE ROTATING-COIL MAGNETOMETER

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ABSTRACT. This paper describes a sensitive rotating-coil magnetometer in which the flux due to the earth's field or to a magnet is neutralized by that due to a current passing through a fixed concentric compensating coil which forms with the rotating coil a variable mutual inductance. It shows how the first-order correcting term due to the length of the magnet may be made to vanish if the angle of contact be suitably chosen, and that the second-order correction may be eliminated by a correct choice of the dimensions of the coil. Many experiments bearing on the theory and uses of the magnetometer are described. In particular, the ohm may be measured very simply, in terms of a mutual inductance and a period, by Weber's method of damping.

§ 1. INTRODUCTION

IT is well known that in the absolute determination of the ohm by the British Association method (suggested by Weber and put forward independently by Lord Kelvin) a small correction has to be made for the mutual energy between the magnet and the rotating coil, the correction involving the moment of the magnet and the galvanometer-constant of the rotating coil.

It is here shown that such a rotating coil, forming a variable mutual inductance with an outer fixed coil and fitted with suitable commutating contacts joined to a low resistance galvanometer, may be used as a magnetometer for the rapid and accurate comparison of magnetic moments of small magnets, for their absolute measurement, and for the determination of hysteresis curves, of the magnetic inclination, and of local variations of the earth's horizontal field-component. Furthermore, it is pointed out that the magnetometer is specially suitable for the complete elimination of the quantities which give trouble when the ohm is determined absolutely by Weber's method of damping, and it is suggested that the method will ultimately be available for the study of susceptibilities of feebly magnetic substances.

The present preliminary investigation has been carried out with a small lecture-demonstration rotating coil not specially designed for our purpose. The results obtained, however, have been so encouraging and the sensibility so good that they have led us to look more closely into the theory of the method and to deal with the problem of the length of the magnet.

33-2

§ 2. PRINCIPLE OF THE METHOD

m

G, θ

F

If a small magnet of moment *m* has its centre coincident with that of a rotating coil of galvanometer-constant *G*, and its axis inclined at an angle *θ* to the plane of the coil, the flux *F* through the coil due to the magnet is given, to a first order, by

$$F = mG \sin \theta \qquad \qquad \text{......(1)}.$$

Thus, if contact is made with a galvanometer connected to strips which are touched by the terminals of the rotating coil over an angle *θ* on each side of the position of zero flux, a total flux change of 4*F* takes place in the galvanometer circuit during every complete revolution.

C, M

This flux may be neutralized by that due to a larger concentric compensating coil carrying a current *C* and possessing a mutual inductance *M* with the rotating coil at the instant of make and break. If the plane of the fixed compensating coil, always normal to the magnet, is adjusted to lie in the magnetic meridian, balance may be tested by rotation of the movable coil, the sensibility being adequate even at low speeds. When the current *C* is so adjusted that, on rotation, no current traverses the galvanometer, we have for equilibrium

$$mG \sin \theta = CM \qquad \qquad \text{......(2)},$$

and thus

$$m = KC,$$

K

where the constant

$$K = M/G \sin \theta \qquad \qquad \text{......(3)},$$

and is known as the moment-constant. It may be determined by the following methods, all of which have been used in this research:

q, γ

(*a*) A small search-coil of total area *q* and carrying a current *γ* may be substituted for the magnet and its flux through the rotating coil may be neutralized by a current

C'

C' in the compensating coil, balance being tested by rotation: whence

$$K = q\gamma/C' \qquad \qquad \text{......(4)}.$$

M'

(*b*) Such a search coil may be substituted for the magnet, and its mean mutual inductance *M'* with the rotating coil at the contact edges may be compared with *M*, the corresponding mean mutual inductance between the rotating coil and the compensating coil: whence

$$K = M/G \sin \theta = qM/M' \qquad \qquad \text{......(5)}.$$

If a standard variable mutual inductometer is available this method is excellent, for the ratio *M/M'* changes slowly with *θ* and moreover *q* may be determined with a standard solenoid.

(*c*) *G* may be found directly and *M* measured absolutely, this method being best for large contacts.

Three different sizes of contacts are fitted to the apparatus. With the largest, extending over some 87°, the mutual inductance *M* is accurately

$$M_{\max} \sin \theta, \quad \text{and} \quad K = M_{\max}/G \qquad \qquad \text{......(6)}.$$

For the smallest, used in the absolute determination of the ohm, $\theta = 9\frac{1}{2}°$. Over the range 0–$40°$,

$$M = \mu\theta,$$

where μ is a constant and $\qquad K = (\mu/G)\,(\theta/\sin\theta) \qquad \qquad \ldots\ldots(7).$

The usual contact, however, for the measurement of magnetic moments is given by $\theta = 50°\ 46'$ and is chosen for the following reason. If the length $2l$ of the magnet is appreciable in comparison with $2a$, the diameter of the rotating coil, the expression for the flux, viz.

$$F = mG\sin\theta,$$

is subject to correction by terms involving l^2/a^2 and higher even powers, the correction varying with the inclination θ. It is shown below that if $\sin^2\theta = \frac{3}{5}$, this correction vanishes to a first order and commences with a term not greater than
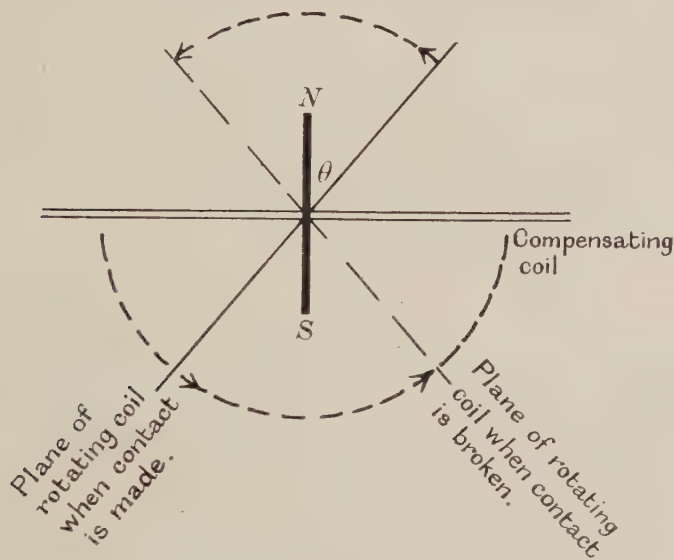
Fig. 1.

$\frac{1}{5}\,l^4/a^4$ which may also be made to vanish if the coil is suitably constructed, the correction being reduced to $\frac{1}{40}\,l^6/a^6$. With the apparatus used in this research

$$F = mG\sin\theta\,(1 - \tfrac{1}{7}\,l^4/a^4),$$

where $a = 10$ cm. when $\sin^2\theta = \frac{3}{5}$. The great advantage of using this angle of make and break, for which the sine law holds best, is strikingly brought out by the curves in Fig. 6 below, for we show that the magnet then lies on a line of iso-mutual inductance—a line along which the axis of a search-coil may be moved without changing its mutual inductance with the main coil. The very long and very short contacts are, nevertheless, useful, not so much in setting a lower and a higher limit to the moment of the magnet as in enabling the effective length of the magnet to be estimated and the correcting constants to be determined.

If the apparatus fitted with the largest contacts is rotated through $90°$, so that

the plane of the compensating coil is now at right angles to the meridian and the contact brushes lie on the middle point of their contacts when the rotating coil is

H

C_H

in the meridian, the effect of the earth's horizontal field-component H may be balanced by a current C_H in the compensating coil. We have then for equilibrium

$$AH \sin \theta = C_H M = C_H M_{max} \sin \theta \qquad \text{......(8)},$$

A

where A is the total effective area of the rotating coil.

The vertical component of the earth's field may likewise easily be received and balanced, and hence the angle of dip may be determined with sensibility, from the ratio of the two balancing currents.

Moments of magnets may also be measured easily with the apparatus in the position for receiving H, either by allowance for the small current needed for field neutralization, or by the taking of the mean of the two balancing currents for the one and the other lie of the magnet in the meridian.

With a Gambrell moving-coil galvanometer of resistance 10 ohms and sensibility 20 cm./μA at a metre scale distance, the apparatus is easily sensitive to 0·2 c.g.s. unit of moment on the 51° contacts and to a change of field of 0·0003 gauss on the large 87° contacts.

Symmetry of contact-setting is very easy to arrange, but slight deviations from symmetry are unimportant.

§ 3. THE THEORY OF THE METHOD

Flux of a very small magnet with centre on the axis of a coil. The flux passing

θ

through a coil from a very small magnet inclined at an angle θ to the plane of the coil and with its centre lying on the axis, Fig. 3, is easily seen to be $mG \sin \theta$,

m, G

where m is the moment of the magnet and G the galvanometer-constant of the coil
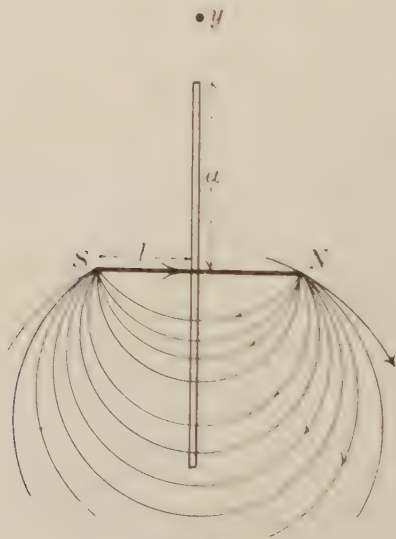
Fig. 2.

at the centre of the magnet, for if a current C traverses the coil, the potential energy of the magnet is numerically $mCG \sin \theta$ and this is equal to the current multiplied by the flux.

Since fluxes and galvanometer-constants are additive this is equally true for a coil having a number of turns in parallel planes.

Flux of a magnet of length $2l$ lying centrally on the axis of a coil. Of the lines of force leaving the N pole, Fig. 2, some thread through the coil in the negative direction and some escape the coil; all return through the magnet. The resultant flux is thus equal to the escape flux.

Since the field at a point on the axis of the magnet at a distance y from the centre is equal to $m (y^2 + l^2)^{-\frac{3}{2}}$ and is normal to the plane of the coil,

$$\text{Total escape flux} = \int_a^\infty m (y^2 + l^2)^{-\frac{3}{2}} 2\pi y \, dy \qquad \ldots \ldots (9).$$
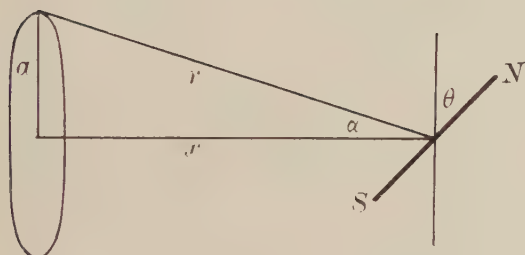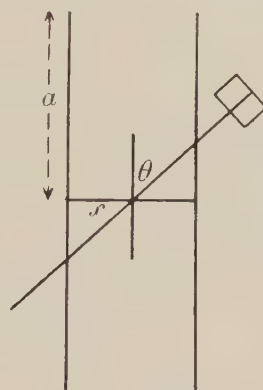
Fig. 3.   Fig. 4.

Thus
$$F = 2\pi m (a^2 + l^2)^{-\frac{1}{2}} = mG (1 + l^2/a^2)^{-\frac{1}{2}} \qquad \ldots \ldots (10)$$
$$= mG (1 - \tfrac{1}{2} l^2/a^2) \text{ approx.,}$$

the true correcting factor lying between the limits

$$(1 - \tfrac{1}{2} l^2/a^2) \quad \text{and} \quad (1 - \tfrac{1}{2} l^2/a^2 + \tfrac{1}{2} l^4/a^4).$$

This method of treatment may easily be applied if the magnet is shifted a distance x along the axis, and complications due to multiple layers only affect the correcting term.

Flux of an inclined magnet of appreciable length $2l$ ($l < a$) with centre on the axis of a coil. Consider a magnet of length $2l$ with its centre on the axis of a circular coil of radius a and at a distance x from the coil centre. Let σ be the pole strength of the magnet.

Let Ω_N, Ω_S be the magnetic potentials at the positions occupied by the north and south poles N and S respectively when a current C passes round the coil in the direction making a positive flux.

F

    Then the potential energy of the magnet is $\sigma\,(\Omega_N - \Omega_S)$, and the flux through the coil from the magnet is F, where

$$F = -\,(\Omega_N - \Omega_S)\,\sigma/C \qquad \ldots\ldots(11).$$

    Now the potential at N is given by

$$\Omega_N = 2\pi C\left[1 - \cos\alpha + \sin^2\alpha \sum_{n=1}^{\infty}\frac{1}{n}\frac{dP_n\,(\cos\alpha)}{d\,(\cos\alpha)}\,P_n\,(-\sin\theta)\left(\frac{l}{r}\right)^n\right] \quad (12),$$

r

where $r = (a^2 + x^2)^{\frac{1}{2}};$

α $\sin\alpha = a/r;$

P_n P_n is the Legendre function of first kind of order $\dot{n}$;

n and n is a positive integer[*].

    Since a similar expression holds for Ω_S in which $-l$ replaces l, we have for the flux through the coil due to the magnet

$$F = 4\pi\sigma\sin^2\alpha \sum_{n=1}^{\infty}\frac{1}{n}\frac{dP_n\,(\cos\alpha)}{d\,(\cos\alpha)}\,P_n\,(\sin\theta)\left(\frac{l}{r}\right)^n \qquad \ldots\ldots(13),$$

where n is a positive odd integer, whence putting n successively equal to 1, 3, 5, 7

m and writing $2l\sigma = m$, we have for the required flux

$$\begin{aligned}
F = m\,.\,2\pi\frac{a^2}{r^3}\sin\theta\Bigg[&1 + \left(1 - \frac{5}{3}\sin^2\theta\right)(a^2 - 4x^2)\frac{3}{4}\frac{l^2}{r^4}\\
&+ \left(1 - \frac{14}{3}\sin^2\theta + \frac{21}{5}\sin^4\theta\right)(a^4 - 12a^2x^2 + 8x^4)\frac{45}{64}\frac{l^4}{r^8}\\
&+ \left(1 - 9\sin^2\theta + \frac{99}{5}\sin^4\theta - \frac{429}{35}\sin^6\theta\right)\\
&\qquad\times(5a^6 - 120a^4x^2 + 240a^2x^4 - 64x^6)\frac{35}{256}\frac{l^6}{r^{12}} + \ldots\Bigg] \qquad \ldots\ldots(14),
\end{aligned}$$

where $2\pi a^2/r^3 = G$, the galvanometer-constant at the centre of the magnet.

Ω We first obtained this expression without the use of spherical harmonics by finding values of Ω_N, Ω_S by Taylor's theorem from Ω, the potential at the centre of the magnet, but the expansion is more laborious when that procedure is adopted.

    Elimination of the effect of the length of the magnet. A rotating coil with provision for an axle and supports for the magnet and calibrating search-coil must necessarily be wound in two sections, and the Helmholtz-Gaugain pattern at first suggests itself. Our fundamental equation (1),

$$F = mG\sin\theta,$$

is essentially additive for different layers of winding in parallel planes, and we measure experimentally the resultant flux and the resultant value of $G\sin\theta$ at the centre of the magnet. The problem before us is thus somewhat different from the problem of simplifying the expression for the couple on a needle where, in galvanometry, a calculation from coil dimensions is required. This contrast is very

    [*] Cf. Andrew Gray, *Absolute Measurements in Electricity and Magnetism*, p. 210 (1921).

noticeable in connection with the effect of multiple layers, which in the galvano-meter problem have a more fundamental effect, but in this case only give rise to small correcting terms.

The first correcting term in l^2 changes sign with θ and vanishes when $\sin^2\theta = \frac{3}{5}$. Since this vanishing is independent of the value of x, the effect of multiple layers is thrown down to the second-order correcting term. In practice the selection of $50°\ 46'$ as the value of θ means only the cutting of a copper contact arc of twice this angle, an operation which not only can be done with accuracy but can be verified, when the contact is *in situ*, by the aid of a variable mutual inductometer.

The second-order correcting term in l^4, which would vanish if θ were $32°\ 35'$ or $64°\ 59'$, has the value $-\frac{81}{100}(a^4 - 12a^2x^2 + 8x^4)\,l^4/r^8$ when $\sin^2\theta = \frac{3}{5}$. This vanishes when $x^2 = a^2(3 \pm \sqrt{7})/4$, the useful solution being $x = 0.2976a$. This value of roughly $\frac{3}{10}$ for the ratio x/a is ample for the axle and support table, and the loss of sensibility due to axial displacement is only 12 per cent. as against a loss of 28·4 per cent. in the Helmholtz arrangement, while there is a marked gain in compactness when rotation is considered.

The elimination of the second-order correcting term by the choice $x/a = 0.298$ is, unlike the elimination of the first correcting term by the choice $\theta = 50°\ 46'$, subject to correction for multiplicity of layers. This complication, however, may easily be removed by means of a design based on a principle due to Maxwell. If a coil of rectangular cross-section has axial breadth $2b$ and radial depth $2d$ and if P_0 is any term in the expression for the action of any kind between the magnet and the central circular filament of the coil, then to a first order the average term for the action of the whole coil is given by

$$\bar{P} = P_0 + \frac{b^2}{6}\frac{\partial^2 P_0}{\partial x^2} + \frac{d^2}{6}\frac{\partial^2 P_0}{\partial a^2} \qquad \dots\dots(15).$$

If
$$P_0 = \frac{a^2}{r^3}\left(\frac{a^4 - 12a^2x^2 + 8x^4}{r^8}\right) \qquad \dots\dots(16),$$

$$\partial^2 P_0/\partial x^2 = -7a^2r^{-15}(5a^6 - 120a^4x^2 + 240a^2x^4 - 64x^6) \qquad \dots\dots(17),$$

$$\partial^2 P_0/\partial a^2 = r^{-15}(30a^8 - 755a^6x^2 + 1650a^4x^4 - 552a^2x^6 + 16x^8) \qquad \dots(18).$$

Thus if
$$\frac{b^2}{d^2} = -\frac{\partial^2 P_0}{\partial a^2}\Big/\frac{\partial^2 P_0}{\partial x^2} \qquad \dots\dots(19),$$

and P_0 vanishes owing to an arrangement that $x^2 = 0.08856a^2$ for the central filament, the entire effect of the multiple layers will be zero for the term we are considering. On substituting for x we find that $b/d = 0.957$.

Whence if the ratio of the axial breadth to the radial depth of the coil channel be chosen as 0.957, and $x/a = 0.298$ for the central winding, the second-order correcting term will disappear. Moreover, slight deviations from this prescription will not have a serious effect. It is interesting to note that our ratio for b/d is different from that, viz. $b/d = 0.733$, which we find would be required to make all the turns have the same galvanometer-constant at $x = 0.298a$ as if they were coincident with the centre turn.

Having made the second-order correction vanish we may alter the contact angle θ, whence our flux-correcting factor will be very approximately

k

$$[1 + k\,(l^2/a^2)\,(1 - \tfrac{5}{3}\sin^2\theta)] \qquad\qquad ......(20),$$

where the constant k can be found experimentally with the aid of a solenoid, a search-coil and mutual inductometer and should differ little from 0·409 if the coil is built as suggested. Knowing k and using the other angular contact pieces on the apparatus we may estimate the effective length of a magnet if this is at all appreciable.

If both the first-order and the second-order correcting terms are made to vanish by the selections $\sin^2\theta = \tfrac{3}{5}$, $b\,d = 0.957$ and $x\,a = 0.298$, the value of the flux F will be theoretically

$$mG\sin\theta\,(1 - 0.025\,l^6/a^6) \qquad\qquad ......(21),$$

while the third-order correction is normally less than the error due to imperfect construction.

Significance of the line of lie of the magnet as a line of iso-mutual inductance. It is easy to show, either from expressions for axial and radial components of field or from the potential in the neighbourhood of a current-bearing circle, that the component N of field along a line inclined at an angle θ to the plane of the circle and passing through the centre is given at a distance l away from the centre by

N

$$N = -\frac{d\Omega}{dl} = C\,\frac{2\pi}{a}\sin\theta\left[1 + \frac{9}{4}\frac{l^2}{a^2}\left(1 - \frac{5}{3}\sin^2\theta\right)\right.$$
$$\left.\pm\frac{225}{64}\frac{l^4}{a^4}\left(1 - \frac{14}{3}\sin^2\theta + \frac{21}{5}\sin^4\theta\right) + ...\right] \qquad ......(22).$$

Hence if $\sin^2\theta = \tfrac{3}{5}$ a small search-coil with its axis on the line will have a constant mutual inductance with the circle until it is moved so far that $l^4\,a^4$ is no longer negligible compared with unity.

If the line inclined at an angle θ to the plane of the circle does not pass through the centre but strikes the axis at a distance x therefrom, additional odd powers of l/r, which vanish when $x = 0$, appear in the corresponding expression for N, and we have no longer a line of equal values of N when $\sin^2\theta = \tfrac{3}{5}$. If, however, the line considered is symmetrically situated between twin coils, the odd-power terms will be neutralized and we have for the field in this direction

$$N = 2\pi t\,\frac{a^2}{r^2}\,C\sin\theta\left[1 + \left(1 - \frac{5}{3}\sin^2\theta\right)(a^2 - 4x^2)\frac{9}{4}\frac{l^2}{r^4}\right.$$
$$\left.\pm\left(1 - \frac{14}{3}\sin^2\theta \pm \frac{21}{5}\sin^4\theta\right)(a^4 - 12a^2x^2 + 8x^4)\frac{225}{64}\frac{l^4}{r^8} \cdots\right] ...(23),$$

t

where t is the total number of turns wound on the twin coil.

If then the coil be designed as above with $b\,d = 0.957$ and $x\,a = 0.298$ we shall, when $\sin^2\theta = \tfrac{3}{5}$, have a line along which $N = CG\sin\theta$, giving a uniform mutual inductance of $qG\sin\theta$ with a small search-coil of area q. The same simple expression will serve fundamentally for the mutual inductance between the coil and a thin

solenoid lying along this line and having its centre at the origin, but the correcting terms will have the smaller coefficients of the expression (14) for the flux of a magnet.

The curves of mutual inductance shown in Fig. 6 below, and obtained with a travelling search coil, illustrate the force of this theory, though our rotating coil has only $x/a = 0.15$. Such tests performed on a designed coil may well afford a practical estimate of its value, for the method is sensitive to deviations, the first-order term in the correction of the length of the magnet being increased threefold for a search coil, and the second-order term fivefold. By simple area measurements the percentage error for any length of magnet for any contact angle might be established.

The determination of the ohm absolutely in terms of a mutual inductance and a period. The apparatus is specially suitable for the determination of the ohm by the method of damping, in that it enables the quantities $m^2 G^2/I$ to be eliminated, and also the effect of the length of the magnet, which does not appear to have been considered previously. In addition, the small effect of self-inductance may yet further be reduced by virtue of this elimination of moment of inertia.

When a magnet is making small oscillations at the centre of a coil such as that just described and its position of rest lies in the plane of the coil, we may write for the flux

$$F = mG \sin \theta \, (1 + A_1) = Q \sin \theta \qquad \text{......(24)},$$

A_1, Q

where the correcting factor A_1 is a constant for small angles. If self-inductance, which has a very small effect, is considered to a first order, the equation of motion for small oscillations is

$$(I - LQ^2/R^2)\, \ddot{\theta} + (p + Q^2/R)\, \dot{\theta} + \mu\theta = 0 \qquad \text{......(25)},$$

where p is the air damping coefficient, I the moment of inertia of the oscillating system, L the self-inductance of the coil, R the total resistance of the coil and closing connections, and μ the restoring couple per unit angular displacement, μ being equivalent to $(mH + c)$, where c is the very small torsion effect. This equation leads without approximation to

p, I
L, R
μ
c

$$\frac{Q^2}{R} = 4I \left(\frac{\lambda}{T} - \frac{\lambda_0}{T_0}\right) - \frac{4\lambda}{T} \cdot \frac{LQ^2}{R^2} \qquad \text{......(26)},$$

where λ_0, T_0 are the logarithmic decrement and period respectively on open circuit, and λ, T the corresponding quantities when the coil is closed through the total resistance R. Hence since the effect of λ is very small,

λ_0, T_0
λ, T

$$R = m^2 G^2 \, (1 + A_1)^2/4I \, (\lambda/T - \lambda_0/T_0) + 4\lambda L/T \qquad \text{......(27)}.$$

The elimination of the undesirable quantities which have subjected the method to criticism is effected thus: First we have the open-circuit equation of oscillations

$$\mu/4I = (\pi^2 + \lambda_0^2)/T_0^2 \qquad \text{......(28)}.$$

Again, using the smallest contacts so that the length-correcting factor is still

C   essentially $(1 - A_1)$, we have, on balancing the flux of the magnet by a current C in the compensating coil,

$$mG \sin \theta \, (1 + A_1) = CM \qquad \ldots\ldots(29),$$

M   where M is the mean mutual inductance between the rotating and the compensating coils at the contact edges.

Lastly, using the rotating coil as a sine galvanometer so that the length correction is still the same, setting it in turn on the contact edges, and finding (by means of mirrors on the magnet and a Gauss eyepiece with object lens) the currents of mean value $\bar{c}$ which restore the magnet to the plane of the rotating coil, we have

$$\bar{c}mG \, (1 + A_1) = \mu \sin \theta \qquad \ldots\ldots(30),$$

whence, all three equations (28), (29), (30) being multiplied together,

$$\frac{m^2 G^2 (1 + A_1)^2}{4I} = \frac{\pi^2 + \lambda_0^2}{T_0^2} \cdot M \cdot \frac{C}{\bar{c}} \qquad \ldots\ldots(31),$$

so that

$$R = M \frac{C}{\bar{c}} \cdot \frac{\pi^2 + \lambda_0^2}{T_0^2 (\lambda/T - \lambda_0/T_0)} + \frac{4\lambda L}{T} \qquad \ldots\ldots(32),$$

or with an accuracy of 1 part in 1000,

$$R = M \frac{C}{\bar{c}} \frac{\pi^2}{T_0 (\lambda - \lambda_0)} \qquad \ldots\ldots(33).$$

The current ratio $C/\bar{c}$, as taken on a Cambridge thermo-electric potentiometer, depends essentially on the ratio of two resistances.

It will be noticed that the minute effect of self-inductance is virtually to reduce the moment of inertia to a value

$$I' = I - L (1 + A_1)^2 \, m^2 G^2 \, R^2 \qquad \ldots\ldots(34).$$

Since I' is eliminated, we may write

$$R = m^2 G^2 (1 + A_1)^2/4I' \, (\lambda/T - \lambda_0/T_0) + 4\lambda_0 L/T_0 \qquad \ldots\ldots(35),$$

and hence, using the alternative expression

$$\mu/4I' = (\pi^2 + \lambda^2)/T^2$$

for equation (28) and again multiplying, we have

$$R = M \frac{C}{\bar{c}} \cdot \frac{\pi^2 + \lambda^2}{T^2 (\lambda/T - \lambda_0/T_0)} + 4 \frac{\lambda_0}{T_0} L \qquad \ldots\ldots(36),$$

which, though reducing the effect of self-inductance, introduces a period more difficult to measure.

It is here pointed out that, though by using the small contacts we eliminate the effect of the length of the magnet almost completely, this would not necessarily be the best procedure if the method were pushed to its limit of accuracy.

The sensibility in relation to the dimensions and winding of the coil. Taking the deflection of a galvanometer as proportional to the current and to the square root of the galvanometer resistance R_g, we have for the mean current C when the flux cut is due to a magnet,

$$C = 4mG \sin \theta . n_r/(R_g + R_c) \qquad \ldots\ldots(37),$$

where n_r is the number of revolutions per second and R_c the resistance of the coil. If a total length l_1 of wire of radius y fills the coil channel of volume V and mean radius a and if ρ is the specific resistance of the material of the wire and g the galvanometer-constant per unit length, then, insulation being neglected,

$$V = l_1 y^2, \quad R_c = \rho l_1^2/V, \quad G = gl_1,$$

n_r, R_c

l_1, y, V

a, ρ, g

whence for the resulting deflection D we have

D

$$D = kgl_1 R_g^{\frac{1}{2}}/(R_g + \rho l_1^2/V) \qquad \ldots\ldots(38),$$

where k is constant for a given speed of revolution.

k

This deflection is easily seen to be a maximum when $R_g = R_c$, and when this condition is fulfilled we have

$$D \propto gV^{\frac{1}{2}} \qquad \ldots\ldots(39),$$

so that the gauge of wire is of consequence only as regards the choice of a galvanometer of resistance equal to the coil, and essentially it is the weight of wire that is important. Though $g = 1/a^2$, the reduction of sensibility on increase of a will not be great, for on theoretical grounds we have for the breadth and depth of the channel $2b = k_1 a$, $2d = k_2 a$ and the volume of the channel will be proportional to the cube of the radius. We may thus take the moment-sensibility to be inversely proportional to the square root of the coil-radius, if the coil is constructed according to the best theoretical conditions.

It is easy to show that if such a coil is exposed to H, then

$$D \propto aV^{\frac{1}{2}} \qquad \ldots\ldots(40),$$

and is again independent of the gauge of wire. The field-sensibility is thus greatly increased with the radius, being proportional to $a^{\frac{5}{2}}$.

§4. THE APPARATUS

The general form of the main apparatus will be gathered from Fig. 5. The rotating coil A, which had been made solely to illustrate the B.A. determination of the ohm, is of mean diameter 20 cm. and of resistance 2·8 ohms. It is wound with a total of 289 turns of enamel-covered wire of s.w.g. 18 (diameter 1·2 mm.), filling two channels each of radial depth 1 cm. and of axial breadth 2 cm., the central circular filament of each section being about 1·5 cm. from the axis of rotation FY. The coil rotates about two spindles EF and XY which are fixed to the framework of the instrument and is provided at its base with a pulley which enables it to be belt-driven from a hand wheel.

The fixed circular concentric compensating coil B is nearly 40 cm. in diameter and of narrow channel; it is wound in two portions with double silk-covered wire of s.w.g. 22 (diameter 0·71 mm.). The inner winding consists of 17 turns, having a calculated galvanometer-constant of 5·50 cm.$^{-1}$; it enables the galvanometer-constant of the rotating coil to be found experimentally by the method of field

neutralization. The outer winding consists of 38 additional turns. Normally all 55 turns are used in the compensation, the maximum mutual inductance with the rotating coil being 1814 microhenries.

Within the rotating coil and on the spindle XY can be fixed either a table D, graduated in degrees, capable of rotation when necessary, and adapted to support centrally any magnet or solenoid under test; or an ebonite carrier holding any of a series of search-coils C (not shown), of known dimensions, arranged with their central planes coincident with that of the compensator B. Leads from such search-coils pass outwards to terminals through the hollow spindle EF.
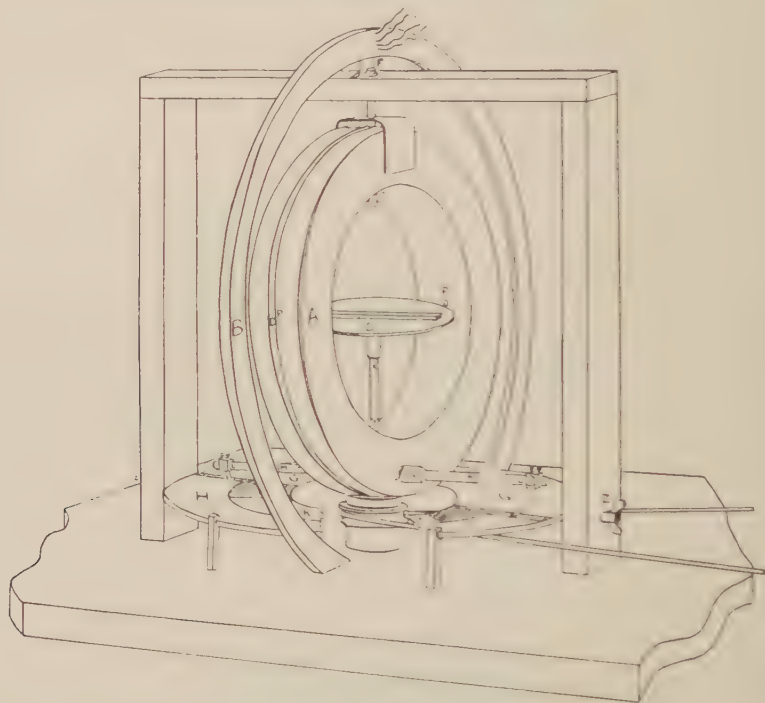
Fig. 5.

The ends of the windings of the rotating coil are brought to special pillar terminals NL projecting outwards as shown and carrying adjustable spring brushes of manganin or phosphorbronze. These brushes touch on copper sectors G and H or K which are fixed on a circular horizontal ebonite table calibrated in degrees and capable of being rotated when necessary and then fixed in position by means of the clamping screws Z, Z. The wide sector G subtends nearly 180° and almost covers half the table, while the narrower outer and inner sectors H and K subtend respectively 175° and 101° 30′ and fix these angles of contact. It will be seen that when the brushes are in the position L they make contact alternately with G and H, while in the position M they make contact with G and K. To ensure smoothness of running the gaps between the sectors are filled with ebonite of the same thickness as the copper.

The sector G and the sectors H and K are normally connected through a tapping-key to a low resistance Gambrell galvanometer provided also with a shorting-key and shunts. Alternatively by means of a mercury rockover switch the rotating coil may be thrown into series with the primary of a standard variable mutual inductometer and a valve oscillator. Likewise the compensating coil B or the search coil *in situ* may be joined to suitable battery circuits or connected so as to be in series in either direction with the secondary of the inductometer and a telephone.

Symmetry of contact is attained as follows. First the rotating coil is turned so that the mutual inductance between it and the compensator is zero—a very sharp setting. The sector turntable is next rotated so that the centre line of the contact H or K, as the case may be, is under one of the brushes. The coil is then rotated through 180° until the mutual inductance is again zero, when the other brush is adjusted to lie on the same centre line. The settings are checked as follows: the four mutual inductances are measured in the positions of the coming on and going off of each brush, so that it is possible to see whether their magnitudes are sufficiently close. Any further necessary improvement is then effected by means of a slight rotation of the sector table.

The mutual inductometer is also used to set a search-coil C coplanar with the compensator B. The rotating coil and compensator are set for zero inductance. The search-coil C is then turned until its mutual inductance with the rotating coil is zero also.

A third sector, not shown in the diagram, and subtending an angle of 19°, occasionally replaced the sector K, as in the tests on the determination of the ohm. For this purpose the magnet, fitted at its ends with plane mirrors, was supported in a stirrup hung at the coil centre on a fibre attached to a torsion rod passing through the hollow spindle EF. A Gauss eyepiece attached to a tube passing through the rotating coil at P enabled the arrangement to be used as a sine galvanometer.

When the plane of the compensating coil is in the meridian the resultant effect of the earth's field is zero if the contacts have been set symmetrically as described above. When this position has been found by means of rotation tests, the framework of the instrument is securely fixed to the bench. Though no resultant deflection ensues there are nevertheless, especially with large contacts, alternating jerks of the galvanometer spot, particularly at low speeds. These jerks are of little consequence except when weak magnets are being dealt with, but in any case they are easily suppressed by the use of a large square frame (not shown in the diagram) wound with some 40 turns of insulated wire of s.w.g. 28 (diameter 0·38 mm.), set symmetrically over the compensator at right angles to the meridian, and carrying a small current sufficient to neutralize the earth's flux approximately. When, on the other hand, as in the experiments on the ohm, the plane of the compensator is set at right angles to the meridian, there are no such jerks and a small current through the compensator neutralizes the earth's flux.

All currents used either in the compensator or in the search-coil circuits can

be switched through a series of calibrated standard resistances which range in value from 0·01 to 10·0 ohms and can be connected at will by potential leads to a Cambridge thermo-electric potentiometer reading from 0 to 90 millivolts. Such currents can thus be measured or compared with accuracy and auxiliary ammeters can at any time be tested.

The areas of search-coils in all later work have been found with the aid of the mutual inductometer and a standard solenoid 1 m. long.

§ 5. EXPERIMENTAL TESTS

Determination of G. The calculated value of the galvanometer-constant G' of the 17 turns of the large compensating coil is 5·50 cm.$^{-1}$. With the planes of both the compensator and rotating coil in the meridian, currents are sent through both in opposite directions so as to produce no deflection of a very small magnet fitted with a mirror and hung by means of a fibre at the common coil-centre. Correcting for the length of the magnet we find

$$G/G' = 31·60,$$

whence $G = 173·8$ cm.$^{-1}$.

t, l
d_1, d_2 Alternatively various search-coils of t turns, axial length l, and inner and outer diameters of winding d_1 and d_2, were set at the centre of the rotating coil and the maximum mutual inductances were compared with that between the search-coil and a standard solenoid of field-constant 310·9 gauss per c.g.s. unit of current. Table 1 shows the results obtained.

Table 1. Determination of G with different search-coils.

Search-coil	t	l	d_1	d_2	Mutual inductance with standard solenoid	Mutual inductance with rotating coil	G
S	1400	1·5	1·0	2·6	1233·5	690·0	173·9
T	1700	1·8	1·0	2·7	1483·0	828·5	173·7
U	1100	0·8	1·0	2·24	712·2	398·2	173·8
V	1513	$\dfrac{l}{d} = \dfrac{\sqrt{3}}{2}$			498·0	278·0	173·6

Determination of H G. The plane of the compensator coil is set at right angles to the meridian and the period T of a small oscillating needle at its centre is taken with various steady currents C passing through the 17 turns of the compensator in such a direction as to oppose H. A plot of C as ordinate against T^2 as abscissa yields a straight line whose intercept on the y axis gives $H G'$ with precision. When the needle is reversed the torsion head is twisted in the same direction through 180°. Incidentally the current C' which reverses the needle and gives the same period as that given by H alone is easily found, and G' for the compensator is 5·50 cm.$^{-1}$; so that, since the torsion head has been rotated, $H G' = C' 2$. Since $C' = 0·07084$ c.g.s. we have $H/G' = 0·03542$, $H = 0·1948$ gauss and $H/G = 0·001121$.

Determination of A, the area of the rotating coil, a, the mean radius and t, the *number of effective turns.* When the coil was set with the plane of the compensator at right angles to the meridian, a balancing current C of 0·00985 c.g.s. was needed through the full 55 turns which were of maximum mutual inductance M with the rotating coil. Whence, since $AH = CM$ and $M = 1814\mu H$ (or alternatively without an inductometer $H/G = 0·001121$, and $M/G = 10435$ as explained below), $A = 91720$ cm.², so that for 289 turns $a = 10·05$ cm.

If, however, we use the equations $A = \pi a^2 t$, $G = 2\pi t (a^2/r^3)$ we have $r^3 = 2A/G$, so that $r = 10·18$, whence, since $x = 1·5$, we have $a = 10·07$ and hence $t = 287·8$.

Thus the first flux correcting term $(1 - \frac{3}{5}\sin^2\theta)(a^2 - 4x^2)\frac{3}{4}l^2/r^4$ is for our experimental coil $(1 - \frac{5}{3}\sin^2\theta) \times 0·00648l^2$.

A, a, t
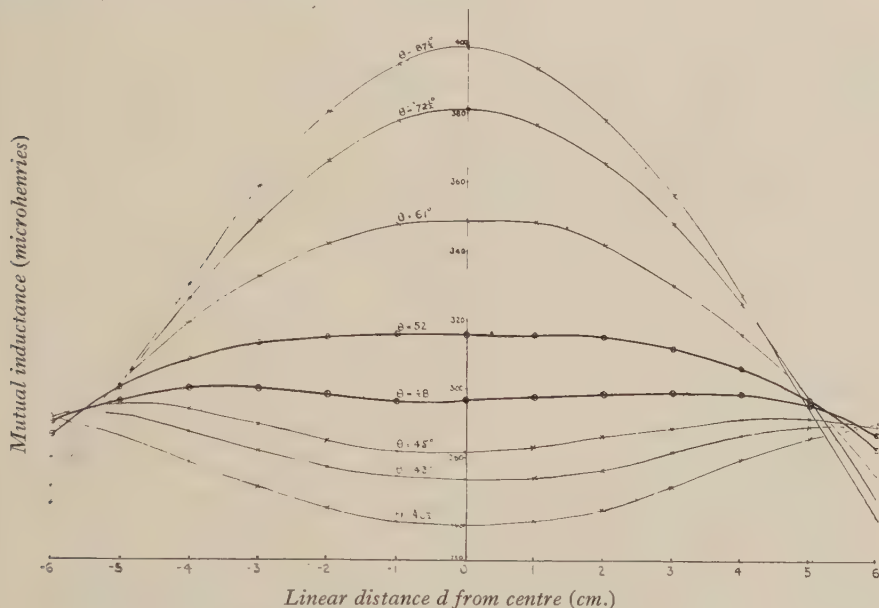
Fig. 6. Mutual inductance between main coil and search coil.

Experimental illustration of the advantage accruing from the use of the contact *angle* 50° 46′. The search-coil used was wound on an ebonite former, 1 cm. in diameter, drilled with a hole so that it could be made to slide along a straight glass tube. The coil consisted of 1100 turns of double-silk-covered copper wire of s.w.g. 38 (diameter 0·15 mm.) and the diameter of the coil rose to a maximum of 2·24 cm., its axial length being 0·8 cm. and its total area as found by the standard solenoid 2290 cm.². The glass tube carrying the search-coil had a linear scale inside it and was fixed horizontally so as to pass through the centre of the rotating coil. The mutual inductance between the search-coil and the rotating coil was then measured at various axial distances from the coil centre ranging from − 6 to + 6 cm. The rotating coil was then turned and fixed with its plane inclined at some other angle θ to the glass tube, an angle easily determined since, when the search-coil is at the centre, its mutual inductance with the rotating coil obeys a simple sine law. The mutual inductance is again measured at various distances along the guiding tube.

It suffices to show the data graphically in Fig. 6 which very clearly supports the general theory of the method and in particular equation (23). When it is realized that with the standard magnet of actual length 7·65 cm. and moment 1385 c.g.s. units, the error with $87\frac{1}{2}°$ contacts is less than 4 per cent., it will be seen how insignificant the length-error must be when the $51°$ contacts are used, and the reason why solenoidal bobbins yield good values of the moment constant K for this particular angle will readily be understood.

Determination of K, the moment constant, by current-ratio method. The current-ratio method is illustrated with a search-coil whose area $q = 4770$ cm.^{2} while it carries a current of 0·2165 amp. This was balanced, as tested by rotation of the coil, with the current C' equal to 0·09845 amp. on the $87\frac{1}{2}°$ contacts and equal to 0·10925 amp. on the $51°$ contacts, whence from equation (4) we find $K_{87} = 10490$ and $K_{51} = 9450$.

Determination of K by mutual inductance measurements. An inductometer test made with a search-coil whose area $q = 4770$ sq. cm. gave the results shown in Table 2 for the large $87°$ contacts.

Table 2. Determination of K for the $87°$ contacts.

Position of rotating coil	Mutual-inductance M' between ro-tating coil and search-coil	Mutual inductance M between ro-tating coil and compensator	Ratio M/M'
Coplanar (for maximum mutual inductance) ...	828·5	1814·0	2·189$_5$
1st brush coming on ...	826·4	1809·5	—
1st brush going off ...	825·5	1810·0	—
2nd brush coming on ...	826·5	1809·5	—
2nd brush going off ...	826·5	1809·5	—
	3304·9	7238·5	2·190$_2$

The sine law holds for both inductances in this neighbourhood and the constant

$$K_{87} = (M/G \sin \theta)(q'/q) = qM_{max}/M'_{max} = 10440.$$

The data of a typical experiment with the same search-coil determining K for the $51°$ contacts is given in Table 3.

Table 3. Determination of K for the $51°$ contacts.

Position of rotating coil	Mutual inductance M' between rotating coil and search-coil	Mutual inductance M between rotating coil and compensator
1st brush going off	645	1278
2nd brush coming on	642	1269
2nd brush going off	644·5	1277
1st brush coming on	642·5	1268

Hence $K_{51} = qM/M' = 9435.$

As a result of experiments with four different search-coils we find $K_{87} = 10435$, $K_{51} = 9430$.

Effect of using solenoidal search-coils in the finding of K_{51}. That the search-coils whose particulars are given in Table 1 might have been considerably larger for the determination of K_{51} is shown by the following extreme cases. For a solenoidal bobbin of length 6·1 cm. $K_{51} = 9480$ and for a solenoid of length 10 cm. $K_{51} = 9530$.

Variation of moment-constant K over the whole possible range of contacts. The search-coil whose area $q = 4770$ cm.² was mounted so as to be coplanar with the compensator and the mutual inductances M and M' were measured at many
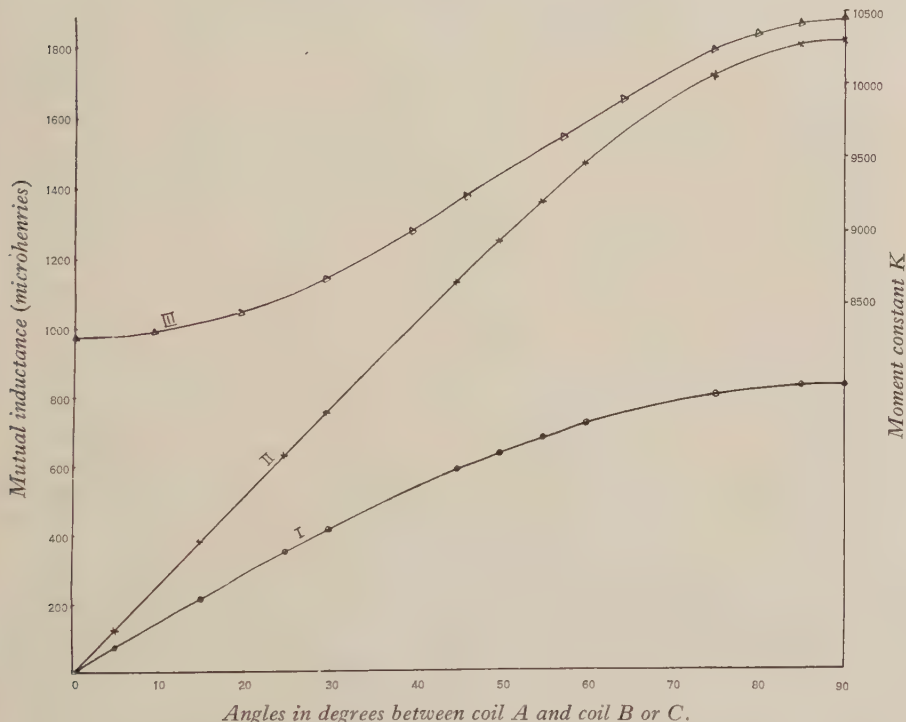
Fig. 7. I, Mutual inductance between coils A and C. II, Mutual inductance between coils A and B. III, Moment-constant K.

angles over a complete revolution of the rotating coil. Fig. 7 shows a plot of the average experimental values over a quadrant. Curve I for the rotating coil and the search-coil is a sine curve within the limits of experimental error. Curve II for the rotating coil and the compensator is practically straight up to 45˚, the mutual inductance increasing at the rate of about 25·1 μH per degree. Curve III is the ratio of the ordinates M/M' multiplied by the area 4770 cm.² so as to show the actual variation of K, the moment-constant, over all possible angles of contact. It will be seen that K changes from the lower limit 8290 for $\theta = 0$ to the higher limit 10440 for $\theta = 90°$. As the rate of change is always slow it matters little in the determination of K for any actual contact if the brushes are not exactly set at the coming-on and going-off positions.

Determination of the moment of a magnet. A flat cobaltcrom steel magnet of actual length 7·65 cm. was found by ordinary magnetometer methods to have a moment of 1370 ± 10 c.g.s. units.

On the 87 contacts it required a balancing current of 1·280 amp. yielding a moment of 10435 × 0·1280 = 1336 c.g.s. units uncorrected for length.

On the 51° contacts of constant $K = 9430$ it required a balancing current of 1·469 amp. yielding $m = 1385$ c.g.s. units. These data yield for the approximate effective length of the magnet $2l = 5·7$ cm.

Variation of the moment of a magnet with small changes of temperature. Small contacts having θ equal to $9\frac{1}{2}$ had been prepared for the determination of the ohm. Even with these least sensitive contacts the change of moment with temperature could be observed.

Solenoid current (amp.)
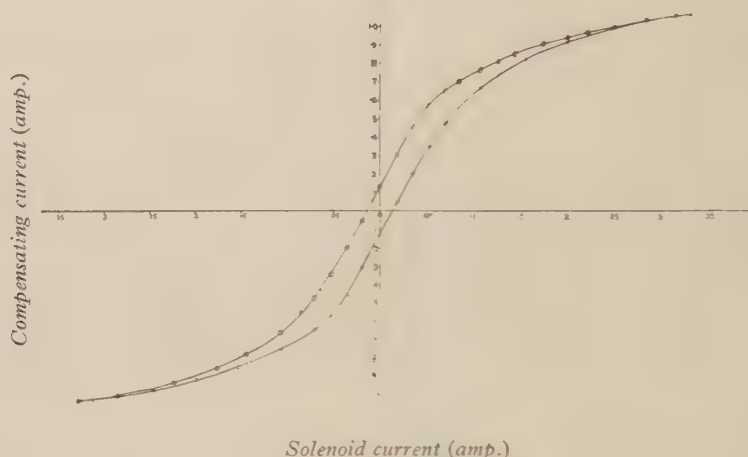
Fig. 8. Hysteresis curve for nickel obtained with apparatus described.

An ordinary steel magnet 7·9 cm. long gave a balancing current of 1·0728 amp. at 18° C. and 1·0518 amp. at 50 C., yielding a temperature coefficient of − 0·00061.

A cobalt steel magnet about 7·2 amp. long required a balancing current of 1·7814 amp. at 18 C. and 1·7718 amp. at 38 C., giving a much smaller temperature coefficient of − 0·00027.

Hysteresis shown by nickel. A solenoid of 2992 total turns of s.w.g. 28 insulated copper wire (diameter 0·38 mm.), spread over a length of 10·1 cm., was mounted on the magnet table shown in Fig. 5, normal to the plane of the compensator. The solenoid current passed also, but in the contrary direction, through the 38 turns of the compensator which was so shunted that on rotation of the coil no deflection of the galvanometer spot took place even when the maximum current used in the experiment was flowing. A few pieces of nickel wire of s.w.g. 18 (diameter 1·2 mm.) and of length 7·5 cm. were placed in a glass tube at the centre of the solenoid. These were subjected to various magnetizing fields and the resulting magnetic moments were neutralized by current flowing through the 17 turns of the compensator. A full cycle involving 70 points of observation was worked through and

the hysteresis curve shown in Fig. 8 was obtained. In view of the length of the wire and shortness of the solenoid there is little point in calculating values of I and H with the present apparatus. Excellent curves were also obtained for pianoforte steel wire, but the field available was not sufficient for saturation.

Determination of the magnetic inclination. The neutralizing currents for the earth's horizontal and vertical field-components were 0·0985 amp. and 0·1853 amp. respectively, whence $\tan \theta = 1853/985$ and the angle of dip was given as 62° 0′.

The laboratory dip circle at the same place gave a mean value of 61° 33′.

Absolute determination of resistance with large standard magnet. An experiment was first made with the standard magnet of moment 1385 c.g.s. units and actual length 7·65 cm. and the following data were obtained:

Mean value of M at extremities of $9\frac{1}{2}°$ contacts	$= 244·5 \,\mu\mathrm{H}$
Balancing current C for magnet, with $9\frac{1}{2}°$ contacts	$= 1·738$ amp.
Mean sine galvanometer current $\bar{c}$ deflecting magnet to contact edges	$= 0·001761$ amp.
Period T_0 of magnet fitted with mirrors	$= 6·013$ sec.
Logarithmic decrement λ_0, on open circuit	$= 0·00115$
Logarithmic decrement λ, on shorting of coil alone	$= 0·1416$
Logarithmic decrement λ, on shorting of coil through 7 ohms	$= 0·04245$

Whence from the formula:

$$R = M \, (C/\bar{c}) \, \pi^2 / T_0 \, (\lambda - \lambda_0) \qquad \ldots\ldots(33),$$

we obtain 2·82 and 9·59 ohms for the two resistances tested, the actual resistances across the shorting key being 2·85 and 9·84 ohms respectively, as measured by Wheatstone bridge.

Resistance determination with a short strong magnet. With a strong cylindrical cobaltcrom steel magnet of actual length 5 cm., diameter 1·6 cm. and approximate moment 700 c.g.s. units, the following data were obtained:

Mean mutual inductance M at extremities of $9\frac{1}{2}°$ contacts	$= 244·5 \,\mu\mathrm{H}$
Balancing current C for magnet, with $9\frac{1}{2}°$ contacts	$= 0·8609$ amp.
Mean sine galvanometer current $\bar{c}$ deflecting magnet to contact edges	$= 0·001794$ amp.
Period T_0 of magnet fitted with mirrors	$= 4·578$ sec.
Logarithmic decrement λ_0, on open circuit	$= 0·00132$
Logarithmic decrement λ, on shorting of coil alone	$= 0·08965$

Whence, from (33), $R = 2·86$ ohms, the resistance, as measured independently being again 2·85 ohms.

The current ratio $C/\bar{c}$ was taken on a thermo-electric potentiometer, the electromotive force being drawn off 0·01 and 10·00 ohms respectively. Self-inductance was negligible in these tests, having an effect of only 0·07 per cent. at most.

§ 6. PROPOSED FURTHER DEVELOPMENTS

A weakness in the present type of apparatus is that any flux through the rotating coil due to a magnet or a solenoid must, of necessity, be neutralized by a flux from the compensator through the same coil and in the contrary direction. This introduces a demagnetizing effect which, though not serious in the experiments described, may well be avoided.

It is proposed, therefore, to construct a new rotating coil of larger dimensions and conforming to theoretical conditions and at the same time to remedy the defect alluded to by making the coil in two portions well separated and probably at right angles to one another, the smaller part only receiving the compensating flux. It is proposed also to consider later the very small corrections due to the width and depth of the magnet.

§ 7. ACKNOWLEDGMENT

In conclusion we would express our deep gratitude to Prof. A. Griffiths of Birkbeck College, who has encouraged us throughout this research and has obtained for us the latest Cambridge pattern of variable mutual inductometer which has proved so useful throughout this investigation.

THE FREQUENCY ERRORS OF RECTIFIER INSTRU-
MENTS OF THE COPPER-OXIDE TYPE FOR
ALTERNATING CURRENT MEASUREMENT

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ABSTRACT. Alternating current milliammeters containing copper oxide rectifiers possess frequency errors of an unusual type. The errors are almost independent of the instrument reading, and thus the percentage errors are inversely proportional to the current to be measured, and may be very large for small currents. It is shown that the errors are due to the capacities of the rectifiers, which, in milliammeters, are of the order $0.09\,\mu F$. The peculiar nature of the errors is due to the fact that the resistance of the instrument is approximately inversely proportional to the square root of the current passing through it. It is found to be possible to compensate the frequency errors by providing the instrument with an inductive shunt, the constants of which satisfy a given relation. The perfection of the compensation depends on the resistance of the instrument, and, therefore, on its range. The compensation was practically perfect for a 1.5 mA instrument up to $4{,}000 \sim$, and for a 7.5 mA instrument up to $10{,}000 \sim$.

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§1. INTRODUCTION

THE development of the copper oxide rectifier has made it possible for instrument makers to produce portable instruments for the measurement of alternating current, reading directly in milliampères or microampères. Such instruments can now be purchased in this country, and they have been found to be very useful for many purposes in the electrical laboratory. Their handy form,
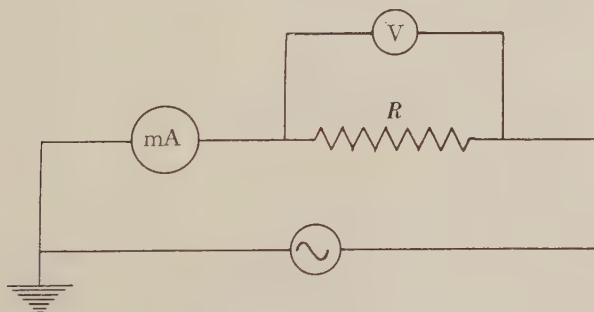
Fig. 1.

almost instantaneous action, and general robustness, are very valuable features, but experiment has shown that they are liable to suffer from a defect which has not yet received sufficient attention; their frequency errors, even at audio frequencies, may be very large.

Three instruments of this type, with ranges of 1·2, 6 and 12 milliampères respectively, were calibrated at various frequencies in the range 50–10,000 ~. The calibrating circuit is shown in Fig. 1, where R is a non-reactive resistance which was given values varying from 500 to 10,000 ohms, and V is a reflecting electrostatic voltmeter giving a full scale reading for 7·5 volts. The earth connection shown eliminated the error due to the earth-capacity of the voltmeter. The self-capacity of the voltmeter and connecting leads was measured, and a correction for this was applied where necessary.
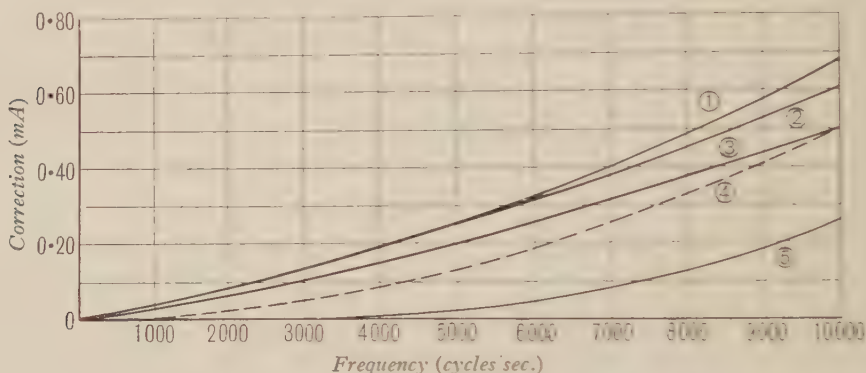
Fig. 2. Frequency errors of rectifier milliammeters.

(1) Instrument of range 2 to 12 mA. (2) Instrument of range 1 to 6 mA. (3) Instrument of range 0·2 to 1·2 mA. (4) Calculated curve. (5) 1·2 mA instrument after compensation.

The results showed the surprising fact that the error of an instrument at any given frequency is approximately independent of the actual reading. Thus the error of the first instrument at 4000 ~, when reading 0·3 mA, was 0·15 mA (an error of 50 per cent.), while the error at the reading 1·2 mA was 0·18 mA (an error of 15 per cent.). The other two instruments, which included rectifiers of the same size, possessed errors of approximately the same amount. Thus the percentage error with these three instruments was inversely proportional to the current to be measured, the smallest value at 4000 ~ being 1·6 per cent. for a reading of 12 mA. The errors for the three instruments are shown in Fig. 2 for the complete range of audio frequencies.

§2. THE INSTRUMENT NETWORK

In seeking for an explanation of these errors, one naturally considers the capacities of the rectifiers. Each rectifier consists essentially of a film of cuprous oxide between two metal plate electrodes, and, as the film is very thin, it is obvious that the capacity must be large. A complete rectifier instrument consists of four rectifiers and a moving-coil d.c. instrument connected as shown in Fig. 3 (*a*). Suppose alternating current is passed between the terminals A and B. For each half-wave of current, two of the rectifiers will pass a conduction current, and these may be

R_1, C_1 represented by a resistance R_1 shunted by a capacity C_1 as in Fig. 3 (*b*). The other

two rectifiers pass practically no conduction current, but they will still pass
capacity current and thus may be represented by capacities C_2. As a first approxi-
mation, the moving-coil d.c. instrument may be represented by an inductance L_1.
When the current changes its direction, the equivalent network, Fig. 3 (b), is
turned round, but otherwise remains unaltered, since the four rectifiers have the
same properties. Since the network is symmetrical, we may, as a further simple
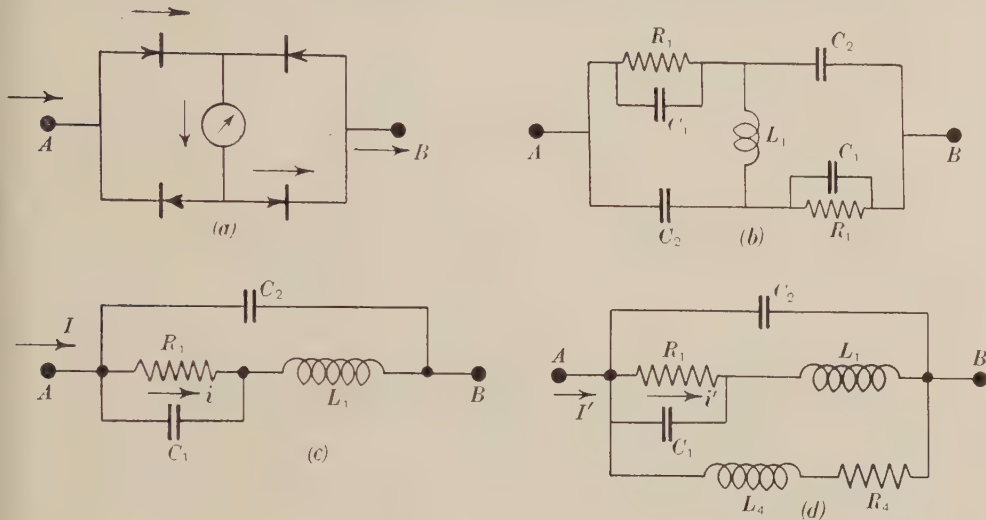
Fig. 3.

approximation, reduce it to Fig. 3 (c) (the values of R_1, C_1, C_2, L_1 will of course be
different in the two cases). Suppose a current I passes through the rectifier instrument
by way of the terminals A and B. A part i of this will flow through the resistance R_1,
and it is this part that the instrument indicates. Applying Kirchhoff's laws to the
network, we find for the ratio I/i at the frequency $\omega/2\pi$ the vector equation

$$I/i = 1 - L_1 C_2 \omega^2 + j\omega \left[C_1 R_1 + C_2 R_1 (1 - L_1 C_1 \omega^2) \right] \qquad ......(1),$$

$$\doteqdot 1 - L_1 C_2 \omega^2 + j\omega R_1 (C_1 + C_2) \qquad ......(2).$$

§3. IMPEDANCE OF THE INSTRUMENT

In order to get some idea of the quantities involved, measurements of the
overall impedance of a rectifier instrument were made by means of the bridge shown
in Fig. 4, which is self-explanatory. The ratio arms R_3, R_4, were equal non-reactive
resistances. The quantities noted were R_e, the effective shunt resistance of the
instrument, and C_e, the effective shunt capacity.

Owing to the rectifying properties of the instrument, perfect silence of the
telephone T could never be obtained; the harmonics of the fundamental frequency
of the source of current were always strongly audible, but the bridge could be
balanced fairly conveniently for the fundamental frequency. The results obtained

represent the constants of the instrument with respect to sine-wave voltage, whereas in using it as an ammeter, we are concerned with its properties with respect to sine-wave current. In considering the results, this limitation must always be borne in mind. The results are given in Table 1.

Table 1. Effective shunt resistance and capacity of rectifier milliammeter.

Current (i) mA	R_e ohms	C_e microfarad	$R_e^2 C_e^2 i$
1	320	0·051	266
2	200	0·057	260
3	155	0·060	260
4	128	0·060	240
5	109	0·060	215
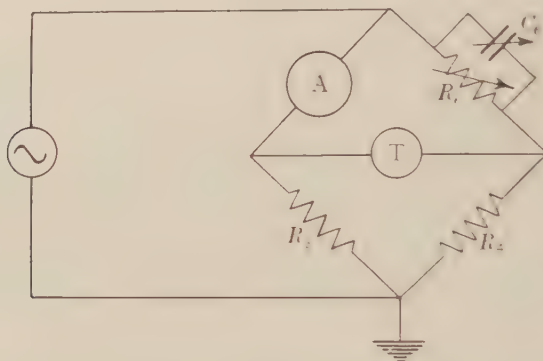
Fig. 4.

Measurements were made at frequencies ranging from 1000 to 4000 ~. The resistance was found to be independent of frequency to the accuracy given. The capacity values seemed to be subject to variations of about 10 per cent. They could not, therefore, be determined with great accuracy, but the observations were sufficient to show that the change with frequency, if any, was not very great.

It is easy to show that the effective shunt resistance R_e and capacity C_e of the network shown in Fig. 3 (c) are given by

$$R_e = R_1 \left[1 + (L_1'/R_1 - C_1 R_1)^2 \omega^2 \right] (1 + R_1^2 C_1^2 \omega^2) \qquad \text{......(3)},$$

$$C_e = C_2 + (C_1 - L_1'/R_1^2)(1 + R_1^2 C_1^2 \omega^2) \left[1 + (L_1'/R_1 - C_1 R_1)^2 \omega^2 \right]$$
$$\text{......(4)},$$

where
$$L_1' = L_1 (1 + R_1^2 C_1^2 \omega^2).$$

When the variation with frequency is small, these expressions must take the approximate forms

$$R_e = R_1 \qquad \text{......(5)},$$

$$C_e = C_2 + C_1 - L_1/R_1^2 \qquad \text{......(6)}.$$

§ 4. THE FREQUENCY ERRORS

We may obtain from equation (2) the ratio of the magnitudes of I and i. Thus

$$I_m/i_m = [(1 - L_1 C_2 \omega^2)^2 + \omega^2 R_1^2 (C_1 + C_2)^2]^{\frac{1}{2}} \qquad ......(7),$$

where I_m, i_m are the magnitudes of the vectors I and i respectively. I_m, i_m

Substituting the approximate expressions for R_e and C_e, we get

$$I_m/i_m = [(1 - L_1 C_2 \omega^2)^2 + \omega^2 (R_e)^2 (C_e + L_1/R_e^2)^2]^{\frac{1}{2}} \qquad ......(8).$$

When the terms involving ω are small compared with unity, this reduces to the approximate form

$$I_m/i_m \fallingdotseq 1 + \tfrac{1}{2}\omega^2 R_e^2 C_e^2 + L_1 (C_e - C_2) \omega^2 + \tfrac{1}{2} L_1^2 \omega^2/R_e^2 \qquad ......(9).$$

The frequency correction of an instrument at any reading i is therefore given approximately by

$$\omega^2 \left[\tfrac{1}{2} R_e^2 C_e^2 + L_1 (C_e - C_2) + \tfrac{1}{2} L_1^2/R_e^2\right] i \qquad ......(10).$$

It seems probable that the first term is the most important one, and, since this correction is found to be approximately independent of i, the product $R_e^2 C_e^2 i$ should be approximately constant. This quantity is calculated from the observed results in Table 1 and is found to be approximately constant. The value of this constant being taken as 260 (Table 1), the corresponding values of the correction $\tfrac{1}{2} R_e^2 C_e^2 \omega^2$ were worked out and plotted in Fig. 2, where they are represented by the dotted line. It is evident that this supplies an explanation of the greater part of the frequency correction, although the agreement between calculated and observed values is far from complete. No doubt the other terms in (10) account for part of the difference, but the discrepancy must also be due in part to the limited validity of the networks, Fig. 3 (b) and (c), and of the results given by the impedance bridge, since the observed values of the correction are not proportional to the square of the frequency whereas the calculated ones are so, if R_e and C_e are independent of the frequency as indicated by the bridge results. It is possible that the resistance R_1 might vary with the frequency, if it were measured under the condition of sine-wave current instead of sine-wave voltage.

§ 5. THE COMPENSATION OF THE FREQUENCY ERRORS

A device for the reduction of the frequency errors immediately suggests itself, viz. an inductive shunt across the instrument terminals. This will obviously cause the complete instrument to pass relatively more current at the lower frequencies, and thus will in some measure compensate for the increase of current at the higher frequencies represented by the errors under discussion. Let the network of Fig. 3 (c) be provided with a shunt across the terminals AB, of resistance R_4, and self R_4 inductance L_4, as in Fig. 3 (d). The complete expression for the ratio of the total L_4

I', i'

current I' to the current i' through R_1 is now somewhat complicated, but if $L_1\omega$ is small compared with R_1 it may be shown that the ratio is given approximately by

$$I'/i' = 1 - L_1 C_2 \omega^2 + j\omega R_1 (C_1 + C_2)$$
$$+ R_1 R_4/(R_4^2 + L_4^2 \omega^2) - j\omega\, R_1 L_4/(R_4^2 + L_4^2 \omega^2) \ \ldots\ldots(11).$$

If we neglect the term $L_1 C_2 \omega^2$, due to the inductance of the moving coil, which is not likely to be large, we may write the equation for magnitudes

I_m, i'_m

$$\frac{I'_m}{i'_m} = \left[\left(1 + \frac{R_1 R_4}{R_4^2 + L_4^2 \omega^2}\right)^2 + \omega^2 R_1^2 \left\{(C_1 + C_2) - \frac{L_4}{R_4^2 + L_4^2 \omega^2}\right\}^2\right]^{\frac{1}{2}} \ldots(12).$$

For frequencies such that $L_4\omega$ is small compared with R_1, we may make a further approximation and obtain

$$\frac{I'_m}{i'_m} = \frac{R_1 + R_1}{R_4}\left[\left\{1 - \frac{L_4^2 \omega^2}{R_4^2}\left(\frac{R_1}{R_1 + R_4}\right)^2\right\}^{} + \left(\frac{R_1}{R_1 + R_4}\right)^2 \omega^2 R_1^2 \left\{(C_1 + C_2) - \frac{L_4}{R_4^2}\right\}^2\right]^{\frac{1}{2}}$$
$$\ldots\ldots(13).$$

By expanding this in ascending powers of ω^2, and equating the coefficient of ω^2 to zero, we obtain, as the condition under which the frequency correction is zero, to the first order of small quantities:

$$\frac{2R_1}{R_1 + R_4}\frac{L_4^2}{R_4^2} - \frac{R_4^2}{(R_1 + R_4)^2} R_1^2 \left[(C_1 - C_2) - \frac{L_4}{R_4^2}\right]^2 = 0.$$

which reduces to

$$C_2 + C_1 = \frac{L_4}{R_4^2}\left[1 + \frac{(2(R_1 + R_4))^{\frac{1}{2}}}{R_1}\right] \ \ldots\ldots(14).$$

It must be remembered that R_1 varies with the current, so that, strictly speaking, compensation can only be effected for one scale reading; but the smaller the ratio R_4/R_1 the less will be the variation with R_1 of the value of $L_4 R_4^2$ required to compensate a given capacity, and thus the more perfect will be the compensation over the whole instrument scale. R_1 cannot be made very small without undue reduction of the sensitivity of the instrument. The instruments referred to above were provided with experimental shunts, in which the mean ratio of $R_4 R_1$ at the most important part of the scale was about 2. The value of the resistance having been adjusted, the instrument was re-calibrated at 50 ~. The frequency was then raised to, say, 4000 ~, and inductance was added to the shunt until the calibration was the same as at 50 ~. The constants of the shunts required for the three instruments were as shown in Table 2.

Table 2. Constants of the shunts required for frequency compensation.

Instrument range mA	R_1 ohms	R_4 ohms	L_4 mH	$L_4 R_4^2$	$C_2 + C_1$
7·5	130	300	2·3	·025 × 10⁻⁶	·090 × 10⁻⁶
15	130	370	3·2	·024 × 10⁻⁶	·091 × 10⁻⁶
1·5	500	1350	38	·021 × 10⁻⁶	·078 × 10⁻⁶

The corresponding values for $(C_1 + C_2)$ calculated from (14) are given in the last column. It will be seen that these capacities and the values of L_4/R_4^2 are approximately constant, the capacities being about 50 per cent. larger than those measured by the bridge. This is to be expected from equation (6).

The final performance of the instruments when fitted with these shunts was very satisfactory. The frequency errors of the 7·5 and 15 mA instruments (6 mA and 12 mA without the shunts) were so small, even up to 10,000 $\sim$, that they could not be detected with certainty. The error was certainly less than 1 per cent. of the maximum reading up to 10,000 $\sim$, and this accuracy held for all scale readings. Mr F. M. Colebrook, of the Wireless Division of the National Physical Laboratory, kindly calibrated the 7·5 mA instrument at frequencies of 20 and 40 kilocycles. The errors were approximately the same for all scale readings and were found to be 0·19 and 1·8 mA respectively for these two frequencies. Thus the useful working range of these two instruments may now be taken as 25 to 10,000 $\sim$. The 1·5 mA instrument was free from frequency errors up to 4000 $\sim$. At 5000 $\sim$ the error was 1 per cent. of the maximum scale reading, and it increased rapidly beyond this point, as may be seen from the curve in Fig. 2. The frequency range of this instrument may therefore be considered to be 25 to 5000 $\sim$.

§ 6. FREQUENCY ERRORS AND TEMPERATURE COMPENSATION

It may be worth while to warn users of commercial models of rectifier instruments that they sometimes possess frequency errors much larger than those discussed in the early part of this paper. The reason is, that it is the practice to compensate the temperature errors of the instruments by providing them with a copper shunt of a suitable resistance. This shunt is unfortunately wound inductively and possesses a self-inductance which may be more than ten times as large as that required to compensate the natural frequency errors of the instrument. The consequence is that the instrument has large errors, of the opposite sign to those found for an unshunted instrument. In a particular case an instrument possessed a correction of $-$ 15 per cent. at 2000 $\sim$, whereas its correction with the temperature compensation removed was only $+$ 0·5 per cent. at the same frequency. The temperature coefficient of the three instruments discussed in this paper was measured, but no constant value was obtained. For one instrument, the coefficient was 0·3 per cent. per 1° C. at a reading of 3 mA, and about half this amount at a reading of 6 mA. For another instrument it was too small to be measured over the range 15° to 20° C. As, moreover, under fixed conditions, the instrument readings were apt to show erratic changes, of the order of 1 per cent. in some cases, it is doubtful whether temperature compensation is worth while. If, however, rectifiers of sufficient stability can be produced, it seems probable that both temperature and frequency errors could be compensated by the shunting of the instrument with a coil of copper wire chosen so that its resistance R_4 is of the value required for temperature compensation, while its inductance L_4 is such that the value of L_4/R_4^2 would compensate for the frequency correction.

The actual performance of a commercial instrument is shown in Fig. 5, which gives in detail the calibration curves for the instrument (1) as originally supplied, (2) with the shunt provided for temperature compensation removed, (3) with the shunt required for frequency compensation fitted.
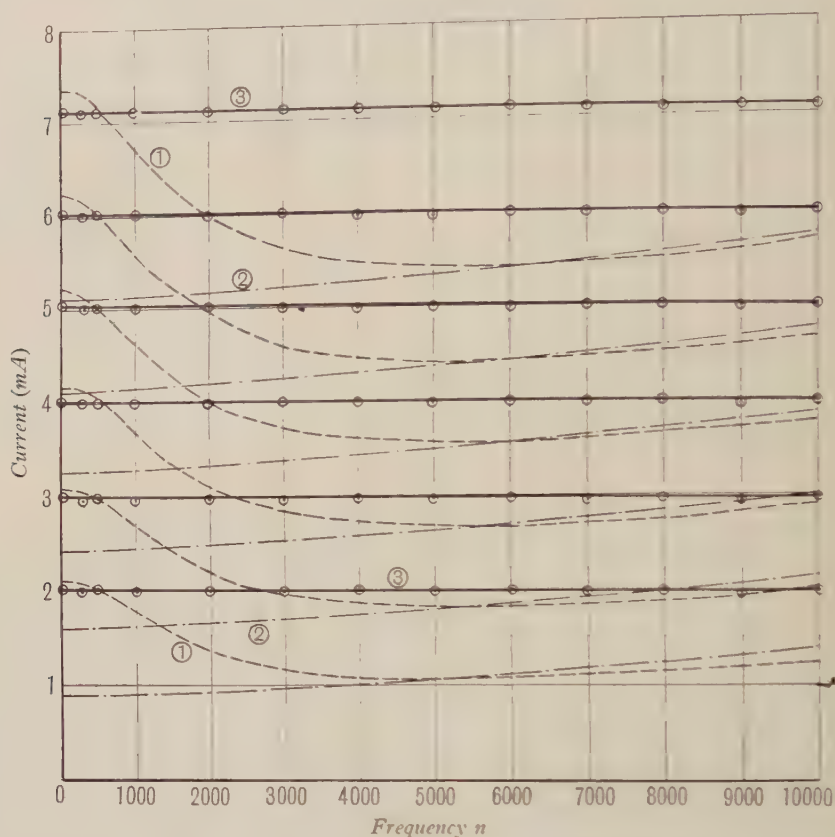
Fig. 5. Frequency errors of rectifier milliammeters. Current frequency curves for constant instrument readings; (1) as originally supplied; (2) with original temperature-compensating shunt removed; (3) with frequency-compensating shunt added.

Each curve represents the variation with frequency of the actual current flowing through the complete instrument for a constant scale reading. Thus, parallelism of the curves indicates that the error is the same for different scale readings, and for a perfect performance the curves become horizontal straight lines.

§ 7. ACKNOWLEDGMENT

DISCUSSION

Mr D. A. OLIVER: Thousands of small a.c. rectifier instruments are now in use and comparatively little attention has been paid to their errors. Dr Hartshorn's paper therefore appears at an opportune time and is, I believe, the first to deal

Wave-form distortion produced by rectifier instrument.

thoroughly with the more serious errors involved. My first introduction to the large frequency-errors of commercial instruments of this type was in taking out an approximate frequency-characteristic of a resistance-capacity-coupled amplifier of which the amplification was found to fall off much more rapidly than was to be expected. Subsequent calibration of the rectifier meter against a thermojunction

ammeter explained the results completely. The simple and effective method of frequency-compensation described in the paper will doubtless find favour in practice. There is, in addition, the question of wave-form distortion. The two oscillograms here shown were taken at the Research Laboratories of the General Electric Company by Mr J. S. Thompson and myself; first a resistance and secondly a bridge-type rectifier ammeter were connected in the low resistance secondary circuit of a step-down transformer. The wave-form distortion recorded is thus the maximum possible and was taken at 50 ∼ with an r.m.s. current of 20 mA. The maximum range of this meter was 50 mA. These oscillograms indicate that the resistances R_1 in the approximate equivalent network must be regarded as non-linear as well as dependent on the value of current flowing. In the light of this result the perfection of the frequency-compensation becomes the more remarkable. Resistance in series with these instruments reduces the wave-form distortion. I have analysed the distorted wave and to the accuracy possible the curve is represented by

$$i = 20\sqrt{2}\,(\sin \omega t - 0\cdot 14 \sin 3\omega t) \times 10^{-3}\ \text{amp.}$$

If this result is assumed to be typical, it appears that up to about 15 per cent. o third harmonic can be introduced by the meter according to the circuit conditions.

Mr R. W. Paul pointed out that the sum of the capacities given in the last column of Table 2 is roughly the same for all the instruments. Would it be practicable to design a standard compensating shunt applicable to all rectifier instruments with a fair degree of accuracy?

Mr E. W. H. Banner (communicated): This paper is of interest to users of rectifier instruments, but it is a pity that some statements and conclusions in it are apt to mislead users to some extent.

Messrs Ferranti, Ltd. have marketed these instruments for over two years and have performed a large number of tests both on rectifier elements of the copper-oxide type and on complete instruments. Unfortunately the author's tests appear to have been made with one make of instrument only, and the generalizations given do certainly not apply to instruments of the above manufacture, although nothing in the paper suggests that the results given are not representative. The usual rectifier voltmeter supplied by Ferranti, Ltd. has a resistance of 667 ohms per volt, or the consumption is 1·5 milliampères r.m.s. at full scale. The copper shunt for temperature correction, which was advocated by the above-named firm in a paper published in 1929*, is a most valuable feature, and although the compensation varies with different ranges it does in general reduce the error from a high one of the order of 0·5 per cent. per C. to the order of 0·1 per cent. per C. or less. At the same time it was realized that inductance was desirable for frequency-compensation but that it could be overdone, and so such copper shunts are wound "commercially" non-inductive. Actually the shunts used on a voltmeter consuming 1·5 milliampères have an inductance of about 30–40 millihenries, which is about

* *Electrical Review*, **104**, 823 (1929).

the figure of 38 given in the paper and not ten times this as stated to be found in commercial instruments. As a result the frequency error is quite small over the range 20–4000 cycles per second, generally within ± 2 per cent., and up to 6000 cycles per second the error is still small although somewhat greater. Numerous tests have been made on Ferranti rectifier instruments by various government departments and testing laboratories and they confirm Ferranti's tests, showing that the copper shunt does not render the frequency error prohibitively high. It has been found that the frequency curve of a rectifier instrument over a wide range of frequency is not a simple curve but one varying alternately about the zero line, and that for correct compensation for high frequencies various networks, each designed to compensate for a given portion of the curve, become necessary; but this is certainly not a commercial proposition. The reference to "erratic changes" is of interest, and although instability of the rectifier can easily be produced by overload such erratic readings when an instrument has been turned out correct are known but inexplicable, so far as I am aware.

AUTHOR'S reply: Mr Oliver's oscillogram is a very useful contribution to the subject. The amount of third harmonic present is surprisingly large in the case chosen (admittedly the worst). It suggests that the wave-form errors might be considerable. This point was investigated, and although I have no quantitative information, I have satisfied myself that in ordinary practical cases the wave-form errors are not large, not more than 1 per cent., say.

In answer to Mr Paul, I am of the opinion that it would be quite practicable to design a standard compensating shunt applicable to all instruments of given range. The only doubtful factor is the degree of reproducibility of the rectifiers, and this could very simply be ascertained by making measurements of the effective resistances and capacities of a batch of rectifiers by the bridge method I have described.

Mr Banner appears to be under the impression that I have not tested the instruments made by Messrs Ferranti, Ltd., and that these are free from the errors that I have discussed. Such is not the case, and I cannot agree that users of such instruments are likely to be misled by any of my statements and conclusions, provided they are read with reasonable care. The statements made in the technical press as to the accuracy of copper-oxide rectifier instruments have shown that manufacturers were quite unaware of the existence of large frequency errors in their instruments, and especially of the large variation of percentage error with the range of the instrument. My object has been to explain these facts and to show how the errors may be corrected, and I shall be surprised and disappointed if the instruments manufactured in the next two years are not very much better than their predecessors. My measurements have shown that the frequency curve of the rectifier itself is a simple curve, and that, whenever the more complicated curve referred to by Mr Banner is obtained, it is due to the presence of too large an inductance.

A METHOD OF ELIMINATING THE EFFECTS OF MAGNETIC DISTURBANCE IN HIGHLY SENSITIVE GALVANOMETERS

By D. S. PERFECT, M.A., D.Phil., F.Inst.P.,

National Physical Laboratory

ABSTRACT. The paper describes a method of making correction for the chaotic fluctua-tions, of magnetic origin, to which the zero of a highly sensitive moving-magnet galvano-meter is susceptible. The method consists in the employment of a second galvanometer with properties adjusted as nearly as possible to identity with, and placed as close as possible to, the first. No current is passed through the second galvanometer, which acts in this respect as a dummy whose sole function is to record the zero changes. As a result of the proximity of the galvanometers, and the interrelation of their dynamical and geometrical conditions, the recording spots of light have an approximately constant separation, in spite of the fluctuations in the position of each. The current to be measured activates one of the galvanometers, and is measured by the accompanying change in the separation of the spots. The paper also describes a compensating method of observing, in which a single beam of light is reflected in succession from the mirrors of the active and dummy galvanometers so that the emergent beam tends to remain stationary.

§ 1. INTRODUCTION

MANY branches of experimental work, in which a galvanometer of very high effective sensitivity is an essential part of the equipment, have called urgently for an improvement in existing sensitivity. Thus the investiga-tions of Prof. A. V. Hill on the production of heat by nerves have necessitated and led to much successful work on the Paschen type of galvanometer by himself and Mr A. C. Downing of University College, London. Infra-red spectroscopy, and indeed radiometry in general, is another important field of research in which it is well recognized that future progress depends to a great extent upon the possibility of securing an increase in effective sensitivity. While in recent years great progress has been made, especially in America, towards greater spectroscopic resolution by the development of the échellette grating with a consequent increase in the amount of radiation available, progress has also been considerable in the improvement of detecting systems. The thermopile has been rendered very stable and rapid in action, while the figure of merit of galvanometers has been considerably increased, particularly by the employment of new types of magnet steel. The demands of the experimenter have however by no means yet been satisfied. Indeed, the progress made with galvanometers is to some extent counterbalanced by the great increase in recent years of the number of mechanical and magnetic disturbances which are produced in the neighbourhood of most laboratories, and have made it necessary,

in order even to maintain the position already reached, to give attention to the improvement of galvanometers.

In order to avoid the loss of effective sensitivity caused by these disturbances many workers have found it desirable to experiment in the small hours of the morning when the disturbances reach a minimum, but the inconvenience and fatigue involved in such a choice of time are undoubtedly an impediment to any protracted research requiring it.

The method here described has been found to give in this laboratory the same order of effective sensitivity during the day-time as is obtained without it under the most favourable conditions at night.

In the earlier stages of a research which required the use of a galvanometer having very high sensitivity, we were fortunate in obtaining an instrument embodying the result of recent work by Mr A. C. Downing and made by him. This galvanometer had the following special features: it removed all difficulties connected with mechanical vibrations, as it was found to be quite immune from such disturbances even when supported on a wooden table without any precautions for the elimination of shocks; it also removed the necessity for magnetic shielding, as the system had been adjusted to be so nearly astatic that it was disturbed only by the introduction of a magnetic field whose non-uniformity was considerable. The presence of a shield, by distorting the field within it, results, not in a gain, but in a loss of stability.

However, this galvanometer, excellent as it is, was still affected to an undesirable extent by the parasitic magnetic fields present in this laboratory. At a sensitivity of the order of 10^{-11} amp./mm. at 1 m. the chaotic movements due to this cause were sometimes as large as 5 mm. The elimination of the effects of these is an important preliminary to our utilizing the highest sensitivity, and forms the subject of the present paper.

§ 2. DESCRIPTION OF THE METHOD AND APPARATUS

If a galvanometer is set up in a certain position in an approximately uniform magnetic field the disturbing forces to which it is subject will be only slightly different at a neighbouring point in space. Consequently the behaviour of the galvanometer would be nearly the same in either of two adjacent positions. If then two absolutely identical galvanometers could be obtained and placed as near together as possible, one would expect them to perform the same series of movements as a result of fluctuations in the surrounding magnetic field. One galvanometer could then be used to measure the electric currents while the other acted as a dummy whose sole function is to record changes of zero, the position of the spot of light from the current-activated galvanometer being measured not from a fixed point on the "scale" (nor from that point of the scale occupied by the spot before the current was switched on) but from the spot of the dummy galvanometer. The success of such a method clearly depends on the nearness of the approach to identity of the two galvanometers and the closeness to each other with which they can be placed.

35-2

D. S. Perfect

Fortunately, a few weeks after the method had been devised a second galvano-meter became available for sufficient time to give the method a trial. The results obtained do not necessarily represent the best which the method is capable of giving. Firstly no attempt had been made to render the properties of the galvanometers identical, and secondly they were placed only so near together as the size of their bases would allow: it was not desirable at the time to construct bases specially adapted to allow a closer approach of the systems. The actual separation used was 25 cm. Experiments made on the mutual action between the two galvanometers indicate that a much closer approach would be possible without this action becoming prejudicial. Although the two galvanometers were not exactly matched,
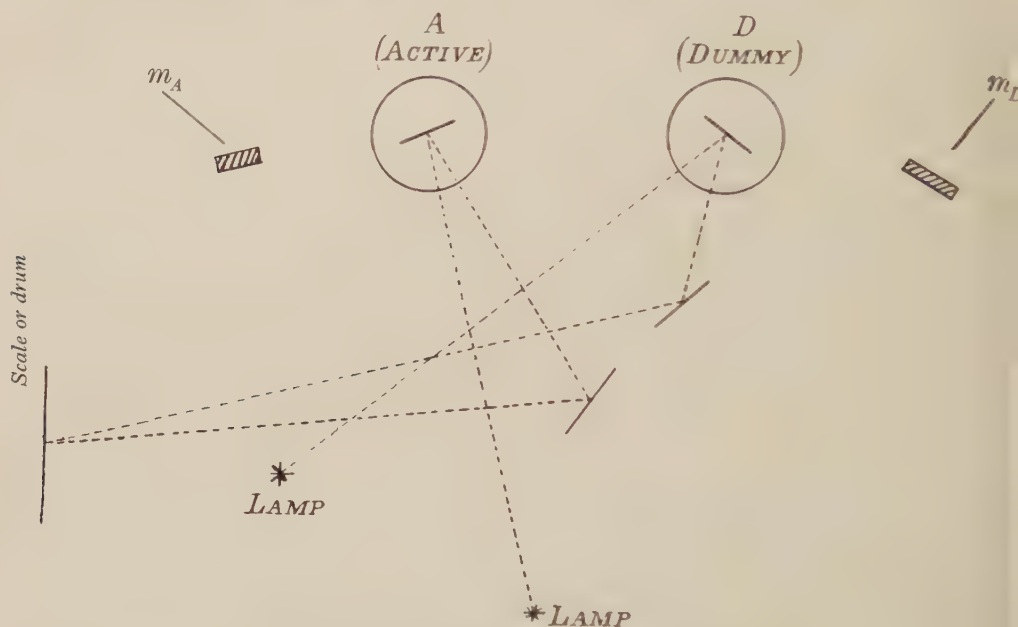
Fig. 1. Arrangement for adjusting twin system of galvanometers.

it was possible by using mechanical control of the orientation, magnetic control of the period, and optical control of the sensitivity, to bring the main relevant factors within the scope of adjustment. Thus, in order to make the two spots behave in the same way in spite of the lack of identity in the galvanometers them-selves, the following method was used:

Firstly the galvanometers were adjusted in azimuth so that when all control magnets were removed each system of suspended magnets was in equilibrium parallel to the planes of windings of its own set of coils, or, what is the same thing, so that each window was parallel to the suspended mirror.

In the second place, when the sensitivity of the measuring galvanometer A, Fig. 1, had been adjusted to the desired value by means of a controlling magnet m_A placed at the side of A remote from D, the period of D was adjusted to equality

with that of A by means of a controlling magnet m_D remote from A. This disposition of the control magnets gives a reasonably independent control of each galvanometer. The equality of periods can be judged with fair nicety by watching the two spots swing together. Both galvanometers were almost critically damped. In order to make these adjustments it is almost essential to mount m_A and m_D so that they are capable of fine adjustment. The arrangement used in practice is shown in Fig. 2. The magnet m is mounted on a vertical rod which can be raised or lowered, adjusted to and kept at any desired height by the collar c, and clamped to the vertical pillar by the screw s. Coarse adjustment of m in rotation
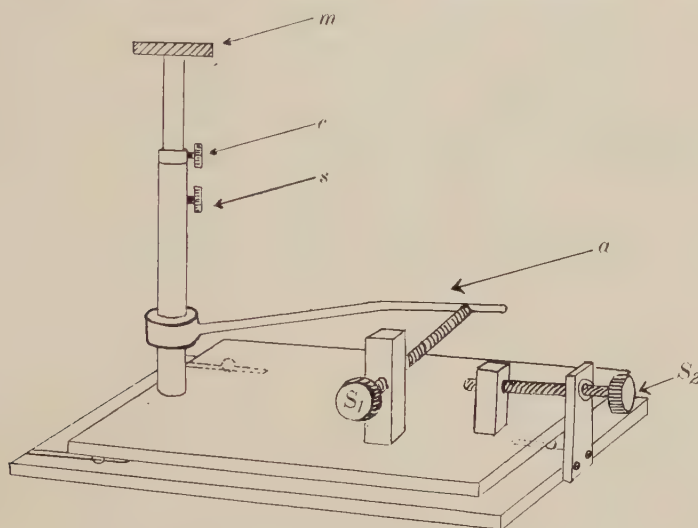
Fig. 2. Mounting for adjusting control magnets.

about a vertical axis is given by unclamping at s and rotating by hand: fine adjustment is given by operation of the screw S_1, which bears on a radial arm a. Coarse adjustment of the distance of m from the galvanometer is given by the sliding of the whole mounting along the table, and fine adjustment by the turning of the screw S_2 which moves a slide on which the pillar carrying m is mounted.

Finally, the periods having been adjusted to equality, we can make the effective sensitivities equal by adjusting the relative lengths of the optical paths between the galvanometers and the scale. To effect this and also to direct each beam on to the scale it may be necessary to employ two reflections of each beam. This was in fact necessary in the first trial given to the method, but fortunately in a subsequent set-up the galvanometers happened to be so orientated that a single reflection (given by a 90° prism) was sufficient to fulfil the necessary conditions for each beam.

§3. EXPERIMENTAL DETAILS

Before proceeding to describe the results obtained it is of interest to examine the actual methods by which observations may be made. It is clear that it would be unprofitable to attempt to make visual determinations of the simultaneous

positions on the scale of two fluctuating spots. It is possible, however, to judge
with accuracy when the two spots are superimposed or when their separation changes,
if this is small, by using such a type of spot as is shown in Fig. 3. A null method
is therefore very easily applicable. The deflections of the spot due to passing
a current through the activated galvanometer may be compensated to a measured
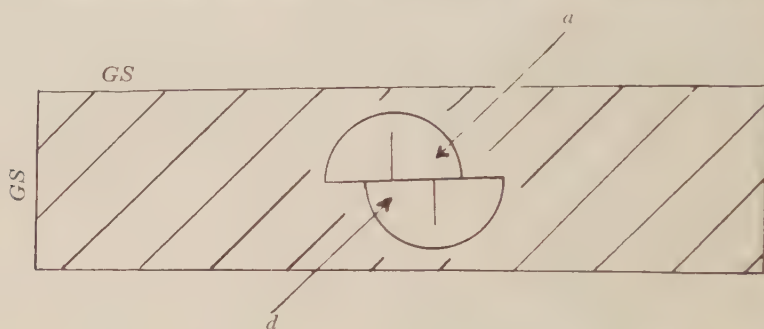
Fig. 3. Appearance of scale and spots for use with null method. *a*, spot from galvanometer *A*;
d, spot from galvanometer *D*; *GS*, galvanometer scale.
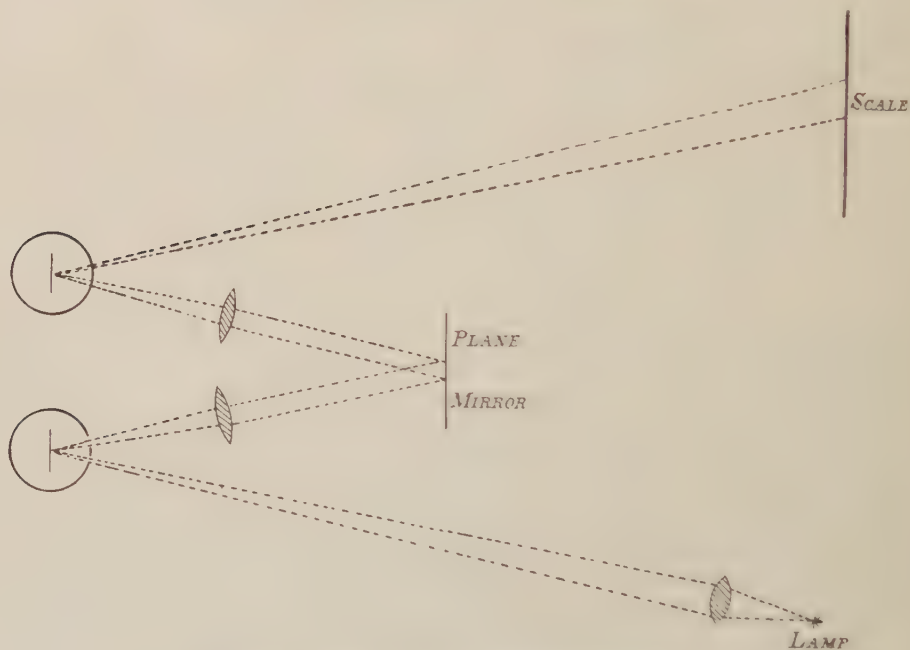
Fig. 4. A system used for optical compensation.

extent either electrically by means of a potentiometer or optically by the use of
a rotatable mirror after the manner of a sextant. The electrical method is preferable
for the reason that its use eliminates any non-linearity of the current scale of the
galvanometer and avoids the need for its calibration.

Another method of optical compensation suggested by my colleague, Mr J.
Guild, was developed and tried, in which only one beam of light was used and

passed from one galvanometer mirror to the other on its way to the scale, with the result that no displacement of the spot occurs if the two galvanometer mirrors are rotated through equal angles. This procedure was subsequently suggested in principle by several other physicists to whom the outlines of the general method had been shown. In particular a similar arrangement is suggested by Mr A. B. D. Cassie* to whom the system of galvanometers had been shown by Mr Downing, but who was unaware that the method of optical compensation had already been applied. An arrangement employed in this laboratory is illustrated in Fig. 4, which is self-explanatory. The method proved quite satisfactory in use but does not give greater precision than is obtained by the employment of two independent spots; moreover it involves the use of additional optical parts, and the aperture tends to be more limited. Further the absence of movement disguises the disturbances which are being compensated. It was found preferable, therefore, at least during the stage of preliminary investigation, to revert to the previous method, which shows clearly the details of the disturbances themselves. The method of compensation is of course applicable to a deflection method as well as to a null method, and for some purposes this is advantageous.

Use has been made by several workers of amplification of galvanometer deflections by means of a relay. The advantage gained consists in the relative ease of working, since the galvanometer gives the same effective sensitivity when in more stable adjustment. There seems no reason why a similar method should not prove applicable to the method of twin-galvanometers described in the present paper. The investigation has not reached a stage at which it has been profitable to develop this refinement, but it may be of interest to give a brief outline of one type of scheme which appears feasible.

A rectangular opaque screen XY, Fig. 5, is placed in the path of the beams of light reflected from the galvanometers so that the spots are focussed on it. The spots might be circular or rectangular, but whatever their shape the separation of their vertical axes is made approximately equal to the width of the screen XY. This result may be accomplished by controlling either the separation of the spots or the width of XY, whichever is more convenient in practice. The portions of the beams unobstructed by XY are directed by prisms on to a photo-electric cell C, the current through which is measured by any suitable means. If we assume that the energy is uniformly distributed over the cross-sections of the beams, it is clear that with such an arrangement a common displacement of the two beams will produce no change in the photo-electric current, while a relative displacement will do so. The method of optical compensation giving a single spot would be immediately adaptable to types of relays already designed for such purposes.

The method finally adopted to investigate the behaviour of the two galvanometers was to take photographic records on a revolving drum. This method has indeed advantages which render it in some respects preferable to the visual null method not only for such preliminary investigations as that in which it was here

* *Journal of Scientific Instruments*, **7**, 21 (1930).

employed but for actual measurements. It creates a permanent record which can be re-measured at leisure, eliminates the personal judgment which enters into the visual settings, and gives a continuous record of the behaviour of the galvano-meters during the time occupied in the measurements, instead of a discrete series of observations selected more or less arbitrarily by the observer. In the final stages of measurement these advantages outweigh the disadvantages due to the indirectness of photographic methods and to the inapplicability of the null method.
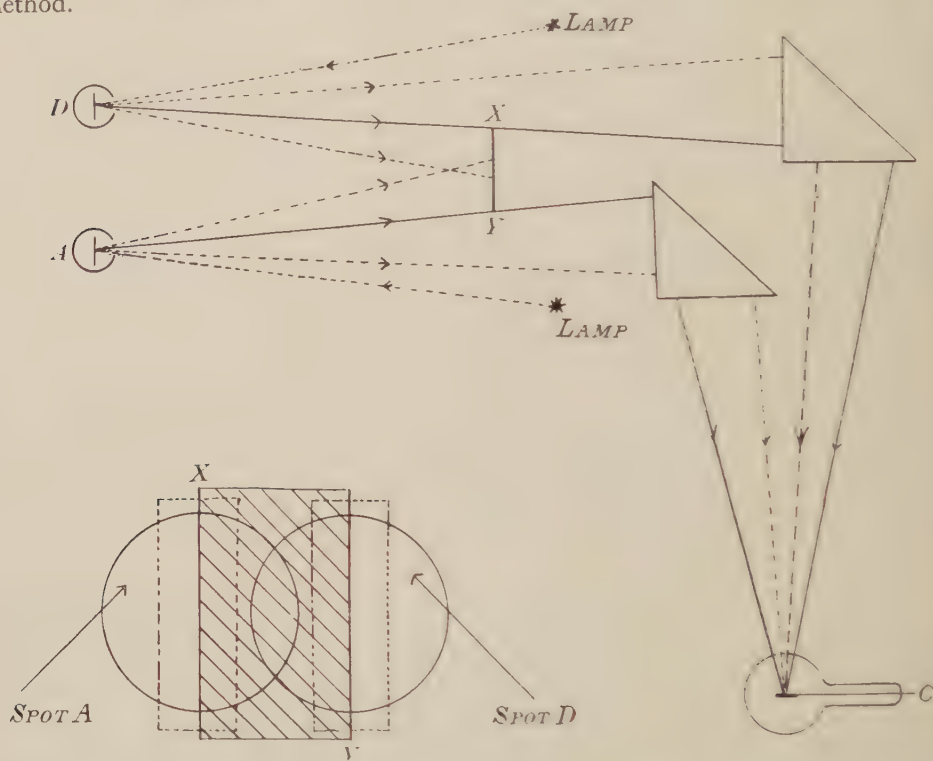
Fig. 5. System suggested for use with relay.

§ 4. TYPICAL RESULTS

Typical records are reproduced in Fig. 6 (a), (b), (c). Records (a) and (c) were taken during laboratory working hours, whereas (b) was taken at 1.0 a.m. The vertical lines are time-traces obtained by flashing on a pea lamp for about 1 second. These traces are marked with numbers representing the time in minutes from the first trace. The time scale is the same in (a) and (b): in (c) it is much reduced, so that a longer time is included but at the expense of the finer structure in the records.

It will be noticed that in (a) and (b) one trace is repeatedly displaced in suc-cessively opposite directions. These displacements are due to the switching on and off of a current of 10⁻⁹ amperes which is made to pass through one galvanometer. In (a) the track of the current-activated galvanometer will be found to cross the track

of the dummy galvanometer on several of the occasions of switching on and off. In other regions there is nearly exact superposition of the tracks. The relative smallness of the disturbances at 1.0 a.m. is clearly brought out in (*b*). In this record, where the irregularities are themselves much smaller, the relative irregularities are more apparent.

No current was passed through the galvanometer during the making of record (*c*), which thus represents the "natural" behaviour of the galvanometers for about 1 hour. At various points of the record will be seen discontinuities in the traces, on
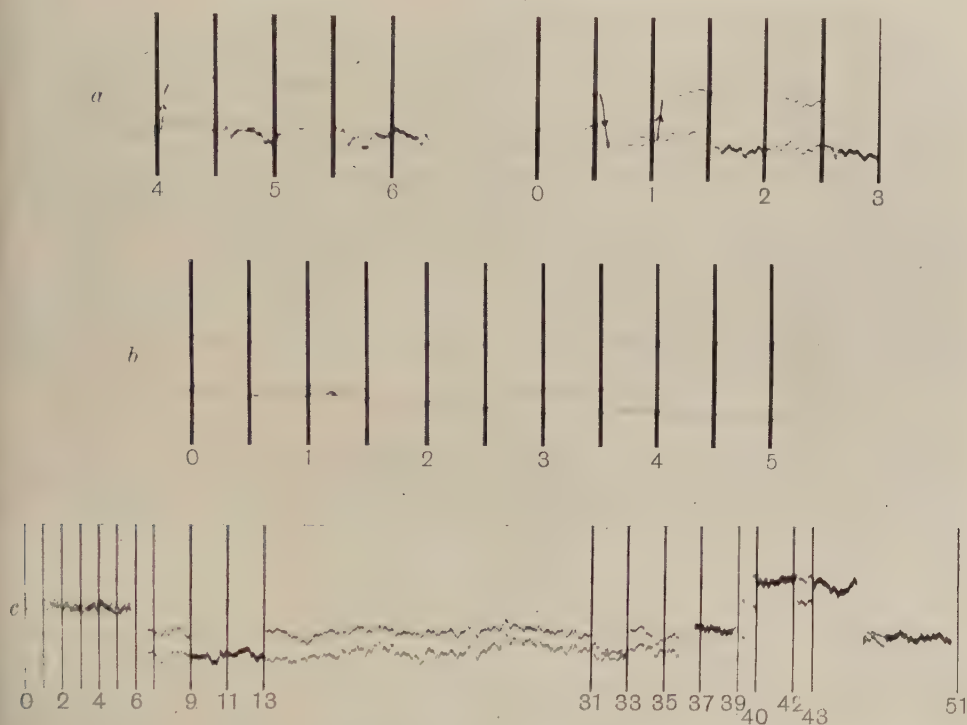
Fig. 6. Typical records obtained with the twin system.

either side of which the separation of the traces differs. These discontinuities were produced by the action of a heavy luggage lift situated about 10 yards from the galvanometers, and, as it happened, in the most favourable position for producing a relative disturbance; that is, nearly in line with them. The changes in the relative and absolute positions of the traces after a discontinuity are due to the changes in height of the lift and its counterpoise.

In each record slow changes occur in the separation of the two traces. This is particularly striking in (*b*). These changes may be accounted for in several ways, and probably are removable. They do not however appreciably affect the accuracy of measurement of current. The function of the dummy galvanometer is,

so to speak, to bridge the time-gap required by the activated galvanometer for settling down after activation, and to indicate the change of "zero" which has occurred after this gap. If the activated galvanometer had infinitesimal period the correction for change of zero would be infinitesimal and the dummy galvanometer would be superfluous.

In practice the time-gap is of the order of 10 seconds and in this time the secular change is negligible. When the disturbances reach so small a magnitude as in (b) we can obtain with sufficient accuracy the natural or steady scale reading by drawing a mean straight line through the trace, and the length of trace necessary to define the mean line with sufficient accuracy is short enough for the secular change to be negligible over its length. In these circumstances the dummy galvanometer is probably superfluous under the conditions of this particular experiment. It does not follow its twin well enough to enable the small irregularities to be corrected any better than they could be by drawing the mean line. If the experimental conditions were improved, however, even these minor disturbances might be more faithfully followed. In a record such as (a) the mean straight line cannot be drawn accurately in a length of track reasonably free from secular change, but the dummy galvanometer follows these grosser irregularities sufficiently well to justify its presence in securing, by measurements of separations of single points on the tracks, the same accuracy as is obtainable without it in (b) by measurement of the separations of mean lines.

§ 5. CONCLUSION

As regards its general applicability it should be noted that the method described in this paper is useful only with galvanometers whose construction is so good that the magnetic disturbances are small enough to make it desirable to dispense with magnetic shields. It is fairly certain that the correlation between the movements of two independently shielded instruments would be very small, and it is unlikely that success would attend the use of a common shield of reasonably small dimensions. Further, it is clearly superfluous to apply the method at all until the effects of disturbances of mechanical origin have been made relatively small either by the internal construction of the galvanometers, or by the use of anti-vibration supports.

In conclusion it may be stated that Mr Downing has meanwhile been carrying out promising experiments with the object of producing two systems very nearly identical. The results already obtained and here described make it appear reasonable to hope that the use of such improved systems, and a decrease in their working separation, will still further increase the utilizable sensitivity of this type of galvanometer.

ADDENDUM

Since the above paper was written my attention has been called to a paper by Dr G. A. Shakespear*, in which some similar arrangements are described.

* G. A. Shakespear, "Experiments on the temperature coefficient of a Kew collimator magnet," *Proc. R. S. A*, **39**, 220 (1913).

CLASSICAL QUANTUM THEORY AND X-RAY EXCITATION BY CANAL RAYS AND ALPHA-PARTICLES

By WILLIAM BAND, M.Sc.

ABSTRACT. This paper shows, by application of the classical quantum theory and simple equations of energy, that it is not possible for canal rays of normal experimental energy to remove K-electrons from the atoms of a metal target either by capture or by simple removal into free space. No particular mechanism is assumed, but a safe maximum for the possible transfer of momentum to the ejected electron is taken to be the transfer which takes place in a head-on collision, electric attraction being neglected. The fact that alpha-particles are observed to excite X-rays shows that the mechanism must be almost as efficient as this. The process of capture is shown to be no more probable than that of simple removal into free space. The minimum energy for canal rays and the minimum velocity for alpha-particles, for K-radiation from copper, are calculated as at least 72 million volts and $1\cdot7 \times 10^6$ cm./sec. respectively. The theory claims to give a satisfactory explanation of all the experimental data so far available.

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§ I. THE PROBLEM: EXPERIMENTAL DATA

IN an early investigation, J. J. Thomson appeared to have detected X-radiation excited by canal rays impinging on a metal target, but his results were explained away by an observation of C. Gerthsen* that canal rays when reflected from the target surface often have their charges neutralized; they were thus able to pass through the electric-field-filter which was used by Thomson to shield the plate from the reflected rays. Gerthsen, using a more refined method, detected an exceedingly weak effect due possibly but not certainly to soft radiation excited by the canal rays.

J. J. Thomson, using a specially assumed model for the collision process, showed theoretically that a proton possessed of the energy actually available could not remove an electron from any X-ray level of the target atoms; but his model also required that the alpha-particle should be unable to effect the removal. His model is therefore unsatisfactory, for alpha-particles have been observed to excite X-rays†.

In a recent piece of work Prof. Barton‡, using a highly sensitive instrument, shows that there is negligible radiation due to even heavy currents of protons up to 25 kV, the efficiency relative to the excitation by electrons being at the most of the order 10^{-5}.

* C. Gerthsen, *Ann. der Phys.* 85, 881 (1928).
† W. Bothe and H. Franz, *Zeit. für Phys.* 52, 446 (1929).
‡ H. A. Barton, *Journ. Frank. Inst.* p. 1 (1930).

We therefore require a theory which while it allows the alpha-particle to excite radiation, must render it practically impossible for protons of the above energy to do so. This paper shows that such a theory may be erected from the quantum theory without any assumption as regards the mechanism whereby the momentum is transferred to electrons.

§2. QUALITATIVE SUGGESTIONS

Barton apparently supposes that there is no essential difference between the processes of excitation of radiation by positive and by negative particles. But excitation by electrons approximates to the collision between two elastic spheres, on account of the mutual repulsion between the incident and the ejected electrons, whereas excitation by protons or alpha-particles must be essentially due to the K-electron being pulled out from the atom by the attractive force exerted by the close approach of the incident particle.

Accepting Thomson's results provisionally, that the mere transfer of momentum during close approach is not sufficient to effect the complete removal from the atom, we are left with the one alternative that the proton or alpha-particle captures the electron and retains it in an appropriate quantum orbit after emerging from the atom. This process may quite possibly be more efficient than complete removal of the electron into free regions, for in the quantum orbit round the capturing nucleus the electron will still have a negative energy, whereas in the free regions it cannot have less than zero energy. The kinetic energy of the capturing nucleus will modify this result to an ascertainable degree.

We are thus led to the following physical picture. An electron revolves in the inner quantum orbit of a metal atom. An incident nucleus, H or He, enters the field of the metal nucleus, and with diminished energy due to the repulsion of the latter approaches very closely to the electron. The electron is deflected from its normal course and sweeps in an open curve round the intruder. Work has been done on the electron by the field of the incident particle, and this is sufficient to carry the electron out into one of the inner orbits of the new nucleus, by which time the latter has carried its prize well out of the field of the original atom. Since the intruding nucleus is of less atomic number than the metal atoms, the energy of the electron has been increased, and by an amount that can be calculated by the rules of the quantum theory.

The possibility of capture thus depends primarily on the amount of energy that must be given to the electron in order to get it into the innermost orbit round the capturing nucleus, and this will be greater for the proton than for the He nucleus. Hence we can already see that the latter will have a much greater chance of capturing the electron than will the proton, quite apart from its greater energy and momentum.

It will thus be seen that, if we assume the quantum theory, simple equations of energy will suffice to solve the problem, without any other assumptions as to the mechanism of the transfer of the momentum. We shall proceed accordingly,

taking for simplicity the simple circular quantum orbits of elementary quantum theory.

§3. PROTON COLLISION

Let M be the mass of the incident proton; M
Let V be its initial velocity on entering the target; V
Let V' be its velocity after escape from the target atom; V'
Let e, m be the electronic charge and mass; e, m
Let a be the radius of the K-orbit in the target atom; a
Let b be the radius of the orbit about the proton after capture; b
Let Z be the atomic number of the target atom; and Z
Let E be the kinetic energy transferred to the electron during the close encounter, E
all units being in the c.g.s. system.

We assume that the proton has negligible effect on the target nucleus. Then the equation of energy of the electron alone is

$$E - Ze^2/2a = \tfrac{1}{2}mV'^2 - e^2/2b \qquad \ldots\ldots(1),$$

and that of the complete system is

$$\tfrac{1}{2}(M + m)\, V'^2 - e^2/2b = \tfrac{1}{2}MV^2 - Ze^2/2a \qquad \ldots\ldots(2).$$

If b is the radius of the innermost orbit round the proton, then

$$b = Za,$$

and by (2) therefore

$$\begin{aligned} V'^2 &= e^2\,(1/b - Z/a)/(M + m) + MV^2/(M + m) \\ &= MV^2/(M + m) - e^2\,(Z - 1/Z)/a\,(M + m) \qquad \ldots\ldots(3). \end{aligned}$$

Adding together (1) and (2) we get

$$E = \tfrac{1}{2}MV^2 - \tfrac{1}{2}MV'^2 \qquad \ldots\ldots(4),$$

which by (3) gives

$$E = [\tfrac{1}{2}MV^2 + \tfrac{1}{2}(M/m)\,(e^2/a)\,(Z - 1/Z)]\,.m/(M + m) \qquad \ldots\ldots(5).$$

Taking Z as 29, and canal rays of 25 kV, or $\tfrac{1}{2}MV^2$ as 4.10^{-8} ergs, we calculate that

$$E = 1\cdot7.10^{-8} \text{ ergs} \qquad \ldots\ldots(6).$$

§4. ALPHA-PARTICLE COLLISION

In terms of the same symbols as in the last section, the mass of the alpha-particle is $4M$ and its charge $2e$; the equations of energy become for the ejected electron

$$E - Ze^2/2a = \tfrac{1}{2}mV'^2 - 2e^2/2c \qquad \ldots\ldots(1'),$$

and for the complete system

$$\tfrac{1}{2}(4M + m)\, V'^2 - 2e^2/2c = 2MV^2 - Ze^2/2a \qquad \ldots\ldots(2').$$

c Here c is the radius of the orbit round the He nucleus, so that

$$2c = Za.$$

From (1′) and (2′) we get

$$V'^2 = 4MV^2/(4M + m) - e^2 (Z - 4/Z)/a (4M + m) \qquad \text{......(3′)}$$

and

$$E = 2MV^2 - 2MV'^2 \qquad \text{......(4′),}$$

so that finally

$$E = [2MV'^2 - 2 (M m) (e^2 a) (Z - 4 Z)].m (4M - m) \qquad \text{......(5′).}$$

Assuming for the velocity V of the alpha-particle that

$$V = 2.10^9 \text{ cm. sec.,}$$

we calculate

$$E = 1\cdot6.10^{-8} \text{ ergs} \qquad \text{......(6′).}$$

n
a′ To remove an electron from the $n = 2$ orbit, we merely substitute for a the value $a' = n^2a$ with $n = 2$. This gives

$$E_2 = 0\cdot54.10^{-8} \text{ ergs} \qquad \text{......(6″).}$$

It is worth noting that the negative energies of the electron in the two states a and a' are $1\cdot54.10^{-8}$ and $0\cdot36.10^{-8}$ ergs respectively, so that the high kinetic energy of the alpha-particle renders the process of capture no more efficient than that of simple removal.

§5. MINIMUM VELOCITIES AND VOLTAGES

The energy of the alpha-particle with the above velocity is $1\cdot3.10^{-5}$ ergs. so that the above results show very definitely how much more probable is the capture, or removal, by an alpha-particle than by a proton of 25 kV. In fact the latter will have to give up nearly half its initial kinetic energy to the electron.

The above work tells us what values of E are necessary for the excitation process. Let us now suppose that we can obtain a safe maximum for the possible values of E by calculating them for head-on collision, neglecting the effect of electrical attraction, and assuming perfect elasticity. This method gives for the alpha-particle the value $E_{\max} = 1\cdot4.10^{-8}$ ergs and for the proton the value $E_{\max} = 9.10^{-11}$ ergs. Since the latter is of a much smaller order than the required value for the proton, we conclude that no mechanism could possibly be constructed which would enable protons of 25 kV to eject a K-electron from a Cu atom. For alpha-particles we may profitably reverse the argument: since X-rays of very slight intensity, due to alpha-particles, are actually observed, the efficiency of the mechanism in this case must be practically as great as that of head-on collision.

To estimate the minimum voltages required to give the proton sufficient energy to excite X-rays we must put our formulae in a more general form. Thus, for the general expression for the possible maximum value of E as a function of the velocity V of the incident particle we have the following equations:

$$MV = MV' + mV'',$$
$$MV^2 = MV'^2 + mV''^2,$$

V″ where

$$\tfrac{1}{2}mV''^2 = E_{\max} \text{ defines } V''.$$

These give both $E_{\max}$ and V', thus

$$V' = V(M - m)/(M + m),$$
$$V'' = 2V(M + m)^2/M^2,$$

or

$$.\ E_{\max} = 2mV^2(M + m)^2/M^2 \qquad \ldots\ldots(7).$$

We may now write (5) in the form

$$\tfrac{1}{2}MV^2 = 2mV^2[(M + m)/m](M + m)^2/M^2 - \tfrac{1}{2}(M/m)(e^2/a)(Z/n^2 - 1/Z)$$

or $\quad \tfrac{1}{2}MV^2 = \tfrac{3}{2}(M/m)(e^2/a)(Z/n^2 - 1/Z)$ ergs $\qquad \ldots\ldots(8).$

This gives the voltage as $7\cdot2\cdot10^4$ kV., with $n = 1$.

For the alpha-particle we shall have

$$E_{\max} = 2mV^2(4M + m)^2/(4M)^2 \qquad \ldots\ldots(7'),$$

and by (5)

$$2MV^2 = 2mV^2[(4M + m)/m](4M + m)^2/(4M)^2 - 2(M/m)(e^2/a)(Z/n^2 - 1/Z)$$

or $\qquad V = [(1/12m)(e^2/a)(Z/n^2 - 1/Z)]^{\frac{1}{2}} \qquad \ldots\ldots(8').$

With $Z = 29$, for Cu targets, this gives the following results:

$$n = 1;\ V = 1\cdot65\cdot10^9 \text{ cm./sec.,}$$
$$n = 2;\ V = 0\cdot65\cdot10^9 \text{ cm./sec., etc.,}$$

which are normal values of the velocity of alpha-particles.

As another example, put $Z = 78$ for *Pt*; then we get

$$n = 1;\ V = 7\cdot10^9 \text{ cm./sec.,}$$
$$n = 2;\ V = 2\cdot10^9 \text{ cm./sec., etc.,}$$

so that the alpha-particles will hardly be able to excite *K*-radiation from a platinum target.

With regard to the head-on collisions it should be remarked that if we assume that superposed on the inverse-square attraction between positive and negative particles is an inverse-fourth-power repulsion, in accordance with Bieler's* deductions from experiments on the wide-angle scattering of alpha-particles by light nuclei, then it follows that such head-on collisions will be exactly equivalent to head-on collisions between perfectly elastic spheres, and the actual energy-transfer will in this case be accurately given by the above maximum values. It is thus quite permissible to suppose that the required energy-transfers are within the range of possibility.

Obviously the distance within which the proton and the electron must approach each other, in order to cause that repelling force to become effective, will be very much less than that required when two electrons collide. Consequently the approximately head-on collision between the proton and a *K*-electron will be less probable than that between the electron and either an incident electron, or a presumably larger alpha-particle.

* E. S. Bieler, *Proc. R. S. A*, **105**, 434 (1924).

§6. CONCLUSION

From further experimental data and by the help of the above theory we should be able to elucidate the mechanism of the process whereby the momentum is transferred to the ejected particle. Thus in the theory we have taken a safe maximum value for E, the kinetic energy transferred, and in practice we expect to find that the actual possible value is somewhat less than this; and the required velocities of the exciting particles will thus have to be greater than the calculated ones. The actual velocities as compared with the theoretical ones will then give a measure of the efficiency of the process of transfer.

Further, if the minimum velocities are found in practice to be sharp, or critical, then the momentum at transference must be quantized; if on the other hand the minima are gradual, or non-critical, as is suggested by the results of Bothe and Franz*, then the momentum is not quantized at transfer.

This paper does not of course pretend to be an exhaustive study, which would be premature until more experimental data are at hand; but it may perhaps suggest possibilities of treatment of the problem by simple methods that do not appear to have been employed before in this connection. We may also fairly claim to have shown that we do not need to wait for the further development of wave mechanics in order to obtain a satisfactory explanation of the phenomena so far observed.

* W. Bothe and H. Franz, *Zeit. für Phys.* **52**, 446 (1929).

DIFFUSION INTO AN INFINITE PLANE SHEET SUBJECT TO A SURFACE CONDITION, WITH A METHOD OF APPLICATION TO EXPERIMENTAL DATA

By A. T. McKAY, M.Sc.

ABSTRACT. The solution of the partial-differential diffusion equation for the infinite plane sheet is found by operational methods, subject to a surface condition analogous to Newton's law of cooling. By use of the fact that the area between the theoretical quantity/time curve, its asymptote and the quantity-axis is finite, a method of curve-fitting suitable to the case is suggested. To facilitate the application of the method to experimental data two numerical tables are appended.

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§ 1. INTRODUCTION

THE investigation with which this paper deals originated in the course of experimental research relating to the interchange of moisture between leather and the atmosphere, consequent upon changes in humidity. It has already been shown* that when a piece of leather is immersed in water this entry of moisture is very closely represented by a simple diffusion theory. Other workers† have found similar results for rubber sheets immersed in water.

When leather specimens which were in equilibrium with an atmosphere of fixed relative humidity were transferred into atmospheres of different humidity, the experimental curves obtained, showing a gain or loss of moisture against time, closely resembled those of the previous case and quite naturally were tested against the simple theory. Discrepancies between experimental results and the simple theory were manifested, however, and were of such a magnitude as could not be attributed to errors of observation. It was necessary to try to formulate a new theory to be tested against experiment. Retaining the hypothesis that the mechanism of moisture movement in the leather is that of diffusion, we were led to contemplate the influence of the surface in retarding the flow.

Analogy with heat problems‡ suggested the examination of the solution of the partial-differential diffusion equation for the infinite plane sheet subject to surface conditions expressed by the analogue of Newton's law of cooling. The problem

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* H. Bradley, A. T. McKay and B. Worswick, *Journ. International Society of Leather Trades' Chemists*, **13**, 87–105 (1929).

† D. H. Andrews and I. Johnstone, *Journ. American Chem. Soc.* **46**, 640–650 (1924).

‡ Preston, *Theory of Heat*, ch. 7 (1924).

is not new in the mathematical sense*† but for the present purpose it has to be considered in a rather unusual way, owing to the fact that experiment merely provides a measure of quantity, whereas in the case of heat investigations temperature is recorded. It is hoped that the study of this problem by means of the operational methods of Heaviside‡, extended by Bromwich§, Jeffreys ‖ and others, will be of interest to physicists. To enhance the utility of the work a method is put forward whereby numerical values of the constants are determined from the experimental curve, so that a theoretical curve may be plotted therefrom to test the fit of the theory. This procedure is perhaps worthy of note for it suggests the employment of a similar device when it is necessary to fit data to such a complicated equation as the one in question. To facilitate the practical analysis of data on the lines of the present theory two numerical tables are appended.

§ 2. STATEMENT OF PROBLEM AND DEFINITION OF SYMBOLS

An infinite plane sheet of hygroscopic material, initially in equilibrium with an atmosphere of given relative humidity, is placed in an atmosphere of different relative humidity. Supposing that the absorption is expressed by the usual partial-differential diffusion equation together with the surface condition given by the analogue of Newton's law of cooling, we have to find the manner in which the sheet attains equilibrium with the different humidity.

The term "concentration" is to be taken as the excess of moisture present, per unit volume, over that contained initially. Most of the subsequent work will not be affected by the supposition that the quantity of moisture in a lamina of thickness τ is equal to τ multiplied by the concentration in the lamina.

The following symbols are used:

t t, time measured from the instant when the sheet is placed in the atmosphere of higher humidity.

x x, distance from the central section, measured perpendicularly to the plane of the sheet.

Q Q, quantity of moisture in the sheet, per unit area of face, in excess of that contained initially.

Q_m Q_m, maximum value of Q, i.e. the equilibrium moisture content of the specimen at the higher humidity, less the equilibrium moisture content at the lower humidity.

θ_a θ_a, concentration at the surface at time t.

θ θ, concentration at plane x at time t.

θ_m θ_m, equilibrium value of θ corresponding to Q_m.

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* W. E. Byerly, *An Elementary Treatise on Fourier Series and Spherical Harmonics*, pp. 117–121 (1893).

† H. S. Carslaw, *Phil. Mag.* 39, 603–611 (1920).   ‡ Oliver Heaviside, *Electromagnetic Theory*.

§ T. J. I'A. Bromwich, *Proc. Lond. Math. Soc.* (2), 15, 401–448 (1916); *Phil. Mag.* (6), 37, 407–419 (1919); *Proc. Camb. Phil. Soc.* 20, 411–427 (1921).

‖ H. Jeffreys, *Cambridge Tracts in Mathematics*, No. 23.

$2a$, thickness of sheet.		$\gamma \equiv 2k/ca.$		a, γ
k, diffusivity constant.		$p \equiv \dfrac{d}{dt}.$		k, p
c, surface constant.		$\sigma^2 \equiv p/k.$		c, σ

§3. SOLUTION OF THE DIFFERENTIAL EQUATION

We require to solve the differential equation

$$\frac{\partial^2\theta}{\partial x^2} = \frac{1}{k}\frac{\partial\theta}{\partial t} \qquad \ldots\ldots(1),$$

subject to the conditions

$$\left.\begin{array}{l} (a)\ \text{Symmetry with respect to } x \\[2mm] (b)\ \dfrac{dQ}{dt} = c\,(\theta_m - \theta_a) \end{array}\right\} \qquad \ldots\ldots(2).$$

The requisite solution could, of course, be built up by the usual device of writing $\theta = \theta_m e^{A c + B t}$ and expanding unity as a generalized Fourier series*. This method does not commend itself for the present purposes, so we shall make use of the operative methods of Heaviside and Bromwich†. The equation (1) may be written

$$\frac{\partial^2\theta}{dx^2} = \sigma^2\theta \qquad \ldots\ldots(3).$$

Solving this as an ordinary differential equation and choosing in accordance with condition ($2a$) the solution symmetrical in x, we have

$$\theta = A\cosh\sigma x \qquad \ldots\ldots(4),$$

where A is independent of x but involves σ. Further, $\qquad\qquad A$

$$Q = 2\int_0^a \theta\,dx = 2A\sinh\sigma a/\sigma.$$

Hence, using condition ($2b$) and writing γ for $2k/ca$, we find

$$A = \theta_m/(\gamma\sigma a\sinh\sigma a + \cosh\sigma a).$$

Substituting this in equation (4) we arrive at the operational solution

$$\theta = \theta_m\cosh\sigma x/(\gamma\sigma a\sinh\sigma a + \cosh\sigma a) \qquad \ldots\ldots(5).$$

The operator on the right-hand side of equation (5) now requires interpretation, and we could effect this by expanding the function of σ in partial fractions, combining pairs of terms and making use of the fact that $1/(p+h) = (1 - e^{-ht})/h$. We shall not, however, do this directly, but shall employ the rule due to Bromwich‡, which states that provided $f(z)$ is an analytic function of z, then

$$f(p) = \frac{1}{2\pi i}\int_L e^{tz} f(z)\frac{dz}{z},$$

where L is a curve in the z-plane, and extends from $\xi - i\infty$ to $\xi + i\infty$ (where ξ is positive and finite) and is such that all the singularities of the integrand are on the

* W. E. Byerly, *loc. cit.*   † *Loc. cit.*   ‡ T. J. I'A. Bromwich, *loc. cit.*

A. T. McKay

$-\infty$ side of the path (see Fig. 1). Recalling that $\sigma = p^{\frac{1}{2}}/k^{\frac{1}{2}}$ and applying the above rule, we find

$$\theta = \frac{\theta_m}{2\pi i} \int_L \frac{e^{kzt/a^2} \cosh (xz^{\frac{1}{2}}/a)}{\{(\gamma z^{\frac{1}{2}}) \sinh z^{\frac{1}{2}} + \cosh z^{\frac{1}{2}}\}} \frac{dz}{z} \qquad \ldots\ldots(6),$$

whence, by integration with respect to x, and noting that $Q_m = 2a\theta_m$, we find

$$Q = \frac{Q_m}{2\pi i} \int_L \frac{e^{kzt/a^2}}{(\gamma z + z^{\frac{1}{2}} \coth z^{\frac{1}{2}})} \frac{dz}{z} \qquad \ldots\ldots(7).$$

Equations (6) and (7) give the complete solution of the differential equation in terms of contour integrals. The integrand on the right-hand side of equation (7) is a single-valued function of z with poles at $z = 0$, $z = -\beta_n^2$, where $n = 0, 1, 2$, etc., and β_n is the nth positive root of the transcendental trigonometric equation

$$\cot y = \gamma y \qquad \ldots\ldots(8).$$
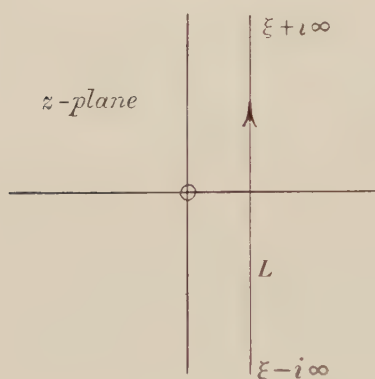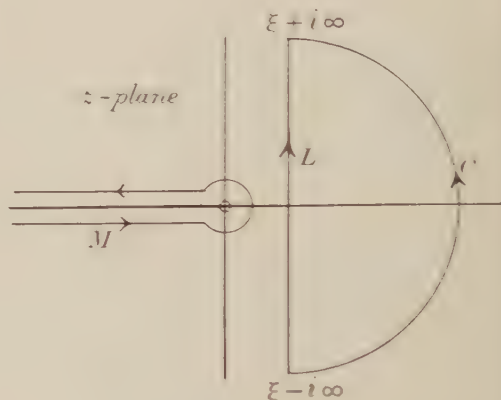
Fig. 1.   Fig. 2.

This equation also arises in connection with the vibrations of a loaded string[*] as well as in the process of expanding a function as a generalized Fourier series, where the roots of equation (8) play a part similar to that of multiples of $\pi/2$ in the ordinary Fourier expansions[†]. Some roots of this equation have been given by Schlömilch[‡], and Jahnke and Emde[§] give a short table and a graph from which values of the first root (β_1) can be obtained. The other roots can be obtained most accurately by the use of iterative methods or of inverse interpolation[||].

Returning now to equation (7), the coefficient of z is always positive or zero and the remaining term in the integrand approaches zero uniformly with respect to $\arg z$ as $|z| \to \infty$ with a possible exception when $-\pi + \epsilon < \arg z < \pi - \epsilon$, where ϵ is arbitrarily small; hence by a slight extension of Jordan's lemma[¶] the path L is equivalent to the path M (see Fig. 2).

[*] H. Lamb, *The Dynamical Theory of Sound*, pp. 82–84 (1910).   [†] W. E. Byerly, *loc. cit.*
[‡] Schlömilch, *Übungsbuch*, 1, 354.
[§] E. Jahnke and F. Emde, *Funktionentafeln mit Formeln und Kurven*, pp. 1–2 (1928).
[||] E. T. Whittaker and G. Robinson, *The Calculus of Observations*, pp. 81–84 (1924).
[¶] E. T. Whittaker and G. N. Watson, *A Course of Modern Analysis*, p. 115, 3rd ed. (1920).

Hence, using the principle of residues, we easily find

$$\theta = \theta_m \left[1 - \sum_1^\infty \frac{2e^{-k^3_n{}^2 t/a^2} \cos(x\beta_n/a)/\sin\beta_n}{\beta_n{}^2 (1 + \gamma + \gamma^2 \beta_n{}^2)} \right] \qquad \ldots\ldots(9),$$

$$Q = Q_m \left[1 - \sum_1^\infty \frac{2e^{-k^3_n{}^2 t/a^2}}{\beta_n{}^2 (1 + \gamma + \gamma^2 \beta_n{}^2)} \right] \qquad \ldots\ldots(10).$$

We shall find it convenient to write $2/\beta_n{}^2 (1 + \gamma + \gamma^2 \beta_n{}^2) = b_n$. It is evident b_n that equations (9) and (10) completely satisfy the conditions laid down and are therefore the solutions sought.

§4. DETERMINATION OF MOMENTS

The curve represented by equation (10), together with its asymptote and the quantity axis, bounds a finite area. We shall refer to this as "the area above the curve."

Let μ_n be the nth positive moment per unit Q_m about the quantity axis, of the μ_n area above the curve. It does not seem possible to find these moments by direct integration of equation (10), so we proceed as follows:

From equation (7)

$$\mu_n = \int_0^\infty \left(1 - \frac{Q}{Q_m}\right) t^n . dt = \frac{1}{2\pi i} \int_0^\infty \int_L e^{kzt/a^2} t^n \frac{f(z)}{z} . dz . dt,$$

where $f(z) = 1 - 1/(\gamma z + z^{\frac12} \coth z^{\frac12})$.

Hence $$\mu_n = \frac{a^2}{2\pi i k} \left[\int_L e^{kzt/a^2} \left(t^n - \frac{na^2}{k} \frac{t^{n-1}}{z} + \ldots \right) f(z) . \frac{dz}{z^2} \right]_{t=0}^{t=\infty} \qquad \ldots\ldots(11).$$

Lower limit. When $t = 0$ the expression in brackets in equation (11) vanishes. The integral is a multiple of $\int_L \frac{f(z)}{z^{n+2}} . dz$. Since $f(z)/z^{n+2}$ is regular on and within the contour LC (see Fig. 2) the path L is equivalent to the path C. Now $|f(z)/z^{n+1}| \to 0$ uniformly with respect to arg z for all points on C as the radius of $C \to \infty$: consequently the integral vanishes.

Upper limit. We consider the path L to be replaced by the path M as before, and note that when t is large the exponential term is only appreciable in the immediate vicinity of the point $z = 0$. We can therefore replace the path M by a closed contour of small dimensions surrounding the origin. Evaluating the residue of the integrand at $z = 0$ we find that when t is large the right-hand side of equation (11) is given by

$$a^2/k \left[(1-n)(\gamma + \tfrac13) t^n + na^2/k \{\tfrac{1}{45} + (\gamma + \tfrac13)^2\} t^{n-1} + \text{terms which vanish} \right.$$
$$\left. \text{when } n = 0 \text{ or } 1 \right].$$

Now plainly we must give n its numerical value before we proceed to the limit, so that

$$\begin{aligned} \mu_0 &= (a^2/k)(\gamma + \tfrac13) \\ \mu_1 &= (a^4/k^2) \left[\tfrac{1}{45} + (\gamma + \tfrac13)^2\right] \end{aligned} \qquad \ldots\ldots(12).$$

These two expressions are, of course, found to agree with those otherwise deduced for the special cases when $C \to \infty$ (simple diffusion) or $k \to \infty$ (simple exponential).

§5. SPECIAL CASES OF THE GENERAL EQUATION

Simple exponential. When the diffusivity is very great (theoretically infinite) the absorption is determined solely on the basis of condition (2b) from which we have

$$Q = Q_m \left(1 - e^{-ct/2a}\right) \qquad \ldots\ldots(13).$$

Table 1. The simple diffusion function.

Values of the function $\psi(v)$, or $\left[1 - (8/\pi^2)\left(e^{-v} + \tfrac{1}{9}e^{-9v} + \tfrac{1}{25}e^{-25v} + \ldots\right)\right]$.

v	0·00	0·01	0·02	0·03	0·04	0·05	0·06	0·07	0·08	0·09
0·00	0·0000	0·0718	0·1016	0·1244	0·1437	0·1606	0·1760	0·1901	0·2032	0·2155
0·10	0·2272	0·2382	0·2488	0·2590	0·2688	0·2782	0·2873	0·2962	0·3048	0·3131
0·20	0·3213	0·3292	0·3369	0·3445	0·3519	0·3592	0·3663	0·3733	0·3801	0·3868
0·30	0·3934	0·4000	0·4064	0·4127	0·4188	0·4249	0·4309	0·4368	0·4427	0·4485
0·40	0·4542	0·4598	0·4654	0·4708	0·4763	0·4816	0·4869	0·4921	0·4972	0·5024
0·50	0·5074	0·5123	0·5173	0·5221	0·5270	0·5318	0·5364	0·5411	0·5457	0·5503
0·60	0·5548	0·5592	0·5637	0·5681	0·5723	0·5766	0·5808	0·5850	0·5892	0·5932
0·70	0·5973	0·6014	0·6053	0·6093	0·6132	0·6170	0·6208	0·6246	0·6284	0·6321
0·80	0·6357	0·6393	0·6430	0·6466	0·6501	0·6536	0·6570	0·6604	0·6638	0·6671
0·90	0·6704	0·6737	0·6770	0·6801	0·6834	0·6866	0·6896	0·6827	0·6958	0·6988
1·00	0·7018	0·7048	0·7077	0·7106	0·7135	0·7164	0·7191	0·7220	0·7249	0·7275
1·10	0·7302	0·7328	0·7355	0·7382	0·7408	0·7434	0·7459	0·7484	0·7509	0·7534
1·20	0·7559	0·7583	0·7607	0·7631	0·7654	0·7678	0·7700	0·7724	0·7747	0·7769
1·30	0·7791	0·7813	0·7835	0·7856	0·7878	0·7899	0·7919	0·7940	0·7961	0·7981
1·40	0·8001	0·8021	0·8041	0·8060	0·8080	0·8098	0·8118	0·8136	0·8155	0·8173
1·50	0·8192	0·8209	0·8227	0·8245	0·8262	0·8280	0·8297	0·8314	0·8330	0·8347
1·60	0·8364	0·8380	0·8396	0·8412	0·8427	0·8444	0·8459	0·8475	0·8489	0·8504
1·70	0·8519	0·8534	0·8548	0·8563	0·8577	0·8591	0·8606	0·8620	0·8633	0·8646
1·80	0·8660	0·8673	0·8687	0·8700	0·8713	0·8726	0·8738	0·8751	0·8763	0·8775
1·90	0·8787	0·8800	0·8812	0·8824	0·8835	0·8847	0·8858	0·8869	0·8881	0·8892
2·00	0·8903	0·8914	0·8924	0·8936	0·8946	0·8957	0·8967	0·8977	0·8988	0·8997
2·10	0·9007	0·9018	0·9027	0·9037	0·9046	0·9056	0·9065	0·9074	0·9084	0·9093
2·20	0·9102	0·9111	0·9120	0·9129	0·9137	0·9146	0·9154	0·9163	0·9171	0·9179
2·30	0·9187	0·9195	0·9203	0·9211	0·9219	0·9227	0·9235	0·9242	0·9249	0·9258
2·40	0·9265	0·9272	0·9279	0·9287	0·9293	0·9300	0·9308	0·9314	0·9322	0·9328
2·50	0·9335	0·9341	0·9347	0·9354	0·9360	0·9367	0·9373	0·9380	0·9386	0·9392
2·60	0·9398	0·9404	0·9410	0·9416	0·9421	0·9427	0·9433	0·9438	0·9444	0·9450
2·70	0·9455	0·9461	0·9466	0·9472	0·9476	0·9482	0·9487	0·9492	0·9497	0·9502
2·80	0·9507	0·9512	0·9517	0·9522	0·9527	0·9531	0·9536	0·9540	0·9545	0·9549
2·90	0·9554	0·9558	0·9563	0·9567	0·9571	0·9576	0·9580	0·9584	0·9588	0·9592
3·00	0·9596	0·9600	0·9604	0·9608	0·9613	0·9616	0·9620	0·9624	0·9627	0·9631
3·10	0·9635	0·9639	0·9643	0·9646	0·9650	0·9653	0·9656	0·9660	0·9664	0·9667
3·20	0·9669	0·9673	0·9676	0·9679	0·9682	0·9685	0·9689	0·9692	0·9694	0·9698
3·30	0·9702	0·9705	0·9707	0·9711	0·9713	0·9716	0·9719	0·9722	0·9724	0·9728
3·40	0·9729	0·9732	0·9735	0·9737	0·9740	0·9742	0·9745	0·9748	0·9750	0·9753
3·50	0·9755	0·9758	0·9760	0·9763	0·9765	0·9767	0·9770	0·9772	0·9774	0·9776

Simple diffusion. This is the term applied to the special case wherein the surface of the sheet instantaneously becomes saturated at the start. This occurs when the surface constant is very great, in which case the roots of equation (8) are

$$\beta_n = (2n - 1)\pi/2$$

and equation (10) reduces to

$$Q = Q_m\left[1 - (8/\pi^2)\left(e^{-\pi^2 kt/4a^2} + \tfrac{1}{9}e^{-9\pi^2 kt/4a^2} + \ldots\right)\right],$$

or if

$$\psi(v) = \left[1 - (8/\pi^2)\left(e^{-v} + \tfrac{1}{9}e^{-9v} + \ldots\right)\right],$$

$$Q = Q_m\,\psi\left(\pi^2 kt/4a^2\right) \qquad \ldots\ldots(14).$$

If, in equation (7), we write $t = 4a^2/\pi^2 k$ and $\gamma = 0$, we find

$$\psi(v) = \frac{1}{2\pi i} \int_L e^{\frac{4}{\pi^2}vz} \frac{\tanh z^{\frac{1}{2}}}{z^{\frac{1}{2}}} \frac{dz}{z};$$

now writing $q^2 \equiv d/dv$ this latter integral can be re-interpreted as an operator by using the Bromwich rule inversely, so that we get q

$$\psi(v) = \tanh(\pi q/2)/(\pi q/2).$$

It is possible to show* that this last expression can be expanded provided we interpret $q^{2\alpha} = v^{-\alpha}/\Gamma(1-\alpha)$. When v is small q is therefore of order $(\pi v)^{-\frac{1}{2}}$, whence

$$\psi(v) = 2/\pi q \text{ when } v \text{ is small, or}$$
$$= 4v^{\frac{1}{2}}/\pi^{\frac{3}{2}}.$$

This latter is a very good approximation up to $v = 0.5$ which is roughly as far as $\psi(v) = 0.5$. The error when $v = 0.5$ is less than 3 in 5000, and for lower values of v is smaller still. Table 1 gives values of $\psi(v)$ from $v = 0$ to $v = 3.50$ at intervals of 0.01. Intermediate values in the lower part of the range should be determined by the above approximation, as direct interpolation is not very reliable.

§6. EQUATION CONSTANTS AND CURVE CRITERION

Returning to the equations (12), we find that

$$\text{Diffusivity } k = a^2/3 \, (5\mu_1 - 5\mu_0^2)^{\frac{1}{2}} \qquad \ldots\ldots(15),$$
$$\text{Surface constant} = 2k/a\gamma = 2a/[\mu_0 - (5\mu_1 - 5\mu_0^2)^{\frac{1}{2}}] \quad \ldots\ldots(16).$$

We now define the following curve criterion:

$$R = \mu_1/\mu_0^2,$$

so that $$1 \leqslant R \leqslant 1.2$$

is a necessary condition attaching to equation (10). $R = 1$ corresponds to the R special case of equation (13), and $R = 1.2$ corresponds to equation (14). Further, we note that

$$\gamma = \tfrac{1}{3}[1/(5R - 5)^{\frac{1}{2}} - 1],$$

so that the roots of equation (8) and the coefficients (b_n) of the exponential terms in the general equation are merely functions of R. The initial slope per unit Q_m is given by

$$\text{Initial slope} \div Q_m = 1/\mu_0 \, [1 - (5R - 5)^{\frac{1}{2}}] \geqslant 1/\mu_0.$$

Table 2 gives values of (γ), $(\mu_0 k/a^2)$, (b_1), $(\mu_0 k \beta_1^2/a^2)$ for a series of values of R, and these are usually sufficient to enable a large portion of the theoretical curve to be plotted and the constants to be determined when μ_0, μ_1, Q_m are known.

* W. E. Sumpner, *Proc. Phys. Soc.* **41**, 405–425 (1929).

§7. EXAMINATION OF EXPERIMENTAL DATA

We have now to consider how the foregoing theory can be utilized to study data, the general trend of which is not at variance with the form of the diffusion curves contemplated herein.

Table 2. Quantities for use in curve-fitting.

Criterion R	γ	$\mu_0 k'a^2$	b_1	$\mu_0 k \beta_1^2 a^2$
1·000	∞	∞	1·0000	1·0000
1·005	1·7749	2·1082	0·9948	0·9947
1·010	1·1574	1·4907	0·9891	0·9894
1·015	0·8838	1·2172	0·9833	0·9841
1·020	0·7208	1·0541	0·9774	0·9789
1·025	0·6095	0·9428	0·9716	0·9737
1·030	0·5273	0·8607	0·9658	0·9685
1·035	0·4635	0·7968	0·9600	0·9633
1·040	0·4120	0·7454	0·9543	0·9582
1·045	0·3694	0·7027	0·9487	0·9531
1·050	0·3333	0·6667	0·9431	0·9480
1·055	0·3023	0·6356	0·9376	0·9430
1·060	0·2752	0·6086	0·9322	0·9380
1·065	0·2514	0·5847	0·9268	0·9331
1·070	0·2301	0·5634	0·9215	0·9283
1·075	0·2110	0·5443	0·9162	0·9236
1·080	0·1937	0·5270	0·9111	0·9189
1·085	0·1780	0·5113	0·9060	0·9142
1·090	0·1636	0·4969	0·9010	0·9096
1·095	0·1503	0·4837	0·8960	0·9051
1·100	0·1381	0·4714	0·8913	0·9006
1·105	0·1267	0·4600	0·8866	0·8962
1·110	0·1161	0·4495	0·8819	0·8919
1·115	0·1062	0·4396	0·8774	0·8875
1·120	0·0970	0·4303	0·8730	0·8832
1·125	0·0883	0·4216	0·8685	0·8790
1·130	0·0801	0·4134	0·8641	0·8749
1·135	0·0724	0·4057	0·8597	0·8709
1·140	0·0651	0·3984	0·8556	0·8669
1·145	0·0581	0·3915	0·8515	0·8630
1·150	0·0516	0·3849	0·8475	0·8591
1·155	0·0453	0·3786	0·8435	0·8552
1·160	0·0393	0·3727	0·8395	0·8513
1·165	0·0337	0·3670	0·8356	0·8476
1·170	0·0282	0·3616	0·8319	0·8439
1·175	0·0230	0·3563	0·8282	0·8402
1·180	0·0180	0·3513	0·8245	0·8365
1·185	0·0133	0·3466	0·8210	0·8329
1·190	0·0086	0·3420	0·8175	0·8294
1·195	0·0042	0·3376	0·8140	0·8260
1·200	0·0000	0·3333	0·8106	0·8225

To make a test of fit it is necessary to find Q_m and the moments μ_0 and μ_1. There seems to be no entirely satisfactory way of determining a reliable estimate of Q_m otherwise than by experiments carried out under uniform conditions for a sufficiently long period. Further, if the readings are taken at equidistant intervals of time, the determination of μ_0 and μ_1 is much simplified, for the numerical integration can rapidly be effected by using Gregory's formula*, or any other similar formula†. If it is impracticable to take observations at equal intervals of

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* E. T. Whittaker and G. Robinson, *loc. cit. ante*, p. 143.

† A. Henry, *Calculus and Probability for Actuarial Students*, pp. 114–124 (1927).

time, there is nothing for it but to pass a smooth curve through the experimental points. It is not satisfactory, generally, to do this graphically and the only really good method is to employ Lagrange's formula* for all or a selected number of the observations. Having obtained μ_0 and μ_1 to the same degree of accuracy as the experimental data are recorded, we can evaluate the criterion R. Now we have seen in a previous paragraph that a necessary though not sufficient condition for a fitting by the general curve (10) is that $1 \leqslant R \leqslant 1.2$, so if the calculated R lies outside this range no fit is possible and one or more of the components of the general hypothesis, as given in the statement of the problem, must be invalid. On the other hand, a value of R within the above range implies the possibility of fit; we can only say "possibility" for conceivably there can be many curves yielding a plausible value for R yet differing considerably from equation (10). To complete the test of fit it is necessary to plot a theoretical curve and examine how much the experimental points diverge therefrom. The values of μ_0, μ_1, Q_m being known, the only plausible values of the diffusivity and surface constants are given by equations (15) and (16). Table 2 has been prepared to expedite the calculation of the numerical quantities in the first term of equation (10).

§8. ACKNOWLEDGMENT

* E. T. Whittaker and G. Robinson, *loc. cit. ante*, pp. 28–30.

INTENSITY MODIFICATIONS IN THE SPECTRUM OF MERCURY

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ABSTRACT. Intensity modifications are noted in the high-frequency spectrum of mercury in the region $\lambda 7000$ to $\lambda 2400$. All the singlet series are strengthened. In the triplets the first members of the series are weakened by self-absorption. Those intercombination lines which begin on singlet levels are strengthened, whilst those which begin on triplet levels are weakened. The unclassified lines show three types of behaviour, namely strengthening, weakening, or freedom from change. It is suggested that the strengthened unclassified lines begin on singlet levels, and the weakened on triplet levels, and that the unchanged lines are due to displaced triplet levels in the excited atom. The unclassified lines are thus broadly divided. The degree of intensity modification can be made to vary by alteration of the current density in the discharge tube. Mercury films are produced by the discharge driving mercury into the walls of the tube. These are described and suggestions made as to their nature.

The line $6\,^1P_1 - 8\,^1S_0$, namely $\lambda 4916$, is examined for fine structure with a resolving power of over 3×10^6. It is quite single and very narrow, the half-width being less than $0\cdot004$ A.U. The intensities of the satellites in $\lambda 5461$ are found to be altered by self-absorption.

§1. INTRODUCTION

INTENSITY modifications in the high-frequency discharge in mercury were first observed by Clarke*, who recorded a relative strengthening of a small group of lines. Ponte†, using one internal electrode and exciting the mercury vapour with oscillations corresponding to wave-lengths of from 5 to 10 metres, found that the two singlet series $6\,^1P_1 - m\,^1S_0$, $6\,^1P_1 - m\,^1D_2$ and a few intercombination lines increased in intensity. Frayne and Montgomery‡, using an electrodeless discharge (3 metres), passing through a solenoid wound round a pyrex tube, extended these observations. Observing the region $\lambda 7000$ to $\lambda 3000$ they found an increase in strength, relative to $\lambda 5461$, in all the singlet series in this region, namely $6\,^1P_1 - m\,^1S_0$, $6\,^1P_1 - m\,^1D_2$ and $7\,^1S_0 - m\,^1P_1$, and also in a group of intercombination lines and in three unclassified lines. Intensities were measured with a photometer and compared with those in the arc spectrum.

In §§ 2 to 4 of the present paper, observations are extended from $\lambda 7000$ to $\lambda 2400$. The observations of Frayne and Montgomery on the singlet series are

* J. R. Clarke, *Nature*, **120**, 726 (1927).
† M. Ponte, *Nature*, **121**, 243 (1928).
‡ J. G. Frayne and C. G. Montgomery, *Phys. Rev.* **33**, 549 (1929).

confirmed and in addition many more intercombination and unclassified lines are observed. Some are strengthened and some weakened in the high frequency discharge, whilst some unclassified lines remain unaltered. Self-absorption is found to affect the first members of triplet series, reducing their intensities. Amongst the lines so affected is $\lambda 5461$, so that the next member of this series, $\lambda 3341$, had to be used as the comparison line for intensities. In § 5 the lines $\lambda 4916$ and $\lambda 5461$ are examined with high resolving power for fine structure.

§ 2. EXPERIMENTAL PROCEDURE

Mercury was distilled into small tubes of quartz and pyrex having central capillary portions. They were about 10 cm. long with a 2 cm. bore, the capillary being 1 cm. long with a bore of about 0·25 cm. A good vacuum was produced by means of a mercury vapour pump and charcoal in liquid air. Round the wider parts were wrapped strips of tin-foil, which were coupled to a 7-metre oscillator. The circuit used (due to Gill and Donaldson*) is shown in Fig. 1. V is a Mullard
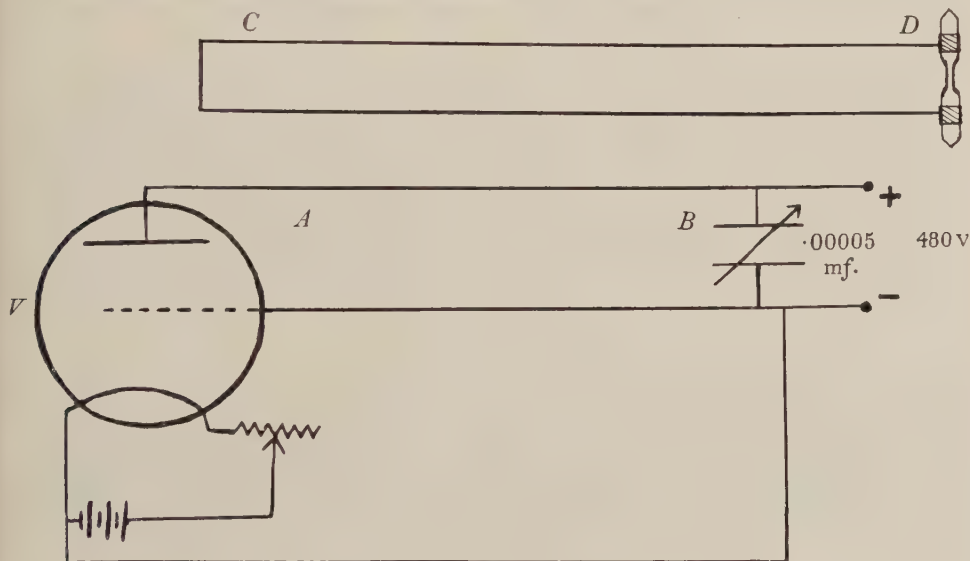
Fig. 1. Circuit for producing high frequency oscillations.

D.O. 40 valve, six volts being required for the filament. A simple Lecher wire system AB, about a metre long, with a tuning condenser is employed. A pair of wires CD of variable length is coupled inductively to AB, the ends being connected to the tin-foil on the tube. The discharge is brightest for a given length of CD. The plate voltage used was 480, the total input being about 40 watts. Full constructional details will be published at an early date by Mr S. F. Evans.

Observations were carried out on the capillary portion. The spectrum was photographed in the region $\lambda 7000$ to $\lambda 2400$ on both a Hilger small quartz and a

* E. W. B. Gill and R. H. Donaldson, *Phil. Mag.* 2, 129 (1926).

large quartz E.I. spectrograph with Ilford Special Rapid Panchromatic and Ilford Monarch plates. The comparison spectrum was that emitted by a Kromayer water-cooled mercury arc taking 3 amps. Different exposure times, varying from one second to half an hour, were given. The high-frequency and arc spectra, for which $\lambda 3341$ was of about the same intensity, were matched for intensity comparisons.

§ 3. RESULTS

The colour of the discharge is very different from that of the arc, being an intense blue. On comparison of the high-frequency and arc spectra it is found that the singlet series $6\,^1P_1 - m\,^1S_0$, $6\,^1P_1 - m\,^1D_2$ and $7\,^1S_0 - m\,^1P_1$ are strengthened, see Plate I (c). In the triplet series, the first members only are affected, being reduced in intensity. Thus the first members of $6\,^3P_{012} - m\,^3S_1$, namely $\lambda\lambda 4046$, 4358, 5461, are considerably weakened with respect to the other members of the series. The diminution in $\lambda 5461$ is very marked, see Plate I (b). These intensity reductions were not recorded by Frayne and Montgomery owing to the fact that they took $\lambda 5461$ as the comparison line. The result was that they found an apparent strengthening of the triplet members of higher order number. This apparent effect almost entirely disappears if $\lambda 3341$ is taken as standard, and it is found that the intensities of all the other members of the triplet series are about the same in the arc and high-frequency discharges.

A reduction in intensity of the first member of the series $6\,^3P_0 - m\,^3D_1$ also is observed, but the effect is smaller than in $6\,^3P_{012} - m\,^3S_1$.

About twenty intercombination lines extending from $\lambda 6070$ to $\lambda 2537$ were observed. These lines form two distinct classes, namely those that are strengthened and those that are weakened in the high-frequency discharge. The following intercombination series (each observed to the third member) are strengthened:

$$6\,^3P_1 - m\,^1D_2,\ 6\,^3P_2 - m\,^1D_2,\ 6\,^3P_1 - m\,^1S_0,\ 7\,^3S_1 - m\,^1P_1,$$

whilst the following are weakened:

$$6\,^1P_1 - m\,^3D_2,\ 6\,^1P_1 - m\,^3D_1 \text{ and } 6\,^1P_1 - m\,^3S_0.$$

The line $\lambda 2537$ ($6\,^1S_0 - 6\,^3P_2$) is very much stronger in the high-frequency spectrum than in the arc. Fig. 2 is a term scheme showing the transitions for the first members of the intercombination series. The lines which strengthen are marked in full, and those which weaken are dotted. The degree of strengthening is of the same order as that of the singlet series.

There is a very marked difference in behaviour amongst the unclassified lines, and this should afford some clue to their classification. Some appear not to be affected, or at most to be affected very slightly. A number are strengthened, and a number are considerably reduced in intensity, the lines in fact disappearing as far as can be observed from the high-frequency spectrum. The strengthened lines lie mostly in the ultra-violet region. Amongst these are

$$\lambda\lambda 2576,\ 2602,\ 2625,\ 2638,\ 2675.$$

In addition the red line $\lambda 6123$ is very strong.

There is a particularly striking group in the region $\lambda 3390$ to $\lambda 3984$ which seems to disappear entirely from the high-frequency spectrum. Its members are all of
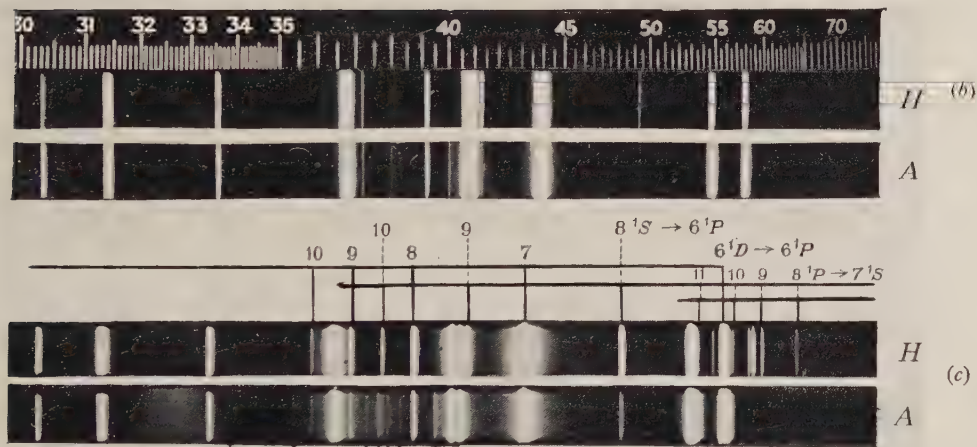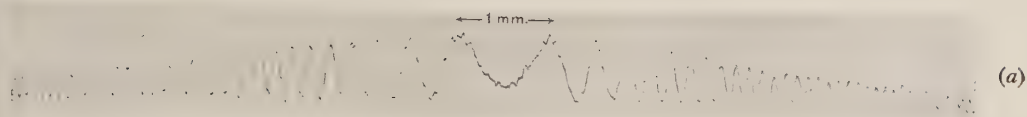
(a) Photometer trace of Fabry-Perot fringes of $\lambda 4916$, plate separation 10 cm.; (b) and (c) high-frequency spectrum of mercury (H) and arc spectrum of mercury (A).

similar intensity and may be related. They are shown in Plate I (c) and are marked with a dash. The members of this group are $\lambda\lambda 3390, 3561, 3752, 3790, 3824, 3984$.

The degree of enhancement could be made to vary considerably by alteration of the current density through the discharge tubes. This was achieved by loosening of the coupling and reduction of the filament current in the oscillator. With tight coupling and normal current the discharge was extremely bright. The capillary became warm although the bulbs remained cold. The spectrum showed all the characteristics described. On reduction of the current the total intensity fell off markedly, but the relative strengthening became greater. The reduction in intensity of $6\,^3P_{012} - 7\,^3S_1$ still appeared. The spectra reproduced in Plate I were taken with full current through the tubes.

The discharge was found to affect the mercury in the tube, driving it very forcibly into the walls. After some twenty hours' running the whole glass, excepting the parts under the electrodes, was covered with a practically opaque film of mercury which showed interference colours, violet predominating. The colours only appeared by reflected light, the film being a dirty brown when seen by transmitted light. The coloration of the film may not be the simple interference effect due to thin films, since its behaviour was different. After an hour's running there appeared a thin film which gave a brilliant violet colour by reflected daylight. This colour was independent of the angle of incidence. On further running the film thickened, becoming more opaque. The reflected colour still remained the same. On still further running new colours appeared, but the original violet was still strong. The phenomenon bore a striking resemblance to the optical resonance of thin films of sodium when viewed by transmitted light. The violet colour may therefore be due to optical resonance of mercury, the later colours being true thin-film interference colours arising as the film thickened. The extent to which the film was driven into the walls is shown by the fact that it was not removed by subsequent heating, even at a dull red heat.

§4. DISCUSSION OF RESULTS

There are three types of lines: strengthened, weakened, and unchanged (or not appreciably affected). The strengthened lines will be considered first. These consist of (a) the singlet series; (b) those intercombination lines which begin in singlet levels, see Fig. 2; and (c) certain of the unclassified lines. The line $\lambda 2537$ is the only line beginning on a triplet level which is strengthened. This exception is explained later. The factor common to the first two groups (a) and (b) is that they begin in singlet levels. It is thus reasonable to expect that the unclassified lines which are strengthened also begin on singlet levels.

The weakened lines are (a) intercombination lines beginning in triplet levels, see Fig. 2; (b) certain of the unclassified lines; (c) the first members of the $^3P - ^3S$ and $^3P - ^3D$ series. Those unclassified lines which are weakened may possibly originate on triplet levels. The weakening of the first members of the triplet series is explained by the results obtained by Ponte*, who passed the light from a Cooper-

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* M. Ponte, *Comptes Rendus*, **187**, 37 (1928).

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Hewitt mercury arc through a tube containing mercury at 100° C., excited by a high-frequency discharge. He found that the singlet series were hardly affected but that members of the series $6\,^3P_{012} - m\,^3S_1$ and $6\,^3P_{012} - m\,^3D_{123}$ were absorbed. The effect was greater for the $^3P - \,^3S$ series, and for each series the first member was much more strongly absorbed than members of higher order number.
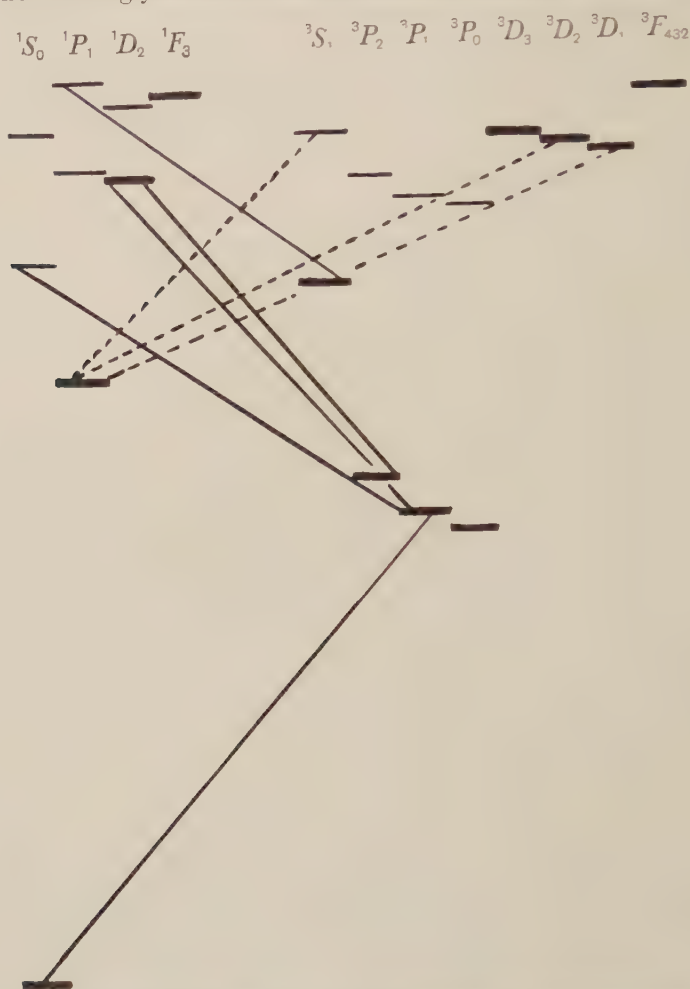
Fig. 2. Term scheme for mercury showing behaviour of intercombination lines.
Strengthened ————— Weakened – – – – –

The following results are due to him, where R is the ratio of intensity when the light passes through unexcited vapour to that when it passes through excited vapour:

	$6\,^3P_0 - 7\,^3S_1$	$6\,^3P_1 - 7\,^3S_1$	$6\,^3P_2 - 7\,^3S_1$	$6\,^3P_0 - 7\,^3D_1$
λ	4046	4358	5461	2967
R	4·4	3·0	4·1	3·2

These are the only lines between $\lambda 5461$ and $\lambda 2650$ for which he gives an absorption ratio greater than 3·0, and they are the only triplet lines found to be weakened in emission. This reduction in emission must, then, be due to self-absorption in the excited vapour, and indicates the existence of a large number of excited atoms especially in the states $6\,^3P_{012}$. The states $6\,^3P_0$ and $6\,^3P_2$ are apparently favoured since they have bigger ratios, namely 4·4 and 4·1. They are metastable states and their greater probability is to be expected since the pressure is low (about 10^{-3} mm.).

The unaffected lines are (a) triplet series members of higher order number, and (b) certain unclassified lines. The triplet lines are not entirely unaffected. There is a relative increase in intensity in the direction of the series limit, but the effect is very slight. If the alterations due to absorption are neglected, then it appears that the triplet series are not affected very much. It is possible that the unchanged unclassified lines are triplet lines, due to displaced terms in the excited atom.

These intensity changes broadly divide the unclassified lines into three types: (a) strengthened, probably beginning on singlet levels; (b) weakened, probably beginning on triplet levels; and (c) unchanged, probably due to displaced triplet terms.

The great increase in the strength of $\lambda 2537$ probably arises from the fact that this is the resonance line. The pressure being low in the high-frequency tube and high in the arc, it follows that the line will be absorbed in the arc (with re-radiation) to a much greater extent, since there are more absorbing atoms. This results in an apparent increase in the intensity of the line in the high-frequency discharge, which explains how this line, although beginning on a triplet level, appears to be strengthened. In reality it is abnormally weakened in the arc.

Since the intensity modifications are greater with lower current density, they are probably due to diminished vapour pressure, because the temperature with smaller currents was decidedly lower, approximating to the room-temperature.

If it is assumed that singlet states are more probable at low pressures, then all the strengthened lines and unchanged lines which occur are explained by this assumption. The existence of the lines which are greatly weakened is not, however, explained by it.

§ 5. EXAMINATION OF $\lambda\lambda 4916$ AND 5461 FOR FINE STRUCTURE

Since the singlet system, especially $6\,^1P_1 - m\,^1S_0$, is so much strengthened, the second member, $\lambda 4916$, of this series was found sufficiently intense to be examined for fine structure with a Fabry-Perot interferometer. The interferometer, by Hilger, has quartz plates with a 6 cm. aperture and variable plate-separation. It was crossed with a Hilger large quartz E.I. spectrograph and fringes were photographed on Ilford special rapid panchromatic plates, an exposure of 10 minutes being necessary for the capillary portion of the quartz tube. This time could have been halved with Ilford soft-gradation panchromatic plates, which are about twice as fast as special rapid plates in this region.

The line was examined with various plate-separations from 0·5 cm. to 10 cm. The resolving power for a 10 cm. gap, for this region, is well over 3×10^6. The line proved to be quite single and extremely narrow. The fringes were so fine that it was estimated that rings would still be seen with a gap of from 40 to 50 cm. They were examined with a self-registering microphotometer, the record being shown in Plate I (*a*). The diameter of the first ring on the actual plate is about 1 mm. The sixteenth ring is distorted owing to an accidental scratch on the plate. The half-width of the fringes, being about one-third of the interval between adjacent orders, is approximately 0·004 A.U. This includes the instrumental widening produced by the interferometer. Since the Doppler half-width for a temperature of 30° C. is 0·002 A.U., there does not appear to be much broadening due to other causes than temperature. This is of interest, as it indicates that no appreciable Stark effect due to the high-frequency discharge exists. It also shows that there is very little tendency to self-reversal, which would broaden the line, and thus confirms Ponte's observation that singlet members are not appreciably absorbed.

The half-width of λ4916 has been estimated by Carrelli*, using the method of limiting visibility. The line was produced by a Hercuss arc taking 3 amp. and the half-width was given as 0·015 A.U. The line is thus very much narrower in the high-frequency discharge than in the arc.

Since the line is well isolated, the nearest bright lines being 500 A.U. away, and as it is bright enough in the high-frequency discharge to be seen with ease after passing through the Fabry-Perot interferometer, it is very suitable for the purpose of adjustment, and, being very easy to produce, it could be used with advantage in interferometry. It is proposed to make an accurate determination shortly of the wave-length and half-width. Fine structure has been observed in the $^1P - {}^1S$ series and this is now being investigated.

The fine structure of the green line λ5461, in emission, was also observed with the pyrex tube and Fabry-Perot interferometer. Gaps of 10 mm. and 5·9 mm. were used, and the line was compared with that from a water-cooled Kromayer mercury arc running on 0·7 amp., which gives quite fine fringes. There was an alteration in the intensities of the satellites, which became more nearly equal. Ponte† has observed the fine structure in absorption by passing the light from a Cooper-Hewitt mercury arc through mercury vapour at 100 C. excited by a high-frequency discharge. He found that the stronger components were relatively more absorbed. The intensity changes in emission must then be due to selective self-absorption.

§6. ACKNOWLEDGMENTS

* A. Carrelli, *Accad. Lincei Atti*, **7**, 1014 (1928).
† M. Ponte, *Comptes Rendus*, **187**, 37 (1928).

A VACUUM-TUBE COMMUTATOR: THE PRODUCTION OF A PERIODIC PULSE OF POTENTIAL OF SQUARE WAVE-FORM

By C. F. POWELL, M.A., Ph.D. and K. H. MANNING,

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ABSTRACT. A type of vacuum tube for producing a periodic pulse of potential of square wave-form is described. It is possible to obtain with it wave-forms in which the ratio of the duration of the pulse t, to the time between pulses T, is of the order of one to twenty, at a pulse frequency of a thousand.

IN the development of the 4-gauze method of measuring the mobility of ions in gases*, we have found it necessary to produce a fluctuating potential of which the wave-form approximates to that shown in Fig. 1. Since it is difficult to build commutators which will give an accurately defined wave-form at high frequencies and behave consistently over long intervals of time, we have developed a vacuum-tube method which has worked very satisfactorily. The method has other advantages besides its consistency and ability to operate at high frequencies, and we are giving this account of it since it seems probable that it may be useful in other fields of research.
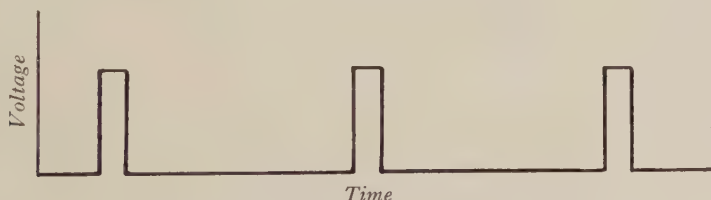
Fig. 1. Required wave-form of potential pulse.

A contact-breaker depending on the use of cathode rays has been described by Lübke† and has been used by him to study the wave-form of alternating currents by Joubert's method. The principle of our method is somewhat similar to his, and our tube is shown diagrammatically in Fig. 2. Electrons from the heavy tungsten filament (emission 100 milliampères) are accelerated to the grid and anode. A beam of electrons passes through the rectangular aperture in the anode

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* Tyndall, Starr and Powell, *Proc. Roy. Soc.* **121**, p. 172 (1928). Tyndall and Powell, *Proc. Phys. Soc.* (1929).

† Lübke, *The Electrician* (Sept. 1919, p. 270).

and between two deflecting plates. If an alternating potential is applied between these plates, the beam waves backwards and forwards across the face of the metal screen shown on the right of Fig. 2. The screen is provided with an aperture similar to that in the anode and behind it is a Faraday collector. As a result, electrons enter the collector twice for every cycle of an alternating potential applied across the deflecting plates. The static curve for the instrument is shown in Fig. 3 in which the current to the collector is plotted against the potential difference between these plates. Curves are given for two values of a voltage bias which tends to maintain the collector at a fixed potential relative to the screen. The object
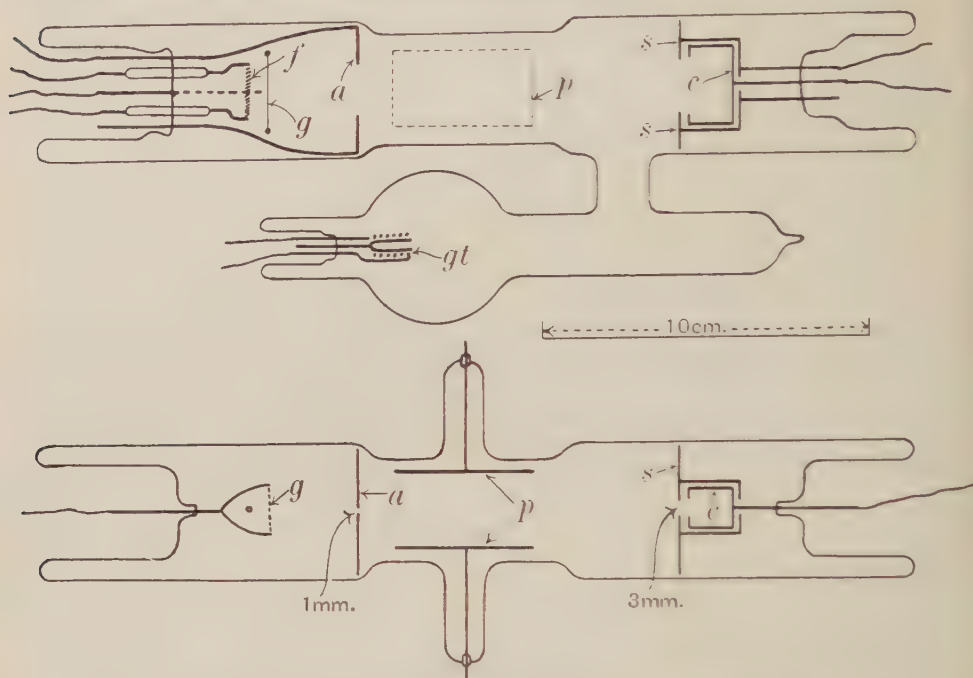
Fig. 2. Construction of tube: *f* main filament, *g* grid, *a* anode, *p* deflecting plates, *s* screen, *c* collector, *gt* getter.

of this bias is to prevent the loss of secondary electrons from the collector. It will be seen that the peak is sharp and that no serious broadening occurs as a result of the diffusion of the beam. The collector current is a maximum for a deflecting potential not of zero but of twelve volts because in our tube the filament and the two apertures are slightly out of line. The asymmetry of the curves is due to the fact that the deflecting plates are not symmetrically placed with respect to the electron beam, so that the beam when deflected in one direction strikes one of the deflecting plates and gives rise to scattered electrons, some of which enter the collector. As will be seen later these faults in construction do not lead to any difficulty in the production of the required wave-form over a wide range of frequencies.

The collector is connected to the grid of a triode and, through a high resistance, to the screen. Variations in the current to the collector cause the grid to fluctuate in potential so that the current through a resistance in the plate circuit of the valve changes. The potential fluctuations across this anode resistance are an amplified copy of those of the grid provided the grid is suitably biassed with respect to the filament.

It is necessary to ensure that the time constant, CR, associated with the capacity, C, of the collector-grid system and the grid leak R shall be small compared with the duration of the current pulse. Suppose that the frequency of the alternating $\qquad$ *C, R*
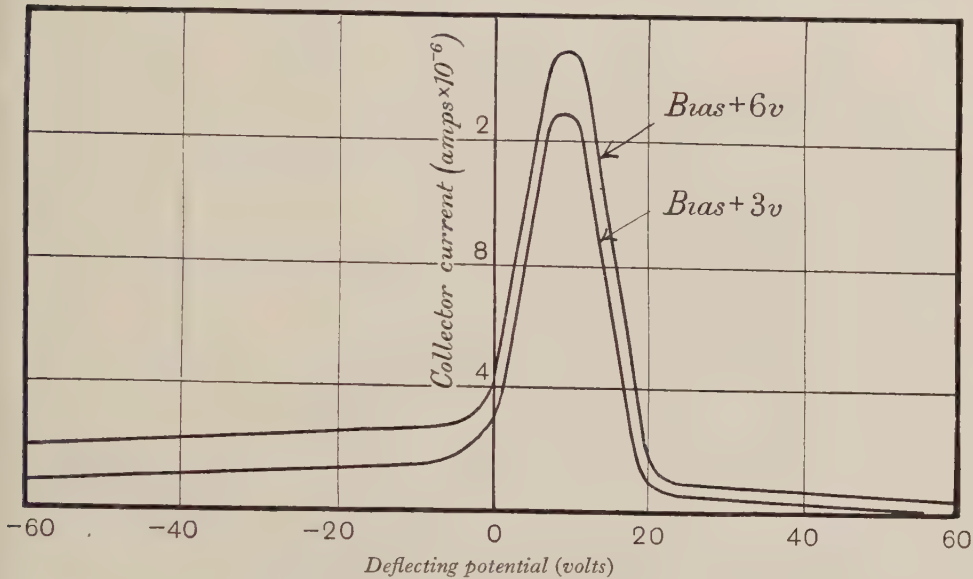
Fig. 3. Static characteristic of tube.

potential applied to the deflecting plates is f and the periodic time T, so that $T = 1/f$. From the calibration curve, Fig. 3, it will be seen that the current to the collector may be represented approximately by a function I of the time t where $\qquad$ *f, T* $\qquad$ *I, t*

$$I = I_0 \sin 2\pi p t \text{ from } t = 0 \text{ to } t = 1/2p,$$

I₀ placed: I_0

the relation between p and f being determined by the peak value of the alternating potential applied across the deflecting plates. When this peak value is large the electron beam suffers large deflections, the time spent in crossing the aperture in the screen is proportionately small, and the ratio p/f is large. If, for example, it is desired to make the duration t of the pulse equal to one-tenth the time $T/2$ between pulses, then p will be equal to $20f$.

The potential, V, of the grid is then given by the equation $\qquad$ *V*

$$I_0 \sin 2\pi p t = C \, dV/dt + V/R.$$

n

The solution of this equation, where $n \equiv 2\pi p$, is

$$V = \frac{I_0}{C\,(n^2 + 1/C^2 R^2)} \cdot \left(n\epsilon^{-t/CR} + \frac{1}{CR} \cdot \sin nt - n\cos nt \right) \quad \ldots\ldots(1),$$

for the interval from $t = 0$ to $t = 1/2p$.

V'   If $V = V'$ when $t = 1/2p$, then for the interval from $t = 1/2p$ to $t = 1/2f$,

$$V = V'\epsilon^{-t/CR} \quad \ldots\ldots(2).$$

Values of V, deduced from (1) and (2), have been plotted against the time in Fig. 4. For this purpose the capacity C has been assumed to be equal to 50 e.s.u., and the resistance R to be 10^5 ohms. The curves have been plotted for three values of the
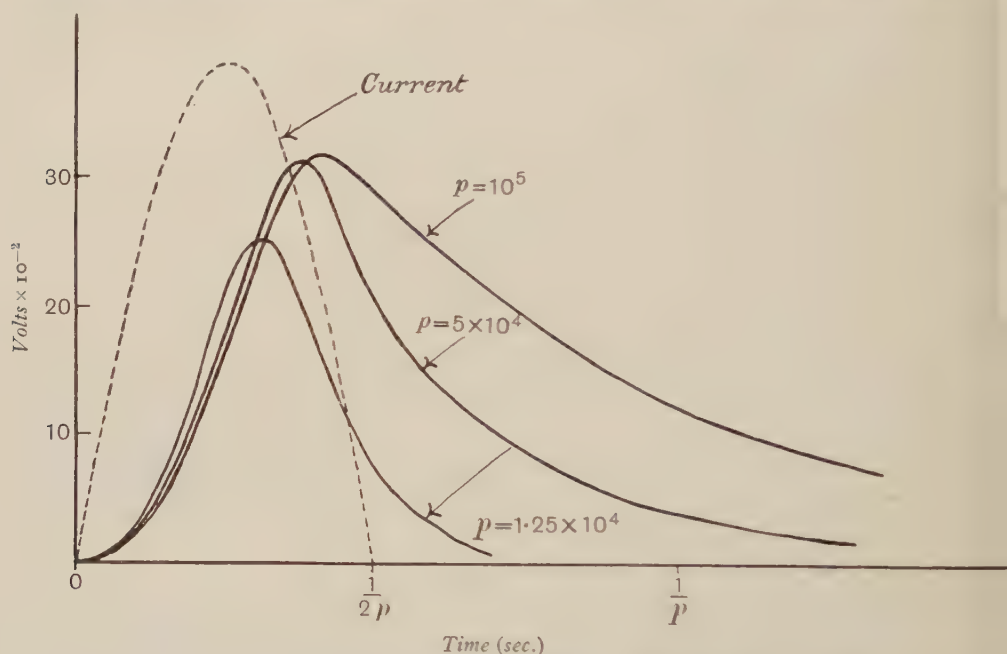
Fig. 4. Curves calculated for various frequencies. Peak current 10^{-6} amp.
$C = 50$ e.s.u. $R = 10^5$ ohms.

quantity p. The time scale for each has been made inversely proportional to the corresponding value of p. As a result the form of the current pulse giving rise to the potential fluctuations is always represented by the same curve whatever the value of p. This curve is shown by the dotted line in Fig. 4. At very low frequencies the changes in the potential of the collector will be very small but they will follow the fluctuations in the current closely. At higher frequencies the potential will tend to lag behind the current. The extent of this lag can be seen in the figure. It is desirable to ensure that the "width" of the potential pulse shall remain the same as the frequency is varied, other quantities remaining constant; that is to say, that the duration of the pulse shall be equal to a given fraction of the periodic

time of the alternations applied to the deflecting plates. It is evident that this condition can be very nearly fulfilled over the range of frequencies provided by our oscillator, namely from 60 to 6000 ∼, for which the maximum value of p is approximately 10^5, if the time constant CR is sufficiently small.

In order to test the conclusions of the last paragraph, the amplified pulse was examined by means of a cathode ray oscillograph possessing two pairs of deflecting plates at right angles. Across one pair of plates was applied a simple harmonic potential in phase with that applied to the deflecting plates of the pulse-producing tube; across the other pair, the potential pulse. This arrangement gives a time base inversely proportional to the frequency somewhat similar to that used in plotting the curves in Fig. 4.
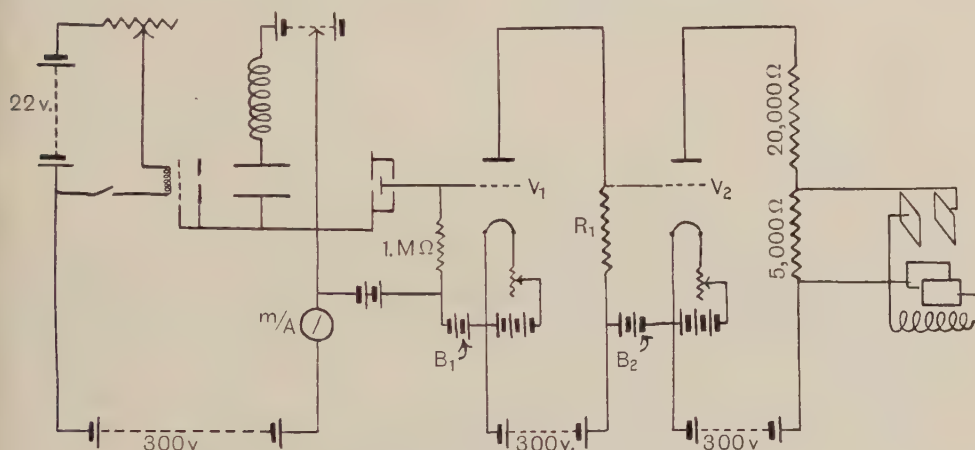
V_1—Marconi L.S. 5*b*. V_2—Marconi L.S. 5.

Fig. 5. Diagram of complete circuit.

The resulting patterns were exactly of the type to be expected from the analysis, but the spread of the curves at different frequencies indicated that the capacity of the grid-collector system had been over-estimated. The time-constant associated with this system was made as small as possible by removal of the ebonite base of the valve. The width of the pulse was then sensibly constant over the whole range of frequencies provided by the oscillator.

For the production of a potential pulse of the required amplitude a second stage of amplification was introduced and the complete circuit employed in this case is shown in Fig. 5.

The potential changes applied to the grid of the first valve are of the form shown in Fig. 6*a*. When the grid potential reaches a certain negative value the anode current ceases. By the choice of a suitable bias B_1 the potential changes across the anode resistance R_1 can, therefore, be made to take the form shown in Fig. 6*b*. This squaring off of the wave-top may be followed very clearly on the oscillograph as the bias B_1 is varied. In a similar way at the second stage
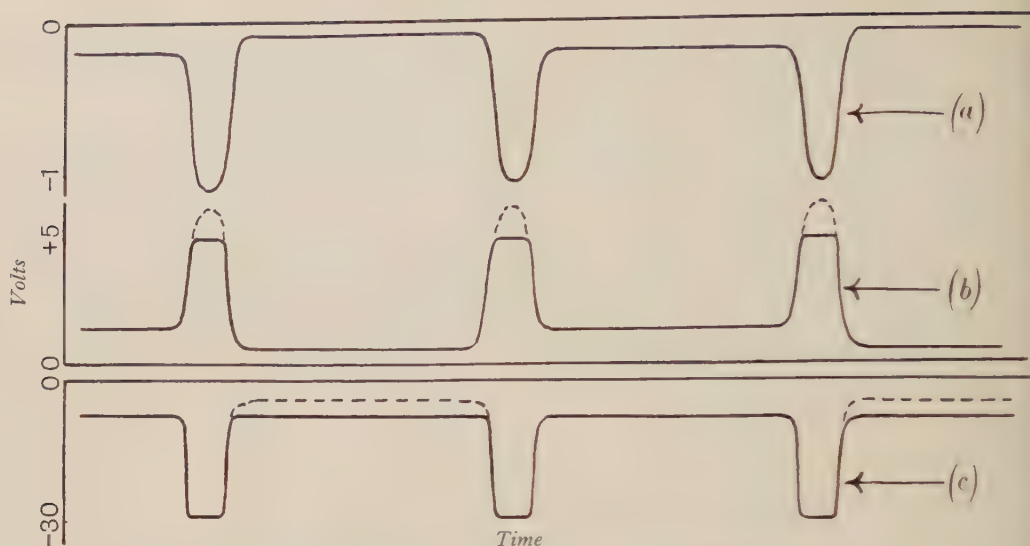
Fig. 6. Voltage fluctuations (a) of grid, (b) across first anode resistance, and (c) across second anode resistance.

of amplification the uneven effect due to the asymmetry in the characteristic may be removed and the final form of the pulses is seen in Fig. 6 c.

Two photographs of the pulses as seen in the oscillograph are shown in Fig. 7. The wave-forms to which they correspond are plotted in Fig. 8 at (a) and (b).

Fig. 7.

They were taken at a frequency of 400 ∼ with two different peak values of the deflecting alternating potential, and they illustrate the very simple way in which the wave-form may be controlled.

In addition, by deflection of the electron beam by a magnet, or electrostatically by the biassing of one of the deflecting plates relative to the other, wave-forms of the type shown at (c), Fig. 8, may be produced.
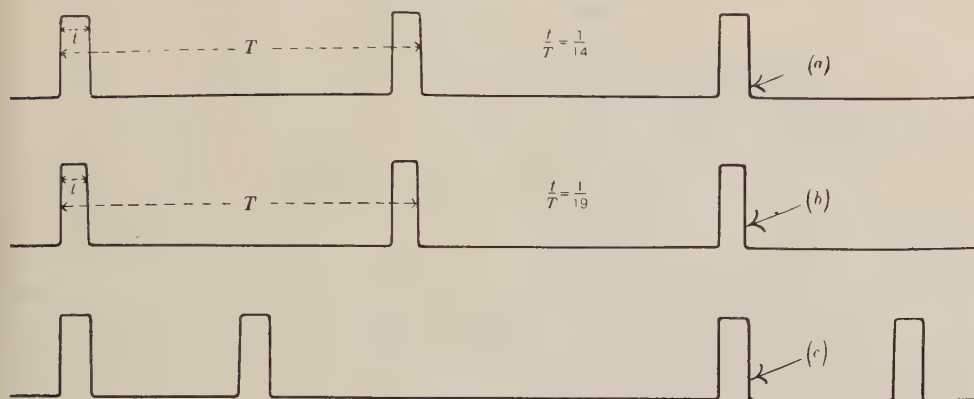
Fig. 8. Types of wave-form at 400 ~.

Up to the present we have not succeeded in getting satisfactory wave-forms at frequencies much above 2000 ~. In the present tube the electron-beam is not electrostatically shielded and the electrode system is not symmetrical with respect to the beam. At high frequencies the asymmetry in the charges on the walls of the tube results in the pulse due to the beam as it swings in one direction being different from that on its return. The pictures we have obtained leave no doubt, however, that the method can be employed at considerably higher frequencies, and we are now constructing a tube with electrostatic shielding which will avoid this difficulty.

We have great pleasure in acknowledging our indebtedness to Prof. A. M. Tyndall for his interest during the course of this research.

THE THERMAL INSULATING PROPERTIES
OF FABRICS

By M. C. MARSH, M.A., B.Sc., A.Inst.P.

ABSTRACT. One of the chief properties of a fabric is its thermal insulation, which prevents excessive heat-loss from the body. The paper gives a critical review of methods used in the past for measuring the thermal insulating properties. These are discussed with a view to the making of a new apparatus for the study of the subject. This apparatus consists of a copper cylinder with guard-ring ends heated electrically to skin temperature. With various fabrics the expenditure of electrical energy necessary to keep it at that temperature is measured. The surrounding conditions are defined by its being enclosed in a larger concentric tube whose temperature is regulated by the circulation of water round it. The humidity of the air can also be regulated. The temperature measurements are all by thermo-couple. The weight per unit area and the thickness of the fabrics are measured. There seems to be no definite relation between thermal insulating value and weight, but there is a relation which is very approximately linear between thermal insulating value and thickness. A detailed analysis of the results is being made.

§ 1. INTRODUCTION

THE value of a material as a thermal insulator is usually judged by its conductivity as measured by well-known methods. There are, however, materials to which such methods cannot be applied to any purpose. Of these, fabrics are a good example for, owing to their compressibility, it is impossible to measure their conductivity in the usual way without causing deformation. It would be possible to put forward a scheme for effecting this measurement without compression but great difficulties are introduced by the lack of homogeneity of the materials and the absence of definite surfaces.

With a discontinuous substance such as a fabric the transmission of heat by convection and radiation as well as conduction must be considered. Further, unless the fabric is in thermal and hygroscopic equilibrium with the surroundings, changes in the moisture content of various parts may cause heat to be transferred. When a fibre takes up water, heat is produced (heat of wetting[1]) in addition to latent heat effects. In this paper, however, only equilibrium conditions are studied, so that conduction, convection, and radiation are the only effects to be considered.

It seemed, therefore, advisable to determine the total heat loss of a body with and without a wrapping of fabric and to divide the work into two parts, viz.: (*a*) A study of the effect of all possible variations of conditions and circumstances on the total heat loss from the body. (*b*) Based on these results, an attempt to define

rigid and repeatable conditions and under these to compare different fabrics approximating as nearly as possible to those obtaining in practical use. Measurements of the physical properties of fabrics would also need to be made.

§ 2. PREVIOUS WORK

A large amount of work of very varied character has been done on this subject, mainly by those interested in the technological aspect, with the result that the physical properties of the fabrics have often been neglected.

A very popular and apparently simple method of comparing fabrics has been to determine cooling curves for bodies wrapped in them[2]. Its simplicity has often led to the neglect of essential precautions such as the stirring of liquid inside the heated body and the defining of surrounding conditions. Some have even worked with damp fabrics and totally neglected the latent heat of water evaporated. The only work approximating to precision involves the use of the Hill katathermometer[3] but the results obtained with this instrument are still the subject of much discussion. The whole method bristles with difficulties. If the temperature range is small, the lag of the thermometer, temperature gradient between various parts, etc. may cause large errors. If the range is large, the change in hygroscopic capacity of the fabric has to be considered, since water will be absorbed as the fabric cools. On the constructional side difficulties again appear. With this method guard-rings cannot be used, so it is necessary to fit the fabric uniformly over the whole surface. A woven fabric cannot be fitted over a sphere. A cylinder is better, but there is difficulty in introducing a stirrer, thermometer, etc.

A very great improvement on the cooling-curve method is one in which a body is kept at a constant temperature by electric heating. With this method the main measurements become electrical; a guard-ring system may be used so that the surface from which the heat is escaping may be very simple in form and the fabric may be left a sufficient time to get into hygroscopic equilibrium with the surrounding atmosphere. Barker and Tunstall[4] have shown recently that the time for an average fabric to reach the equilibrium state is much longer than had been previously supposed.

Haven[5] used a cylinder 60 in. long and $4\frac{1}{2}$ in. in diameter filled with water, with electric heating controlled by thermostat and relay. Instead of guard-rings he had insulating caps, which are less definite thermally and mechanically, on the ends of the cylinder. The thermostat and thermometer were inserted into the cylinder radially, and this involved cutting holes in the specimen. The heat capacity of the tube and water was very great so that three thicknesses of blanket had to be used. For thinner materials the apparatus would therefore have been useless. At the Bureau of Standards[6, 7, 8, 9] experiments were carried out with a square plate let into a board as the hot body. This had the great advantage that the fabric could be laid on without cutting or stitching, but the guard-ring system was very complicated, consisting as it did of four separate sides of the square and another similar heating system on the other side of the board, making nine guard-ring

heaters to be watched and controlled. The temperature measurement was made with a complete set of thermo-couples. The outside conditions were rather vague as the whole apparatus was in an ordinary-sized room and no precautions were taken to avoid draughts. The plane surface makes it very difficult to vary the contact pressure between the fabric and the hot surface. Floyd and Baker [10] have used an oil-filled cylinder and Muller [11] a sphere. The surroundings were carefully defined by outer jackets but with both methods difficulty was experienced in covering the whole body uniformly.

Much good work has been done on measurements of thermal conductivity of fabrics. As has been already pointed out, there is considerable doubt whether thermal conductivity is a criterion for the comparison of discontinuous substances like these, and further, the fabrics are compressed during the experiment, the pressure often being unmeasured. The results are often stated as the thermal conductivity of the fibre (wool, cotton, etc.), whereas what is measured is an average "mass" conductivity of compressed fabric.

In most of this work, a few fabrics are mentioned incidentally among other insulators with the result that full specifications of the material are not given. Under this heading comes the work of Lees [12, 13], Lees and Chorlton [14], Randolph [15] and Griffiths and Kaye [16]. Of those who worked specifically on fabrics the following may be mentioned. Bauer [17] has used an empirical method of little value. Lees' disc method with corrections for change of emissivity with temperature has been employed by Rood [18]. Staff [19], with an apparatus similar in essentials to that of Griffiths and Kaye, studied the change of conductivity with moisture content. Speakman and Chamberlain [20] have recently used a guard-ringed Bunsen ice calorimeter method. The thesis [21] on which this paper is based gives a complete summary of results of all conductivity work on fabrics and loose fibres. Loose fibres strictly do not come into this subject, but it may be noted that Nusselt [22] and Lamb and Wilson [23], using concentric spheres and coaxial cylinders respectively, measured the conductivities of several types of fibres at various densities.

Of other methods, the only one which need be mentioned is by Gregory [24, 25], who worked on the suitability of fabrics as protectors against the radiant heat of the sun.

§3. EXPERIMENTAL DETAILS

After some preliminary trials it was decided to use the constant temperature method and for this a cylindrical "heater," similar to Haven's but much smaller, was constructed. A guard-ring is provided at each end so that all difficulty about end losses is completely removed. Thermo-couples eliminate the need for a mercury thermometer and others are used to detect the temperature-difference between the central part and the guard-rings. The outside conditions are made definite by enclosure of the heater inside another cylinder whose temperature can be controlled. The ends of this cylinder are closed to prevent draughts and to

allow for the control of the humidity of the air. Rheostat-controlled electric heating is used in preference to timed intermittent heating, and a.c. was rejected on account of possible induction currents. For sensitivity it was desired that the heat capacity should be small and for this reason the heater was not filled with oil or water. The result may have been to give a greater lag in response to change of current, but the arrangement is thought to be worth while.

A detailed description of the final form of apparatus will now be given.

The heater A, Figs. 1 and 2, consists essentially of a copper tube containing heating coils. The diameter of the tube is 5·5 cm. approximately, and the central portion is 25·0 cm. long while the guard-rings are about 10 cm. long. The heat capacity of the central copper tube is only 69 gm. of water while its thickness is calculated to be such that temperature-differences round the circumference due to disposition of the heating coils are inappreciable.
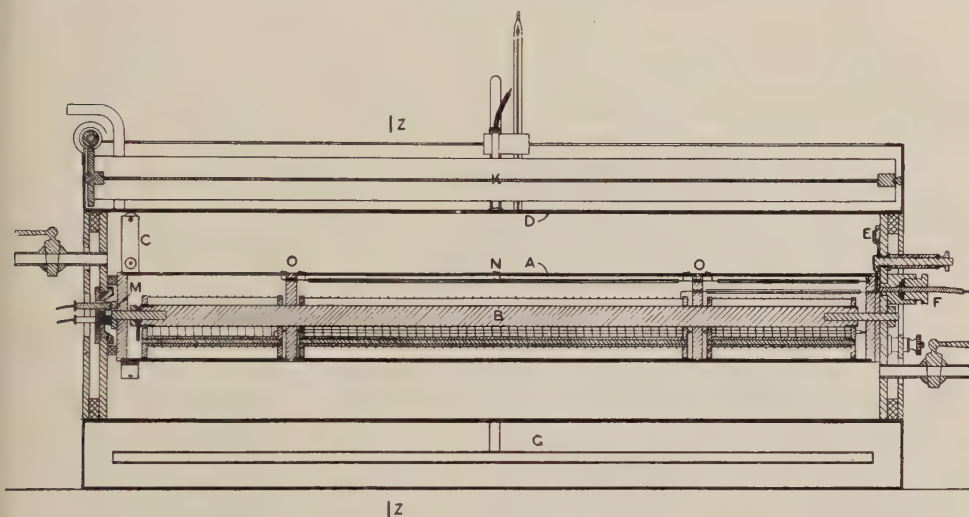
Fig. 1.

Two turned discs of red vulcanized fibre insulate the end tubes from the central one electrically and to some extent thermally. Two similar discs close the ends and are drawn together at their centres by screws projecting from the ends of a rod of vulcanized fibre *B*, thus clamping the whole heater together solidly.

The heater is coated with shellac varnish to give a non-tarnishing surface which could be repeated fairly accurately if necessary.

On the central rod are supported the three heating coils of Eureka wire, each of which is wound on quartz insulators threaded on brass rods. The resistance of the central coil is about 6 ohms and that of the guard-ring coils about 10 ohms. The leads to these coils are heavy copper wires insulated with glass tubes. All the internal connections are soldered in order to reduce contact resistance. The three leads to the coils come to three terminals at the front end of the tube while the common return is soldered to a plate *M* at the other end.

The thermo-couple N for measuring the temperature of the central portion of the heater is at the middle point. As was pointed out by Aberdeen and Laby[26], with a long tube of this kind there will be an approximately constant temperature along a considerable portion of the tube even if the guard-rings are not at exactly the right temperature. There is thus an "internal" guard-ring effect. Lamb and Wilson[23] showed that it was enough to measure the temperature of a cylinder used in this way at one point. In the same radial plane differential thermo-couples OO are provided between the central and end tubes. The thermo-couple leads are all carried inside the tube and insulated and protected from injury by glass tubing.

The far end of the heater is supported on a three-legged spider C which is clamped round it by three screws. Each leg is provided with a small rubber-tyred roller. These hold the heater centrally in the outer tube and allow it to slide in and out without scratching the surface.
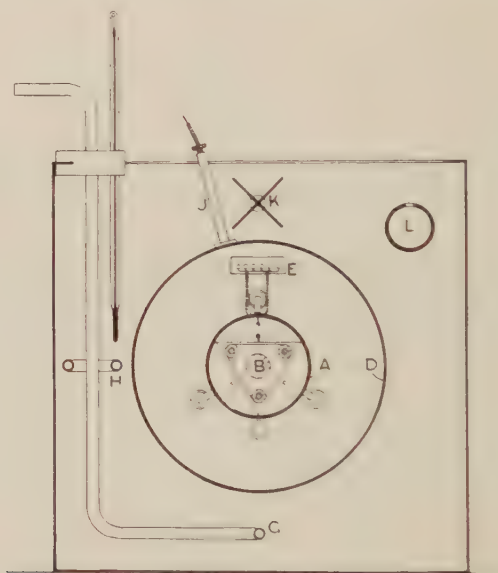
Fig. 2. Cross-section at ZZ, Fig. 1.

The end-plates. This heater, covered by a fabric, is placed during an experiment in a large concentric copper tube D surrounded by water. It was desirable that the ends of the tube should be closed with airtight plates to prevent draughts along it and also to allow the humidity of the air inside to be controlled. As one of them is attached to the heater the description of both may be put in here.

Each plate consists of two stiff Monel-metal discs which are an easy fit into the outer tube. Between them is held a rubber ring of the same outside diameter and this can be squeezed between the discs by six screws (not shown). When squeezed it increases in outside diameter and thus makes an airtight joint with the outer tube. If necessary these plates can withstand considerable differences of pressure. With both plates the inner disc is thicker and everything passing through

t does so with airtight joints while the outer one simply serves to squeeze the rubber ring. Both plates are provided with a large tap to enable air to be circulated through the annular space between the heater and outer tube.

Through the plate attached to the heater it was necessary to carry the three leads for the heating currents and the thermo-couple wires. The current is brought through the plates by three screws which are insulated from the plate and are made airtight by threading over them a piece of thick-walled rubber tubing whose outside diameter is equal to the diameter of the hole in the inner disc. When this rubber is compressed flanges are formed at the ends so as to provide an insulating airtight joint. This method of making such a joint has found several applications and has been described elsewhere[27]. The thermo-element wires from the heater are brought to a small terminal block E on the back of the plate, to which they are soldered. Similar wires run from there and are enclosed in a sheath which passes through the plate by means of a packing gland F.

The plate at the other end carries a central insulating bush for three spring contacts which impinge on the plate M at the end of the heater. To prevent damage to the heater when this end-plate is pushed up against it, a rubber buffer ring is provided on the end-plate.

Surroundings. The outer copper tube D is built into a tank which can contain water. The water is stirred and circulated round D by a long paddle K, above the tube, driven by an electric motor through a worm gear. A convenient method of constructing this worm gear so as to be silent has been described elsewhere[28].

Water can be supplied directly into the tank from the mains by a pipe G running along the bottom and perforated by a number of small holes. A large tube L with many holes prevents overflowing and a drain tap is provided. On the other hand, there is an independent cooling or heating system H running the whole length of the tank.

A thermo-couple is placed on this outer tube but is electrically insulated from it by a very thin mica sheet, as a precaution against stray e.m.f.'s. The couple and mica are held on the main tube by a screw inside a side tube J. For alteration of humidity inside this chamber an air circulation system is provided (see Fig. 3). For this purpose a small centrifugal fan having an aluminium rotor 10 cm. in diameter and mounted on ball-bearings was constructed. The drive is from the motor which works the stirrer. This fan pumps air through the wetting or drying system to the chamber round the heater. The air returns to the fan through a sample bottle for testing the humidity, then through a combined flowmeter and manometer. Conditions are always adjusted so that the manometer shows a pressure inside the chamber slightly in excess of atmospheric, so that all leaks in the system should be outwards. The only inward leak is through the fan bearings, so outside air has to pass through either the wetting or the drying system before reaching the chamber. The drying system consists of several calcium chloride tubes in series, while the wetting system is a number of wash-bottles immersed in the water in the tank. This precaution prevents the air from taking up too much water and depositing it as dew on the walls of the outer tube. The sample bottle Y

was packed with some cuttings of the fabric under test and by removing and weighing the bottle from time to time the observer was able to estimate the hygroscopic condition of the fabric under test. It was sometimes convenient to use the drying system, when no special humidity was desired, to prevent the formation of dew on the outer tube as the result of the giving up of moisture by the fabric when heated. In all cases the air current was cut off for some time before any readings were taken.

The inside of the outer tube was originally bright, but in order to get a surface which would not change greatly owing to atmospheric impurities, it was blackened with a resulting large increase in its heat-absorption.
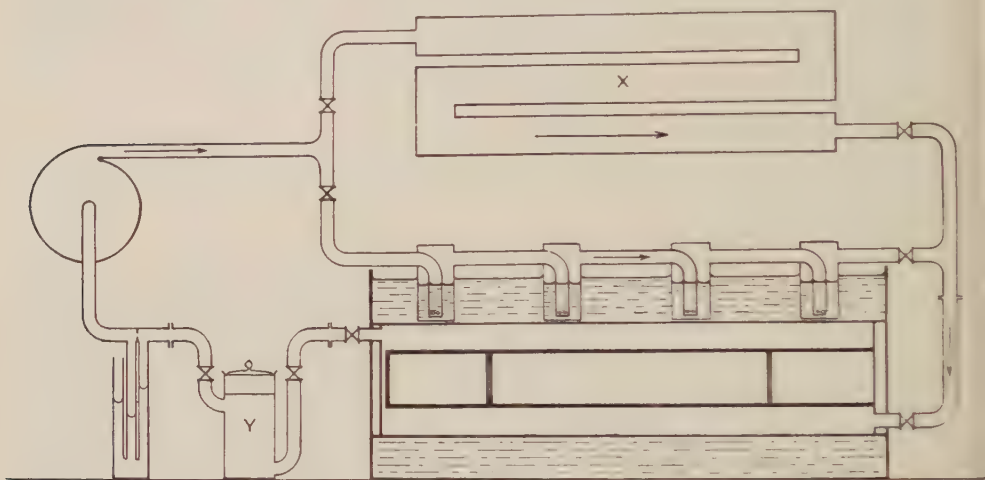
Fig. 3.

The fabrics were initially made into sleeves which fitted over the heater. Later, work was done with a definite air film under the fabric. For this purpose three frames were used made of woven wire of a fine gauge and very open mesh, (i) wrapped directly on the heater surface; (ii) 6·5 cm. in diameter and supported only on the guard-rings; (iii) 8·0 cm. in diameter and supported only on the guard-rings. With the last two the fabric is held completely clear of the heater surface. It is estimated that the wire occupies only 8·5 per cent. of the area of the frame. When the frames are in use an insulating ring is pushed on the heater first to prevent the frame from making contact with any of the electrical leads.

Energy control and measurement. In order to get a very steady current for the heating coils, large-capacity accumulators were used. It was quickly found that variations in the guard-ring circuit currents caused small but perceptible variations in the main circuit. A separate battery was therefore provided for the guard-ring heaters. It was found that the resistance of some of the leads common to both batteries had an effect, so the circuits were separated completely except for the common wire inside the heater, which had no measurable effect. This change involved putting two spring contacts in the insulating bush at the centre of the

back end-plate. These were later increased to three, the third being for a voltmeter lead, as it was found that the contact resistance between the spring plungers and the end-plate of the heater was not small compared with the resistance of the heater coil and that, being variable, it made the voltmeter readings uncertain to an extent rather greater than the experimental error. The third contact completely eliminated this effect. The complete circuit is shown in Fig. 4.

To allow the guard-ring currents to be set at definite values an ammeter is provided which can be switched into either circuit as desired. An equal resistance completes the other of the two circuits.
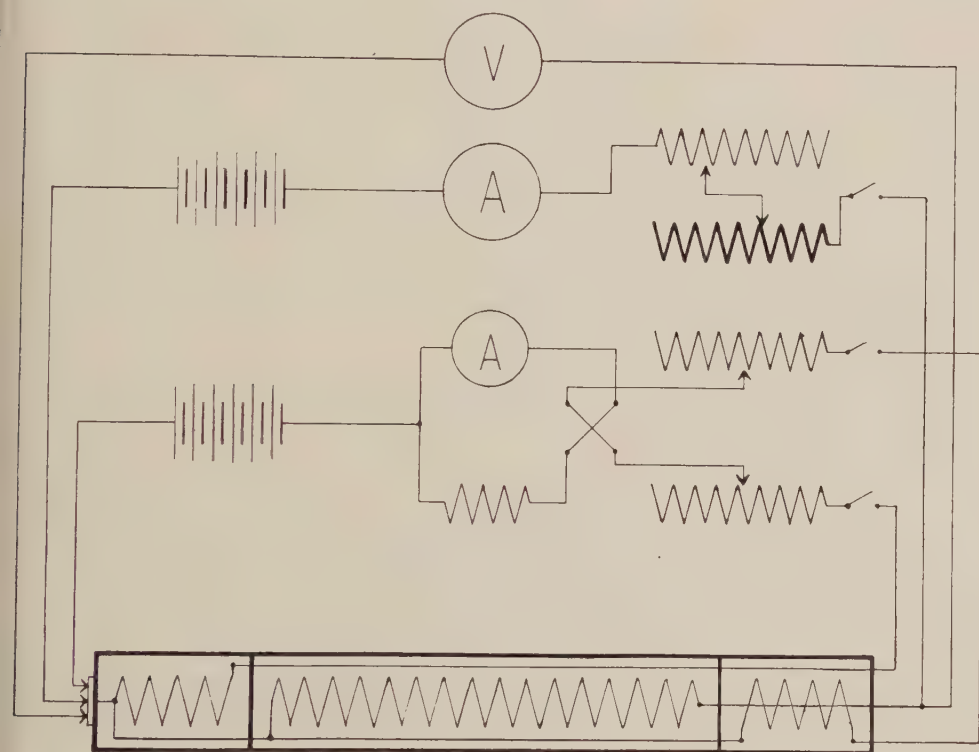
Fig. 4.

The control of the currents is by ordinary rheostat but the circuit for the central section is provided with a carbon compression resistance in series with the rheostat to allow very fine adjustments to be effected.

Temperature measurement. There are two distinct thermo-electric systems in the apparatus. They have the following uses: (i) The measurement of the temperature-difference between the heater surface and the outer tube. It is also important to be able to detect very slow changes in this temperature-difference. (ii) The detection of the temperature-difference between the central and guard-ring parts of the heater.

The temperature of the bath is measured with a N.P.L. certificated mercury thermometer.

M. C. Marsh

The measurement of temperature-difference between the heater and outer tube is made with a calibrated millivoltmeter (see Fig. 5). In order to get rid of resistance effects the millivoltmeter measures an e.m.f. provided by a potentio-meter-type circuit which is balanced against the thermo-electric e.m.f. The balancing is indicated by a sensitive mirror-type galvanometer and this also serves to show any slow change in temperature if the potentiometer setting is kept constant. Such a change indicates that the heating current is not correct. This combination of galvanometer and millivoltmeter proved very convenient.
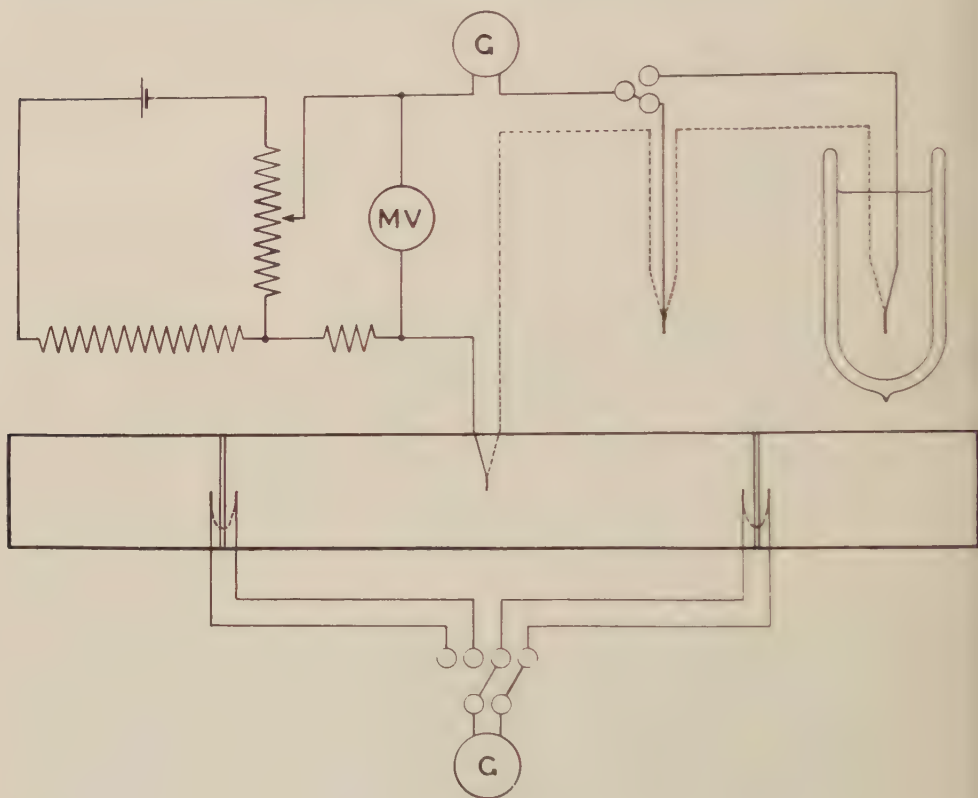
Fig. 5.

As soon as the work was started, it was found difficult to prevent variations in the temperature of the water, and hence of the outer tube, of the order of $0 \cdot 1°$ C. Though these were small compared with the temperature-difference they made it impossible to know whether the heater was varying in temperature. This difficulty was overcome by the use of a large volume of water in a Dewar flask, and during the experiment the temperature-difference between the heater and the water was indicated. When a good balance was obtained the circuit was changed by the mercury switch and the temperature-difference between heater and outer tube measured.

The detection of temperature-differences between the central portion and the two guard-rings was done by differential thermo-couples connected to a galvano-meter by a two-way two-pole mercury switch. The two galvanometers were of the same type and they indicated on the same scale. Their sensitivities were of the order of 2×10^{-7} volt/mm. at 1 metre radius, which corresponds to about 200 mm. divisions per 1° C.

Method of experiment. The sleeve is fitted on the heater with or without a frame, as required, and the spider is put on the end. The whole is put into position in the outer tube and the six nuts on the end-plate are screwed up. The tank is filled with water and the stirrer and fan are started. Heating currents are then switched on and the potentiometer adjusted to give the desired e.m.f. The mercury switch connects the couples in the heater and in the vacuum flask. The heating currents are then adjusted to give a constant temperature on the central portion close to that required, the guard-rings being kept constantly within 0·1° C. of the tem-perature attained. With care it is possible to adjust the current to yield a temperature-change of no more than 0·005° C. per minute, which, with a heat capacity of 70 gm. of water, corresponds to an error of less than $\frac{1}{2}$ per cent. in the energy supplied. When this balance is obtained the mercury switch is changed to measure the temperature-difference between heater and outer tube and all measurements are made.

All measurements were made on the fabrics as supplied by the makers. Any preliminary washing or other treatment may alter the characteristics considerably.

§4. MEASUREMENT OF THE PROPERTIES OF THE FABRICS

In order to specify the physical properties of a fabric with a view to relating them to the thermal insulating properties, it was necessary to make measurements on each.

Material. Most of the fabrics used consisted of one type of fibre only and these were generally specified by the makers. Where there have been mixtures of two fibres, e.g. wool and cotton, the makers' statements of the proportions have not been definite enough and the fabrics have been submitted to the Bradford Conditioning House for standard tests. These give the proportions of various fibres calculated on the clean dry weight.

Weight per unit area. A sample of the fabric was cut out with a square knife under a cloth cutter, which is essentially a hand screw-press. The samples were then placed in a room whose humidity and temperature are controlled automatically. After three days they were weighed without being removed from the room. For all samples the relative humidity was 70 per cent. and the temperature 73° F. (22·8° C.).

Thickness. It was realized from the start that the measurement of thickness would present difficulties, since the thickness depends on the pressure. An apparatus for this purpose should therefore be capable of measuring the thickness under

very small pressures and yet, without the cloths being moved, be able to put on any reasonable pressure and measure the thickness at the same time. An instrument which has proved satisfactory for this purpose has been described in detail elsewhere[29]. Thicknesses can be measured to 0·01 mm. at all pressures from 1 to 100 mgm./cm.² without the fabric being moved.

Light transmission. In order to get some measure of the "openness" of the structure of a fabric to radiant heat, a photo-electric method was used. The apparatus was an adaptation with slight alterations of that used by Barker and Stanbury[30] for the measurement of levelness of yarn. A constant, parallel beam of light is focussed on to a slit, under which is a photo-electric cell connected to a Lindemann electrometer. The deflection of the electrometer is proportional to the quantity of light passing through the slit. If a fabric is placed in the beam the ratio of the deflection to that obtained when there is no obstruction in the beam is the fraction of the fabric which has a clear aperture from side to side. There are quite open fabrics in which the apertures are not perpendicular to the surface, so that exact concordance is not to be expected between air and light transmission.

Permeability to air. After some attempts to compare the resistance of a fabric with standard orifices, tapped viscosity tubes, etc., it was decided that the simplest method would be to draw air of known temperature and humidity through a defined area of fabric, to determine the pressure-drop and to measure the air passing with a calibrated gas meter. In the case of the pressure-drop, a micromanometer was used for measuring differences too small for an ordinary manometer. This micromanometer consisted of a U-tube with wide limbs and narrow connecting tube so that the damping was high. Two hook gauges moved by micrometer heads entered through mercury sealed joints. Large differences were found between samples from the same piece, but these are insignificant compared with the enormous range of resistances measured, and all that is required for the present purpose is the order of the resistance. This section of the work has found other applications and will be described fully in a subsequent paper.[31]

Colour. No measurements of colour have been made.

§ 5. RESULTS

Before the actual results of the experiments are discussed, the mode of expressing them may be noticed. In the preliminary work on the effect of various factors, results are generally stated as the total heat loss from the central portion of the heater in watts.

For the comparison of fabrics or of one fabric under different conditions, it has been found convenient to define a new quantity depending only on the heat lost by the heater uncovered and covered *in a definite specified way*. This quantity, called the thermal insulating value (T.I.V.), is defined to be:

$$\left\{ 1 - \frac{\text{heat lost by covered heater}}{\text{heat lost by uncovered heater}} \right\} \times 100 \text{ per cent.}$$

It would have a zero value if the heat loss were the same with as without the fabric, and a value 100 if there were no heat loss. It has been found that some thin fabrics placed directly on the heater cause an increase in the emission of heat. These will have a negative value when calculated by the above expression.

These negative values may appear strange at first sight but the reason for their occurrence is quite clear. The effective surface for loss of heat is increased when a fabric is put over a smooth varnished surface. Negative values have been obtained with some suitings and hosiery fabrics wrapped directly on the heater surface.
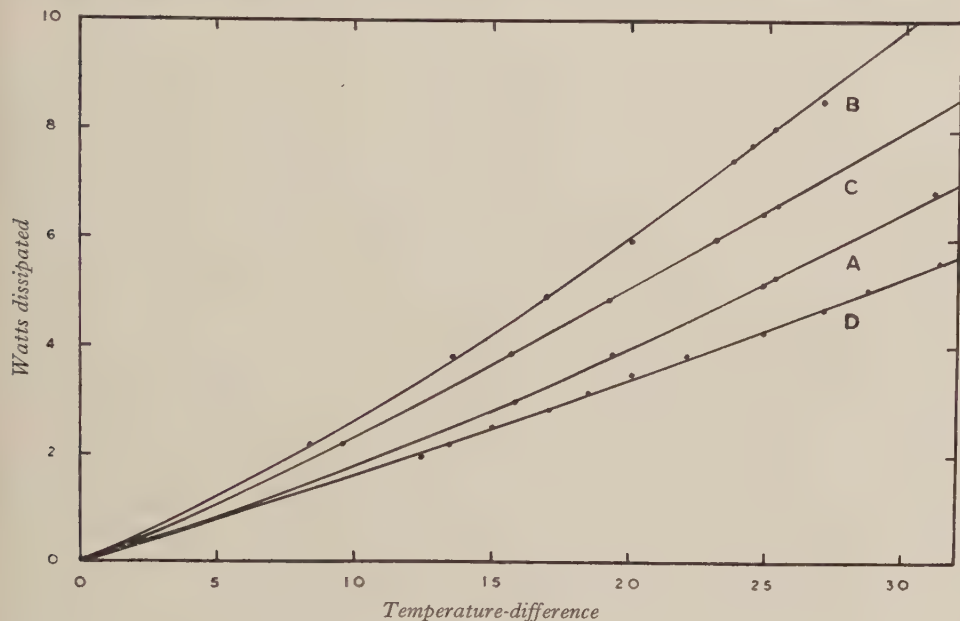
Fig. 6.

The first experiments undertaken were to find the relation between the energy supplied (i.e. total heat loss) and the temperature-difference between the heater and the outer tube. Various surfaces and conditions were used and the results are shown in Fig. 6.

Curves *A* and *B* relate to the uncovered heater when the inner surface of the outer tube is bright and black respectively. These curves show the importance of a definite surface as well as temperature in the boundaries of the space. As bright surfaces very easily tarnish it was decided to use a black surface on the outer tube for all work. Curves *C* and *D* show the relation for the heater covered with a fabric, directly in contact, and supported on a frame, respectively.

It will be noticed that the curves do not differ greatly from straight lines through the origin, such as would be given by Newton's law of cooling. This allows experiments to be done at temperature-differences approximating to a selected standard, small corrections being made on the assumption that the power required is proportional to the temperature-difference with only second-order errors.

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From the curves it máy also be deduced that the ratio of the ordinates of any two curves is practically independent of temperature. It is therefore possible to select any convenient temperature for the comparison of fabrics.

Very large changes of humidity, such as would not be met in normal practice, were tried. These were produced by the continual passage of the air through the wetting system, so that it became saturated at a temperature just lower than that of the cold surface, or by passage of the air over the calcium chloride. They had very little effect on the total heat loss and it was concluded that the differences due to ordinary changes of humidity are less than the experimental errors, so that measurements could be made without the need for humidity control. This result appears to contradict results of previous workers, but it must be emphasized that in practically no previous investigation has the fabric been left for two or three days in the experimental condition to reach hygroscopic equilibrium as it was in the present experiments. Gregory, who was careful to allow his cloth to reach equilibrium, found a small decrease in transmitted radiant heat with increase of humidity.

In all the earlier experiments the fabrics, made up into sleeves, were on the surface of the heater itself. The sleeves were made so that they slipped on and off readily without being slack. No special precautions were taken to make them all fit in exactly the same way, as it was thought that the fitting would have very little effect on the surface density. With these sleeves thermal insulation values could be repeated consistently to 1 per cent. and numerous tests were made in this way.

The effects of multiple layers and combinations of fabrics were being tried when certain anomalies were discovered in the results. After many experiments and the elimination of all possible variables and complications, it was found that, while any sleeve gave results consistently, two sleeves of the same knitted material gave very different results. The two sleeves were closely examined and tested for weight per unit area and no differences capable of explaining the discrepancies were found. It was noticed, however, that one sleeve was more slack than the other on the heater: it gave a much higher insulating value, but when it was tightened the value fell eventually to below that for the other sleeve.

As these results were for a knitted fabric which is easily stretched, it was decided to test woven fabrics, and a flannel and a blanket were chosen. The same effect was again obtained as is seen in the following table:

Effect of tension of fabric on thermal insulation

Description	Thermal insulating value. Tension increasing		
Hosiery (Wool)	10·5	− 3·5	− 6·1
Flannel	10·5	5·0	2·9
Blanket	40·7	31·2	—

It is thus clearly established that it is very important in comparing fabrics either to find a standard or comparative tension for each fabric and work to that, or to eliminate the effect of tension completely.

Since in the above cases the stretching was never more than a few per cent. of the original length (say 3 to 7 per cent.) the fall in insulating value cannot be explained by changes of surface density or the opening of the interstices of the fabric so as to allow a freer passage of air. The only possible explanation appears to be that a fabric resting on a surface ordinarily is supported by the more projecting fibres and that an air film, which is comparable in its insulating power to the fabric itself, exists between the fabric and the surface. When the fabric is tightened on to a curved surface this film is diminished in thickness and the fabric establishes a better thermal contact with the surface and thus the apparent thermal insulation is diminished.

This suggestion was tested out as follows. A more definite air film between the fabric and the heater was created by the insertion of the very open-mesh wire gauze. This gave a much greater insulating value, but there was still a large difference on the fabrics being stretched. It was noticed that it was still possible for the fabric to sag through the meshes and come into contact with the heater.

The two larger frames of wire netting of different diameters were then tried, and further increase in the insulating value of the fabrics on these was found in all cases. But now that the fabric, whether stretched or loose, was quite clear of the heater surface it was found that the insulating value was unchanged on the fabrics being tightened on the frame. This result gave a method of comparison of fabrics *independent of tension* within the accuracy of experiment. In all previous work the effect of this air film beneath the fabric has been completely overlooked. The results are therefore open to very grave criticism as the effect must have occurred in all work involving the wrapping of a body in the fabric. It further confirms all that has been said above as to methods involving the compression of the fabric between two surfaces. Once this effect has been seen it becomes quite obvious, and it is remarkable that it should not have been detected before. Clearly, therefore, a fabric must be measured with both surfaces free from solid bodies as far as possible. This condition was approached by the supporting of the fabric on the very open wire frame, 6·5 cm. in diameter, for all comparative measurements.

§6. COMPARISON OF DIFFERENT FABRICS

From the results given above it is now possible to formulate a scheme for the comparison of different fabrics under defined conditions. It was decided to make the comparisons on the 6·5 cm. frame and at a standard temperature-difference of 25° C. approximately, the values being corrected to 25·0° C., while the cold temperature was kept as near as possible to 12° C.

The thicknesses of the fabrics were measured at three pressures, viz. 0·1, 1, and 10 gm./cm.². The results for thermal insulating value and thickness at 10 gm./cm.² are shown plotted in Fig. 7. At this pressure the points were less scattered than at the other pressures. This would indicate that it is the more solid part of a fabric which is important in thermal insulation, while the projecting fibres are not of great importance. It would have been better to have determined a thickness

pressure curve for each fabric, as from such a curve the point where the solid
structure of the fabric is reached can be seen. But the average of a number of such
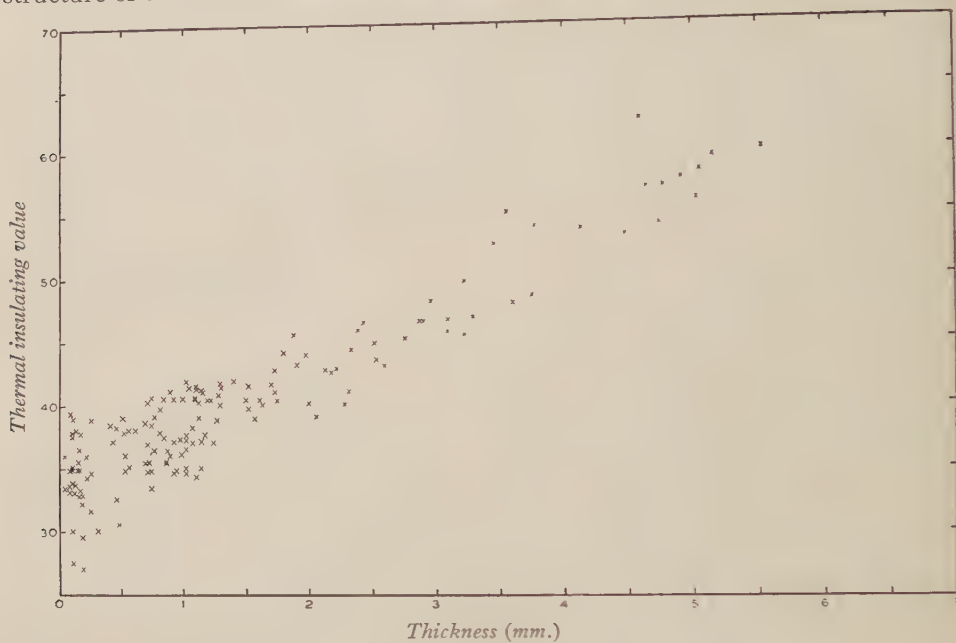
Fig. 7.
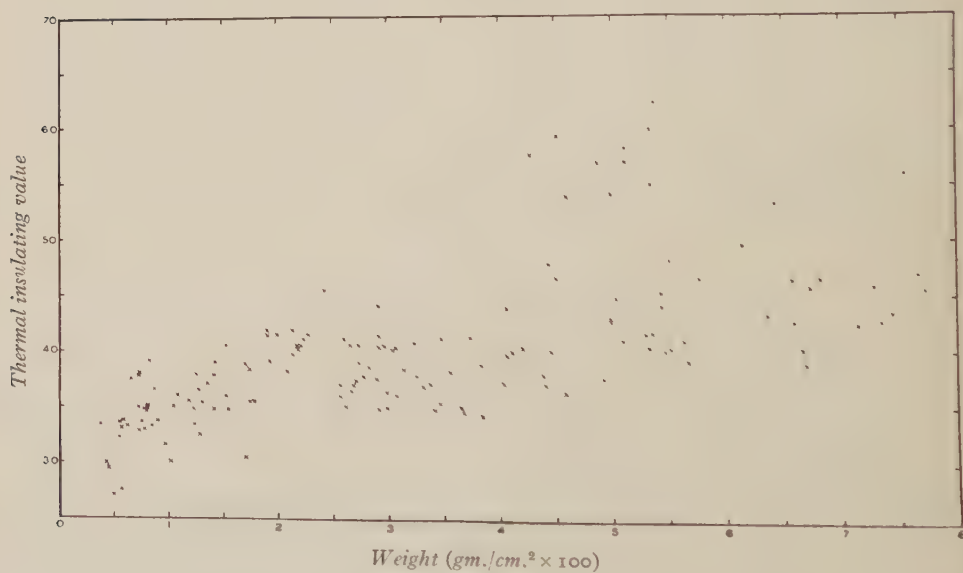
Fig. 8.

curves showed that this point was not very different from that given by a pressure
of 10 gm./cm.², and measurement at a definite pressure is much more convenient.

The points lie very approximately on a straight line which does not pass through the origin. The deviations from the straight line are largely capable of explanation by reference to the other physical properties and thus give definite information as to the method of transfer of heat through a fabric. This will be discussed later.

Fig. 8 shows thermal insulating value plotted against weight per unit area and shows that there is no direct relationship except perhaps a slight increase in thermal insulating value with weight. Again, however, the position of certain fabrics and groups of fabrics, related to their other physical properties, give definite information.

From the thickness at 10 gm./cm.² and the weight per unit area the density has been calculated. No definite relationship appears to exist between thermal insulating value and density for the same weight or thickness, but there is a marked tendency for the less dense fabrics to show a higher thermal insulating value than others.

While textile details need not be entered into here, the general trend of detailed analysis may be given as it has some physical importance. Most of the work on heat flow through fabrics has been done with the object of deciding the relative values of different fibres and many opinions have been expressed. The great difficulty is to get comparable fabrics of two different materials. It is found that there are in general use very few non-woollen fabrics that are more than 1 mm. thick and not many made of wool less than 0·5 mm. thick. Comparison has therefore to be made between rather extreme fabrics of each material. Similarly, as regards weight per unit area, wool occupies practically the whole region above 0·02 gm./cm.². Any other materials which come in this region are not of a comparable type of fabric. In the parts where wool overlaps with other materials, either in weight or in thickness, there is strong evidence that it is superior to cotton and artificial silk. Direct comparison cannot be made with silk or linen.

Cotton, silk, linen and artificial silk all fall into the same region, but there are greater differences between different fabrics of the same fibre than between those of different fibres. Thus, structure is much more important than material for these. Silk, however, is capable of producing a better insulator than the others with a proper structure.

Quality as affecting thermal insulation has only been studied in wool fabrics and shows itself especially in the case of blankets. The finer the wool the higher the insulating value is a general rule.

In general, a large light-transmission and a low air-resistance go together and cause a low insulating value, especially in the thinner fabrics. In the case of fabrics under 1 mm. thick every fabric with a thermal insulating value less than the average has a very low air-resistance. With thicker fabrics air-resistance seems to have little significance. A fabric with a shiny surface appears to be a good insulator, but direct comparison is very difficult. Again, as far as can be ascertained from these experiments, the colour of the fabric has no effect on the thermal insulation. The difficulty of doing direct experiments on this point is the change which necessarily takes place in a fabric on its being dyed.

§7. DISCUSSION OF RESULTS

The (thickness) (thermal insulating value) diagram may be taken as a basis for the discussion of results. From it can be formed a fairly accurate idea of the mode of heat-transfer through a fabric or through insulators of a similar character used with a free air space on each side.

It may be taken that there is approximately a linear relationship between thermal insulating value and the thickness of the fabric excluding projecting fibres. This holds over a very wide range of thicknesses covering every type of fabric.

It should be noted that an average line drawn through these points does not pass through the origin, but that for small thicknesses there is a thermal insulating value of approximately 30 per cent. This would indicate that there is a large heat loss by convection from the body when uncovered and that the thinnest fabric breaks up the air stream, thus affording a considerable amount of insulation. This conclusion is in accordance with the general known methods of reducing heat loss by convection. Confirmation is obtained from the results which show that all thin fabrics with a small resistance to the passage of air have a smaller insulating value. In this case the warm air can pass directly through the interstices of the fabric, thus causing a greater heat loss than would be expected from the thickness measurement.

Similarly those fabrics which have a high light-transmission give a low insulating value, as would be expected since they allow direct passage of heat radiation and convection.

In order that these ideas might be tried out by extreme tests, several unusual coverings were investigated. A thick very open net gave a value much lower than any other fabric of the same thickness. On the other hand, tracing-linen and paper, which are impervious to air, gave high results compared with those of pervious fabrics.

A further very interesting experiment was on thin sheet brass. It gave a thermal insulating value of about 54 per cent. when blackened and about 72 per cent. when polished. Here the conductivity of the metal is so high that the resistance to passage of heat through it is negligible. Under these conditions, on the assumption that the heat passing across the space is proportional to the temperature-difference, the conducting plane should cut down heat loss to 50 per cent. if its surfaces are perfectly absorbing and radiating. Actually the blackened surfaces do not fulfil the condition completely, so that the thermal insulating value is rather over 50 per cent. When the surfaces are bright this effect is increased so that the thermal insulating value is greatly increased. This is a similar effect to that shown in the earlier experiments on the loss of heat from the uncovered heater to polished and black surfaces.

It may now be asked why a good conductor such as brass is, under these conditions, a far better insulator than fabrics, whose conductivity as ordinarily determined is much lower. This cannot be altogether explained by the imperviousness to air of the metals, as some of the fabrics did not allow any passage of air at all.

It is suggested that the effect is due to the larger surface of the fabric in which more heat can be received and lost by convection. Confirmation of this is given by the fact that thin sleeves fitted directly on the heater caused it to lose more heat than it lost when bare. It was also found that a smooth, shiny-surfaced fabric had a high insulating value. Gregory[25] has shown recently that fabrics have an emissivity ten times as great as polished copper.

From a theoretical standpoint the passage of heat through an insulator such as a fabric is very complicated. Smoluchowski[32] has worked out the problem of the conduction of heat through spheres regularly packed, making certain assumptions as to discontinuity of temperature at the surfaces. This theory and the results to which it leads do not fit in at all with the work of Aberdeen and Laby[26], so that even in this relatively simple case there are very great difficulties. When we consider a fibrous insulator such as a fabric the difficulties increase enormously, because: (i) fibres are of irregular shapes and very different cross-section*; (ii) the arrangement of the fibres in the yarn may be somewhat haphazard, and they often project from the yarn; and (iii) the processes subsequent to weaving tend to merge the threads of yarn into each other.

The mathematical treatment of the heat-flow through a fibrous insulator therefore becomes impossible, but an approximate general idea, which is confirmed at many points by the results obtained, may be formulated.

The heat passing directly through the fabric by convection and radiation may be ignored and attention may be concentrated on that part which passes through the solid structure. This solid has a much higher conductivity than the air and thus affords an easy passage to the heat. Many who have worked on the subject have reached the conclusion that it is the air which functions as an insulator and that the fibres simply serve to keep down convectional heat-flow though they increase the conduction effect. This conduction effect may, however, be quite small, as the general direction of fibres will be at small angles with the plane of the fabric. Heat will therefore encounter much resistance due to poor contact between fibres, especially in the case of those with scales, like wool. The higher insulating value of the less dense fabrics would be evidence for this point. It seems there will be little local convection inside the fabric, but a considerable amount of radiation from the warmer parts to the cooler.

From a practical point of view, however, the relations between the insulating-value and the physical properties are so uncertain, owing to the many variables encountered, that the only method of estimation of insulating-value is by direct measurement. No mathematical relations definite enough to be of utility can be deduced. The main point which comes out in this work is the great importance of the surface conditions in measurements of thermal insulation.

* For wool, see the work of Barker and Burgess[33].

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DEMONSTRATIONS

The regional absorption of dyes by growing crystals*. *Demonstration given on May* 9, 1930, *by* Dr A. G. MILLIGAN, Admiralty Research Laboratory.

The purpose of the exhibits was to demonstrate that, when a crystal is coloured by a dyestuff dissolved in the mother liquor from which it is grown, the colour is not uniform throughout the crystal, but is confined to the wedge-shaped regions swept out by certain faces as they grow. When a dye is absorbed upon any face the deposition of solute upon that face is inhibited, and, since by a well-known paradox the faces which tend to predominate are those upon which deposition is slowest, the presence of a dyestuff may profoundly modify the habit of the crystal.

The exhibits included:

1. Crystals of potash alum coloured by chlorazol sky-blue *FF*. In moderate concentrations the dye was absorbed by the cube faces, which were abnormally developed. A wedge of blue ran from the centre of growth to each cube face and the usual octahedral habit of alum crystals was almost lost. In higher concentrations the dyestuff so inhibited the deposition of salt upon the cube faces that vicinal faces developed, and the colour was confined to a lozenge-shaped patch in the centre of each cube face.

2. Crystals of potash alum coloured by methyl violet. This dye coloured the octahedron faces. The normal octahedral habit was enhanced, and cube faces were rare. Where they occurred, however, the regions below them were colourless.

3. Crystals of Rochelle salt coloured by chlorazol sky-blue *FF*. This salt forms rhombic prisms with many faces developed; but the dye took only upon the b {010} prism faces. A transverse section of one large prism was projected on the screen to illustrate the sharp definition of the coloured wedges.

4. Crystals of Rochelle salt coloured by acid green *G*. This dye coloured not only the b {010} faces but also, less strongly, the l {201} faces as well. No other instance of a dye taking upon two forms simultaneously was ever observed.

5. Crystals of Rochelle salt coloured by phloxine. None of the prism faces was coloured but a wedge of intense pink ran from the centre of growth to each of the end faces.

A modified form of Callendar recorder for the automatic control of a high-temperature oil-bath. *Demonstration given on June* 13, 1930, *by* H. R. LANG, Ph.D., F.Inst.P., Imperial College of Science.

The usual method of using a recorder for the temperature control of a bath is to place on either side of the recording pen contacts which operate suitable relays. In the Callendar recorder such a method involves the movement of the bridge-wire contact, and produces hunting of the order of a few tenths of a degree centigrade.

* *Journ. Phys. Chem.* **33**, 1363 (1929).

This has been overcome in the following way. A long arm is fixed to the screw over which the driving-belt for the pen usually passes, and to the end of the arm a short length of platinum wire is soldered. As the screw is turned by the clockwork mechanism, the platinum wire makes contact with a fixed contact-wire, closing a relay circuit. Suitable stops prevent the arm from turning too far. The relay puts a 100-watt lamp into series with a 250-watt heating-coil wound around the stirrer in the oil-bath. Such a method is better than switching the heating-coil off altogether, and tends to reduce hunting. In its present form the hunting is three or four hundredths of a degree, and it would be possible to reduce it further if necessary. By means of a friction drive it would also be possible to make the apparatus recording, if desired. The apparatus possesses all the advantages of the form devised by the late Prof. Callendar, particularly that of non-sticking contacts, which are made and broken by clockwork. There seems to be no reason why this form of thermostat should not operate equally well at any temperature where a resistance thermometer can be used.

The demonstration showed the oil-bath maintained at 180° C. and the period of hunting was arranged to be about two minutes. By means of another platinum thermometer in the bath the magnitude of the fluctuations was shown to be less than 1/25° C.

FIELD INTENSITY MEASUREMENTS AROUND SOME AUSTRALIAN BROADCAST STATIONS

By R. O. CHERRY*

Author's reply to the discussion on the above paper

In reply to Mr R. A. Watson Watt: The accuracy of this method of measuring field intensities depends only on two factors: (i) measurement of the induced voltage and (ii) the constancy of the ratio E/V for the field loop and the accuracy with which this ratio is known. The calibration of the voltmeter immediately after each reading of V in the field disposes of the first factor, as no evidence has been obtained for any sudden change in the calibration of the voltmeter. The absolute value of the ratio E/V depends on several factors which are difficult to determine accurately but may, with care, be kept constant. Thus the relative positions of the apparatus, the car and the observer were all reproduced for each observation. The effective loop-resistance is made up of copper losses, eddy-current and dielectric losses. With the use of solid wire the first two losses should be constant; the very small amount of solid material used in the construction of the loop should diminish the dielectric losses, the most serious of which are probably the losses in the valve socket and base. Any arrangements for measuring this loop resistance in the field would have meant a complication of apparatus, with the possibility, under field conditions, of errors in measurement exceeding the probable change in loop resistance. Frequent laboratory tests failed to show the necessity for this elaboration of the apparatus.

The ratio $V/E = (2\pi An/\lambda)\,(\omega L/R) = \text{const.}/\lambda^2 R$.

Hence E/V is only proportional to λ^2 if R is independent of λ, and this of course is not even approximately the case, even at frequencies far removed from the fundamental of the coil. I do not know of any simple formula which gives the ratio of resistance to frequency, but since $107/86 = 1\cdot24$ and $484/371 = 1\cdot30$ (where 107 is the value of E/V for $\lambda = 484$, and 86 its value for $\lambda = 371$) it appears that in the present case R is roughly proportional to $1/\lambda$.

A small condenser was actually inserted across the other half of the loop after the above work had been carried out. Comparison of the results obtained with and without this condenser showed that no appreciable change was produced in either the maximum induced voltage or in the direction of the minima whose angular separation differed from $180°$ by less than $1°$. These minima were not however exactly zero.

The figures quoted by Mr Watson Watt with regard to the importance of the grid-leak resistance were obtained for very large input voltages. The corresponding figures for an applied voltage of, say, $0\cdot01$ volt would be interesting. I have not seen any account of a valve voltmeter, using 1 valve only, of equal sensitivity and

negligible input conductance. The mode of action of the rectifier is not clear to me, but all the tests that have been performed on the voltmeter show that it fulfils satisfactorily all the requirements of such an instrument.

The anode-bend instrument used in the comparison was a commercial instrument of conventional design.

The information from which Table 4 was compiled is rather scanty and it was only inserted to show the very marked decrease in attenuation with the use of the longer wave-length. Comparisons involving the product $E.d$ are not valid in this table, as the two stations are not equidistant from the areas considered.

Noise levels. Further information obtained during another investigation compels me to withdraw the suggestion that atmospherics are less prevalent in Victoria than elsewhere. Mr G. H. Munro has recently set up a cathode-ray direction-finder near Melbourne and he tells me that during his month's observations, he finds atmospherics more prevalent in Victoria than in England.

In reply to Dr Smith Rose: The estimated accuracy in the determination of the intensity applies only to the standard loop. The two methods of determining $\omega l. R$ for this loop agree to 2 per cent. and for the actual measurement of the intensity an additional resistance of 6·16 ohms was inserted in series with the loop. While the actual loop resistance (0·91$_1$ ohm) is not known to better than 2 per cent. or 3 per cent., the addition of the short length of straight-wire resistance considerably reduces this error. The suggestion made by Dr Smith Rose to explain the results of measurements taken over sea water is original and interesting. The conditions under which these experiments were carried out were far from ideal, but it appeared desirable to make the investigation because in it surface conditions were as uniform as possible. The unexpected nature of the results emphasizes the necessity for repeating this series of observations under conditions where the possibility of coastal reflection and refraction are absent. If the suggestion of coastal reflection were correct, it would affect the measurements taken, on both sides of Port Phillip Bay, to determine the attenuation due to sea water. I regret that I have no further relevant data on these measurements.

THE MASSES OF THE PROTON AND ELECTRON

By H. T. FLINT, D.Sc.*

Addition to the discussion on the above paper

Mr William Band (Yenching University): I am much interested in the article by Dr Flint on the masses of the proton and electron, particularly because his suggestion that the difference between these masses may be equivalent to a difference between the space-gauge to be used for the two types of particles is similar to the idea which started the theory of electron-tracks that has already received preliminary announcement in *Nature* and the *Physical Review*†. In that theory the metric tensor, or in other words the gauge, is an arbitrary factor which must be chosen as an initial boundary condition as it were. The gauge is not a determinate function of position, and is defined only along the track of the particle. It had not occurred to me that this might be connected with the relations between the proton and electron, but obviously in the light of Dr Flint's suggestions, the connection is simple. Thus a particle is defined as a region bounded by zero field (external and particle-field neutralized) and the mass is defined by the equation

$$m = e^2/4\pi a \text{ in "natural" units,}$$

where a is the radius (average) of the region of zero field. This, as Dr Flint suggests, will depend on the gauge used; and the gauge being initially arbitrary in the mathematical theory, we can choose it so that it will give a measure of a that will give the required empirical value of m. But while this seems to verify that the mass-difference is similar to or connected with a gauge difference, it scarcely explains it.

I think that the real explanation is only to be found by admitting that the simple contrastability suggested by polarity of charge is false. We have already, in Dirac's fanciful suggestion that the proton is a hole in the otherwise full negative energy-states of electrons, the suggestion that protons and electrons are of essentially different natures. And we have evidence that protons will not excite radiation in the way that electrons will. If we accept Whitehead's philosophy that an object is an abstract permanency among the relations between events, then an electron is an abstract entity invented to relate quantum jumps of otherwise independent nature occurring in different atoms, whereas a proton is another invention to connect quantum jumps occurring in the same atom and definitely related as regards frequency numbers, etc. There seems no reason that the two objects as so defined should be directly contrastable in the $(+, -)$ manner, and if their charge is so contrasted, the contrast would seem rather fortuitous, and a property of the definition of charge rather than of the nature of the particles concerned.

AUTHOR'S reply: Mr Band's note is a very interesting contribution to the discussion on my paper on the masses of the proton and electron. I had not seen his work on this subject and it is very interesting to find that he has been led to similar views on this important question. It is difficult to say if we shall be led to alter our conceptions of positive and negative electric charges, but in our present groping in the dark we may find some guidance in approaching our difficulties from another point of view.

PRESENTATION OF
THE DUDDELL MEDAL FOR 1929

To Prof. A. A. MICHELSON,
University of Chicago.

The President of the Society, in presenting the medal, made the following remarks:

At the meeting in December last the Council selected from the world of physicists Prof. Michelson as the recipient of the medal for 1929. I think I may say that the general membership of the Society and others outside have cordially approved the selection; indeed messages of warm approval have been received.

The interferometers invented by Prof. Michelson, of which the first was used for carrying out in 1887 the famous Michelson and Morley experiment, have been applied by him, always with complete adequacy of design, to other important and difficult problems, most of them of audacious novelty. These problems included the measurement in 1892 and 1893 of the metre in wave-lengths of light [1]: the measurement of the diameters of stars: the re-measurement of the earth tides: and the testing of the effect of the earth's rotation on the velocity of light [2]. These measurements have had great consequences, of which the following examples may be mentioned:

The difficulties of reconciling the result of the Michelson-Morley experiment with the then prevailing physical conception of the nature of the universe were the direct cause of the enquiry of Albert Einstein [3], which resulted in the theory of relativity. The measurement of the metre in wave-lengths of light [4] resulted in establishing a standard of length free from the uncertainty concerning possible variation which attaches to all material standards. The interferometer for the measurement of the diameter of stars, suggested by Michelson in 1890 [5] and first applied by him to Betelgeuse [6], has not only confirmed the correctness of the previously almost incredible dimensions yielded by indirect means of calculation, but has detected fresh stellar phenomena in the variable diameter of Mira Ceti, and the separation of double stars too close for resolution by the unassisted telescope.

The invention by Prof. Michelson of the echelon diffraction grating [7] provided physicists with a potent tool for the investigation of the fine structure of spectral lines, knowledge concerning which has become of such great importance in modern physics. Prof. Michelson has also designed a ruling-engine with which very large gratings have been ruled.

As a final example of Prof. Michelson's work on scientific instruments for the advancement of knowledge, mention may be made of the completion by him in 1926, with apparatus designed by himself, of a redetermination of the velocity of light [8]. The elaborate precautions taken to secure freedom from error included means whereby the distance of 82 miles traversed by the light was measured to a higher degree of accuracy than had ever been reached in triangulation.

We regret that Prof. Michelson's state of health and his distance from us have conjoined to prevent him from being with us to-night, but we welcome very heartily the official representative of the great American Republic who attends in his stead. May I venture to convey through you, Sir, to the United States of America, the congratulations of this Society that you number among your citizens so distinguished a physicist as Prof. Michelson.

Mr David McK. Key, a Secretary of the United States Embassy, received the medal on behalf of Prof. Michelson and expressed his thanks for this honour.

(1) *Trav. et Mém. du Bur. Intern. des Poids et Mesures*, **12**, 1 (1895).
(2) *Astrophys. Journ.* **61**, 137 (1925).
(3) *Ann. d. Phys.* **17**, 891 (1905).
(4) *Trav. et Mém. du Bur. Intern. des Poids et Mesures*, **12**, 1 (1895).
(5) *Phil. Mag.* **30**, 1 (1890).
(6) *Astrophys. Journ.* **51**, 257 (1920).
(7) *Astrophys. Journ.* **8**, 37 (1898).
(8) *Astrophys. Journ.* **65**, 1 (1927).

OBITUARY NOTICES

A. A. CAMPBELL SWINTON, F.R.S.

Alan Archibald Campbell Swinton was born on October 18, 1863. His father was a professor of civil law in the University of Edinburgh. His father's sister was the mother of Archbishop Lord Davidson. The family could prove direct descent from the royal house of Scotland. His brother Capt. George S. C. Swinton has been Chairman of the London County Council, and of the town planning committee of the new City of Delhi. He was also Lord Lyon King of Arms.

From his earliest days Alan showed a strong bent towards engineering and physics. The headmaster of the first school he went to encouraged his bent, but when he went to Fettes College the masters endeavoured to make him devote most of his time to classical studies. His interests however were mainly connected with constructing telephones and taking photographs. At the age of 15, two years after the telephone was invented, he constructed two which worked excellently. His house master at Fettes sent them home so that they should not interfere with his classical studies. As he disliked games his school life at Fettes was not happy. The training he received left a mark on his after life. He always liked talking about classical history and his visits to Italy he found extraordinarily interesting. The impressions produced led to his writing several poems.

After a short visit in 1881 to France, where he improved his mathematics and his knowledge of French and saw the wonderful electrical inventions at the Paris Exhibition, he was apprenticed to Lord Armstrong in the works at Elswick. In 1887 he left Elswick and set up in London as an electrical contractor and consulting engineer. In the former capacity he installed the electric light in many town and country mansions. In 1895 he was elected a Fellow of the Physical Society. In January 1896, after reading an account in the morning paper of Prof. W. C. Röntgen's discovery of X-rays, Swinton was successful in obtaining the same afternoon a shadow photograph by means of a Crookes tube which he happened to possess. A few days later he obtained a shadow photograph of the bones in his own hand. He at once recognized the great benefit of this discovery to mankind. Yet in those days when one of his photographs was shown to an eminent personage, the latter's comment was "How disgusting!"

In February 1898 he showed many interesting experiments with cathode- and X-rays at the Royal Institution. In conjunction with Sir Charles Parsons he converted diamond into coke by heating it in a vacuum by cathode-rays. As the diamond became carbonized it split up and frothed and became much larger in size. The temperature at which this conversion took place was $1890°$ C.

In 1904 Campbell Swinton gave up contracting work and became exclusively a consulting engineer. He gave many lectures to various societies, including the

Physical Society, and did much to spread a general interest in science. I particularly remember hearing him lecture to the Camera Club, and the sensation he caused, I think in 1896, by throwing an X-ray picture of his own head on the screen when speaking. On another occasion when giving a lecture he passed sufficient high-frequency current through his own body to light a 100-volt 5-candle-power lamp.

He was specially interested in radio work. When Marconi (now the Marchese Marconi) came to England Swinton introduced him to Sir William Preece, and long-distance radio-telegraphy was the result. In 1910 Swinton first heard articulate speech by radio when listening to tests with the Poulsen system. He was associated with the development of the Parsons turbine, and was a pioneer of motoring in England. In 1915 he was elected a Fellow of the Royal Society. He was unmarried. In his private life he was very hospitable. I remember once when he had tests done in the testing laboratories at Faraday House and had received the report which proved that his expectations were wrong, he wrote and thanked not only the Principal but also the assistant who had done the test. He was very interested in the application of a cathode-ray oscillograph to television and hoped that by its help a practical system might be obtained. This perhaps prejudiced him against the Baird system. But only last year, after seeing a practical demonstration of the Baird system working, he wrote to *The Times* withdrawing his strictures on its feasibility.

In 1924 he showed me a sciagram, which had been taken recently, of his own heart and the arteries connected with it. It proved that his life might be cut short at any minute and that he would have to take the greatest care of himself. He did not allow this knowledge to interfere with his work or to prevent his entertaining his friends as cheerfully as he had always done. The end came on February 19 of this year. Physical research has lost one who did much valuable work.

A. R.

WILLIAM R. BOWER

The death occurred on November 20, 1929, at his home at Langley Terrace, Oakes, Huddersfield, of Mr William Richard Bower, who was for over thirty years head of the Physics and Electrical Engineering Department of the Technical College, Huddersfield. Mr Bower was educated at Hartley College, Southampton, and the Royal College of Science, South Kensington, where for a time he acted as demonstrator in physics. After serving on the physics staff at University College, Aberystwyth, he became head of the physics department of the Brighton Technical College, whence he was appointed to Huddersfield. He served there until his retirement at the end of 1926.

Mr Bower was one of the pioneer workers on the application of X-rays to medical work, and installed the necessary apparatus at the Royal Infirmary, Huddersfield.

In addition to his work as a teacher, Mr Bower was well known as a writer on scientific subjects. He was joint author of a popular text-book, Bower and Satterley's *Practical Physics*, and author of *Primary Physical Science*. He contributed also a number of original papers, mainly on optical subjects, to the proceedings of scientific societies. He was a Fellow of the Physical Society (1892–1929) and often attended its meetings in London and the provinces—some of our fellows will remember seeing him at the Birmingham meeting last summer.

In addition to his many other activities Mr Bower was interested in the work of the Association of Teachers in Technical Institutions and for many years served on the national executive council, ultimately becoming president of that association.

He was widely known in his profession, and the announcement of his death will be received with regret by his old students and by a large circle of teachers of physics and Fellows of the Physical Society of London.

A. B. W.

JAMES WALKER

James Walker, born at Dover in 1857, was the only son of Thomas Walker and nephew of the Rev. R. Walker, Professor of Experimental Philosophy in the University of Oxford. Educated from 1872 to 1876 at Rugby, he matriculated at Oxford in 1876, having obtained a junior studentship in Mathematics at Christ Church. He was placed in the First Class in the Final Honour School of Mathematics and from 1883 to 1885 acted as a college lecturer in this subject. In 1883 he was also appointed a demonstrator in experimental philosophy at the Clarendon Laboratory, Oxford, under Professor Clifton, and for 36 years until his resignation in 1919 he was intimately connected with the laboratory and with the general development and teaching of physics in the University. He was a strict disciplinarian, most meticulous in the handling of delicate instruments and in the treatment and discussion of the results of experiments, but students could always rely on sympathetic help from him. Although his interests tended to the mathematical side of physics, as his published papers show, he would never allow difficulties of technique to be slurred over in the laboratory. Deviations and irregularities of observations had always to be explained and traced to their source.

His first published paper appeared in 1887 and was concerned with Cauchy's theory of reflection and refraction. From that year to 1914 papers by him appeared in the *Philosophical Magazine*, the *Proceedings of the Royal Society* and the *Proceedings of the Physical Society*. They were all connected with his favourite study of optics. He was a member of the Council of the Society, and eventually a Vice-President. His constant volumes of reference were the original memoirs of the great physicists from Fresnel to Stokes. His treatise on *The Analytical Theory of Light*, published by the Cambridge University Press in 1904, will always be a memorial to his vigour and power of exposition and logical deduction. In this book he endeavoured "to give an account of physical optics without recourse to any hypotheses respecting the nature of the influence that constitutes light or the character of the medium of propagation."

As a demonstrator in a physical laboratory, he was naturally interested in modern developments of physical theories, but by training and natural inclination his mind was attracted to the logical statement and derivation of the fundamental laws underlying the subject-matter of physics.

He married in 1879 and is survived by his widow, a son and two daughters. Generations of Oxford students will look back with grateful memories of hours spent in discussions with a delightful host and stimulating teacher, and to the generous hospitality which for so long emanated from his house in Oxford.

I. O. G.

REVIEWS OF BOOKS

The Mechanism of Nature, by E. N. Da C. Andrade, D.Sc., Ph.D. Pp. xii + 170. (London: G. Bell & Sons.)

It is not often that he who knows and he who can communicate his knowledge are subsumed under one identity, and the lay world is fortunate in that Professor Andrade possesses both these qualities, and is content to spend his holiday time in making the fundamentals of science as plain as may be to the plain man. He writes, and being Professor Andrade he writes clearly and fascinatingly, about heat and energy, sound and vibrations, light and radiation, electricity and magnetism, the quantum theory and the atom. Even though the book is quite untechnical, it touches many philosophical problems of fundamental importance, and if it raises more questions than it resolves, that is very much to the good.

The book is a model of clear exposition, of apt analogy and illustration; and it is with unalloyed pleasure that we record a complete absence of the heavy and cheap humour characteristic of some attempts to make the truths of modern science evident to the man in the street. But we wish that Professor Andrade would not "claim" where he should "assert."

<div align="right">A. F.</div>

Definitions and Formulae for Students (*Light and Sound*), compiled by P. K. Bowes, M.A., B.Sc. Pp. iv + 36. (London: Sir Isaac Pitman & Sons, Ltd.) 6d.

This book, like the lady's baby in *Midshipman Easy*, is "only a very little one," and its size is the main excuse for its existence. It certainly represents a marvel of compression and, judiciously and critically used, may serve to recall a number of leading facts and formulae, elementary and advanced, in sound and optics. But the pemmicanizing of knowledge is an adventure not to be undertaken light-heartedly—indeed it is easier to write a book on a single topic than to condense that topic into a single definition of unimpeachable correctness. Many of the definitions given are crisp and clear, some are mere expansions of the term defined, others are incorrect.

Fermat's law, for example, is better given as the law of the stationary rather than the minimum path, and the definitions of intensity and illuminating power ("Intensity, see illuminating-power." "Illuminating power—the ratio of light emitted by a source of light to that emitted by a standard source") may be left without comment. In his collection of formulae, the author gives no indication of the sign-convention followed.

<div align="right">A. F.</div>

Das ultrarote Spektrum by Cl. Schaefer and F. Matossi. (Berlin: Springer.) Price 28 RM, or bound 29.80 RM.

This work constitutes Volume **10** of the series of monographs on the structure of matter issued under the editorship of Profs. Born and Franck. The names of its authors, Prof. Clemens Schaefer and Dr Matossi, are so well known in the field of infra-red spectra that one might expect an authoritative treatise on the subject. Indeed the authors themselves announce in the preface that their aim is to produce a work which shall be comprehensive both on the theoretical and on the experimental side. The book undoubtedly fills a much felt want. The first section contains an admirable account of experimental

methods. The second and third deal mainly with standard work on black-body radiation, metallic reflection and cognate subjects, an account of which could not be excluded from a treatise of this character.

Readers will probably, however, be more directly interested in the fourth and fifth sections, which deal with the analysis of infra-red spectra in solids, liquids and gases. These sections are clearly set out and well illustrated, while the theoretical treatment is admirably suited to those whose natural approach to theory is from the experimental side. The rapid growth of such a subject is a source of embarrassment to any author, and readers may well be satisfied with the fact that the literature is covered up to the end of 1929. We can confidently recommend the work as a valuable contribution to the literature of modern physics.

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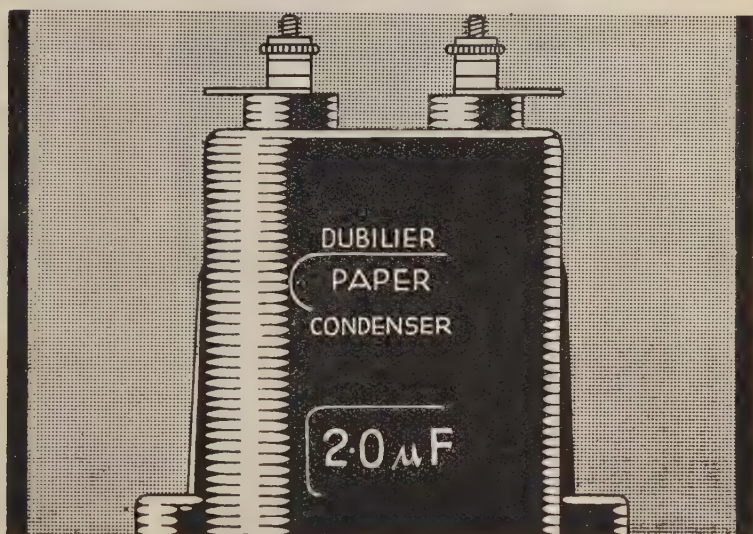
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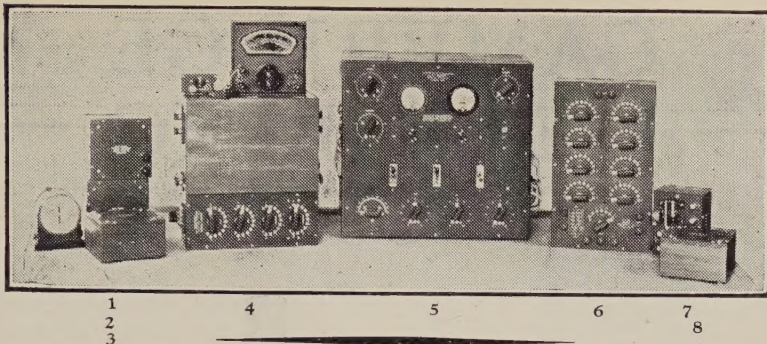
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